Total Chromatic Number of Middle and Total Graph of Path and Sunlet Graph

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Abstract: A total coloring of a graph $G$ is an assignment of colors to both the vertices and edges of $G$, such that no two adjacent or incident vertices and edges of $G$ are assigned with the same colors. In this paper, we discussed the theorems some total coloring of middle graph, total graph of path and sunlet graph. Also, we obtained the total chromatic number of middle graph, total graph of path and sunlet graph.

Keywords: Path; Sunlet graph; Middle graph; Total graph; Total coloring and Total chromatic number.

1. INTRODUCTION

In this paper, we have considered the finite, simple and undirected graphs. Let $G = (V(G), E(G))$ be a graph with the vertex set $V(G)$ and the edge set $E(G)$ respectively. In 1965, the concept of total coloring was introduced by Behzad [1] and in 1967, he [2] came out new ideology that, the total chromatic number of complete graph and complete bi-partite graph. A total coloring of $G$, is a function $f : S \rightarrow C$, where $S \rightarrow V(G) \cup E(G)$ and $C$ is a set of colors to satisfies the given conditions.

(i) no two adjacent vertices receive the same colors,
(ii) no two adjacent edges receive the same colors and
(iii) no edges and its end vertices receive the same colors.

The total chromatic number $\chi''(G)$ of a graph $G$ is the minimum cardinality $k$ such that $G$ may have a total coloring by $k$ colors. Behzad [1] and Vizing [13] conjectured that for every simple graph $G$, has $\Delta(G) + 1 \leq \chi''(G) \leq \Delta(G) + 2$, where $\Delta(G)$ is the maximum degree of $G$. This conjecture is called the Total Coloring Conjecture (TCC). Rosenfeld [9] and Vijayaditya [12] verified the TCC, for any graph $G$ with maximum degree $\leq 3$ and Kostochka [6] for maximum degree $\leq 5$. In Borodin[4] verified the total coloring conjecture (TCC) for maximum degree $\geq 9$ in planar graphs. In 1992, Yap and Chew [14] proved that any graph $G$ has a total coloring with at most $\Delta(G) + 2$ colors, if $\Delta(G) \geq |V(G)| - 5$. In recent era, total coloring have been extensively studied in different families of graphs. Mohan et.al [7] given the tight bound of Behzad and Vizing conjecture for Corona product of certain classes of graph. Muthuramakrishnan et.al [8] proved that Total Chromatic Number of Line, Middle and Total graph of star and square graph of Bistar graph. Vaidya et.al [11] prove that the total chromatic number of middle graph, total graph, shadow graph of cycle and one point union of cycle. Sudha et.al [10] prove that the lower and upper bound for the total chromatic number of $S(n, m)$ graph. In this paper, we determine the total chromatic number of $M(P_n), T(P_n), M(S_n)$ and $T(S_n)$.

2. PRELIMINARIES

Definition 2.1. A Path in a graph is a sequence of vertices such that from each of its vertices there is an edge to the next vertex in the sequence.
Definition 2.2. The middle graph [5] of a graph \( G \) denoted by \( M(G) \) is defined as follows, the vertex set of \( M(G) \) is \( V(G) \cup E(G) \). Two vertices \( x,y \) in the vertex set of \( M(G) \) are adjacent in \( M(G) \) in case one of the following condition holds: (i) \( x,y \) are in \( E(G) \) and \( x,y \) is adjacent in \( G \) (ii) \( x \) is in \( V(G) \) and \( x,y \) is in \( E(G) \) and \( x,y \) are incident in \( G \).

Definition 2.3. The total graph [3] of a graph \( G \), denoted by \( T(G) \) is defined as, the vertex set of \( T(G) \) is \( V(G) \cup E(G) \). Two vertices \( x,y \) in the vertex set of \( T(G) \) are adjacent in \( T(G) \) if one of the following condition holds: (i) \( x,y \) are in \( V(G) \) and \( x \) is adjacent to \( y \) in \( G \). (ii) \( x,y \) are in \( E(G) \) and \( x,y \) is adjacent in \( G \). (iii) \( x \) is in \( V(G) \), \( y \) is in \( E(G) \) and \( x,y \) are incident in \( G \).

Definition 2.4. The \( n \)-sunlet graph on \( 2n \) vertices is obtained by attaching \( n \) pendent edges to the cycle \( C_n \) and is denoted by \( S_n \).

3. Total Coloring of Middle Graph of Path

Theorem 3.1. Let \( M(P_n) \) be the middle graph of path of order \( n \). Then \( \chi''(M(P_n)) = 5, \ n \geq 4 \).

Proof: Let \( V(P_n) = \{v_i: 1 \leq i \leq n\} \) and \( E(P_n) = \{e_i = v_i v_{i+1}: 1 \leq i \leq n-1\} \). By the definition of middle graph, each edge \( \{e_i = v_i v_{i+1}: 1 \leq i \leq n-1\} \) in \( P_n \) is subdivided by the vertices \( \{u_i: 1 \leq i \leq n-1\} \) in \( M(P_n) \). The vertex set and the edge set \( M(P_n) \) is given by \( V(M(P_n)) = \{v_i: 1 \leq i \leq n\} \cup \{u_i: 1 \leq i \leq n-1\} \), where \( \{u_i: 1 \leq i \leq n-1\} \) is the vertices of \( M(P_n) \) corresponding to the edge \( \{v_i v_{i+1}: 1 \leq i \leq n-1\} \) of \( P_n \) and \( E(M(P_n)) = \{v_i u_i: 1 \leq i \leq n\} \cup \{u_i u_{i+1}: 1 \leq i \leq n-1\} \cup \{u_i v_{i+1}: 1 \leq i \leq n-1\} \).

Now we define the total coloring \( f \), such that \( f:S \rightarrow C \) as follows, where \( S = V(M(P_n)) \cup E(M(P_n)) \) and \( C = \{1,2,3,4,5\} \).

\[
\begin{align*}
f(u_i) &= \begin{cases}
3, & \text{if } i \equiv 0 \pmod{3} \\
1, & \text{if } i \equiv 1 \pmod{3} \text{ for } 1 \leq i \leq n-1 \\
2, & \text{if } i \equiv 2 \pmod{3}
\end{cases} \\
f(v_i) &= \begin{cases}
1, & \text{if } i \equiv 1 \pmod{3} \\
2, & \text{if } i \equiv 2 \pmod{3} \text{ for } 1 \leq i \leq n \\
3, & \text{if } i \equiv 0 \pmod{3}
\end{cases} \\
f(u_i u_{i+1}) &= \begin{cases}
4, & \text{if } i \text{ is odd for } 1 \leq i \leq n-1 \\
5, & \text{if } i \text{ is even}
\end{cases} \\
f(v_i u_i) &= \begin{cases}
2, & \text{if } i \equiv 1 \pmod{3} \text{ for } 1 \leq i \leq n-1 \\
3, & \text{if } i \equiv 2 \pmod{3} \\
1, & \text{if } i \equiv 0 \pmod{3}
\end{cases} \\
f(u_i v_{i+1}) &= \begin{cases}
1, & \text{if } i \equiv 1 \pmod{3} \\
2, & \text{if } i \equiv 2 \pmod{3} \text{ for } 1 \leq i \leq n-1 \\
3, & \text{if } i \equiv 0 \pmod{3}
\end{cases}
\]

It is clear that the above rule of total coloring, the graph \( M(P_n) \) is properly total colored with 5 colors. Hence the total chromatic number of the middle graph of \( M(P_n) \), \( \chi''(M(P_n)) = 5 \).

Illustration 3.2: Consider the middle graph of a path \( P_6 \)

![Middle graph of path \( P_6 \)](image)
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By applying the above rule of total coloring pattern as given in the theorem 3.1 the colors \{1, 2, 3, 4, 5\} to the vertices and edges are received as shown in Fig 1. Thus the total chromatic number of middle graph of path is 5.

4. TOTAL COLORING OF TOTAL GRAPH OF PATH

Theorem 4.1. Let \(T(P_n)\) be the total graph of path of order \(n\). Then \(\chi''(T(P_n)) = 5, \quad n \geq 3\).

Proof: Let \(V(P_n) = \{v_i: 1 \leq i \leq n\}\) and \(E(P_n) = \{e_i = v_i v_{i+1}: 1 \leq i \leq n - 1\}\). By the definition of total graph, each edge \(e_i = v_i v_{i+1}: 1 \leq i \leq n - 1\) in \(P_n\) is subdivided by the vertices \(\{u_i: 1 \leq i \leq n - 1\}\) in \(T(P_n)\). The vertex set and the edge set of \(T(P_n)\) is given by \(V(T(P_n)) = \{v_i: 1 \leq i \leq n\} \cup \{u_i: 1 \leq i \leq n - 1\}\), where \(\{u_i: 1 \leq i \leq n - 1\}\) is the vertices of \(T(P_n)\) corresponding to the edge \(\{v_i, v_{i+1}: 1 \leq i \leq n - 1\}\) of \(P_n\) and

\[
E(T(P_n)) = \{\{v_i, v_{i+1}: 1 \leq i \leq n - 1\} \cup \{u_i, u_{i+1}: 1 \leq i \leq n - 1\}\}
\]

Now we define total coloring \(f\), such that \(f: S \rightarrow C\) as follows, where \(S = V(T(P_n)) \cup E(T(P_n))\) and \(C = \{1, 2, 3, 4, 5\}\)

\[
f(v_i) = \begin{cases} 
1, & \text{if } i \equiv 1 \pmod{3} \\
2, & \text{if } i \equiv 2 \pmod{3} \text{ for } 1 \leq i \leq n \\
3, & \text{if } i \equiv 0 \pmod{3}
\end{cases}
\]

\[
f(u_i) = \begin{cases} 
3, & \text{if } i \equiv 1 \pmod{3} \\
1, & \text{if } i \equiv 2 \pmod{3} \text{ for } 1 \leq i \leq n - 1 \\
2, & \text{if } i \equiv 0 \pmod{3}
\end{cases}
\]

\[
f(u_i u_{i+1}) = \begin{cases} 
4, & \text{if } i \text{ is odd for } 1 \leq i \leq n - 1 \\
5, & \text{if } i \text{ is even}
\end{cases}
\]

\[
f(v_i v_{i+1}) = \begin{cases} 
4, & \text{if } i \text{ is odd for } 1 \leq i \leq n - 1 \\
5, & \text{if } i \text{ is even}
\end{cases}
\]

\[
f(v_i u_i) = \begin{cases} 
2, & \text{if } i \equiv 1 \pmod{3} \\
3, & \text{if } i \equiv 2 \pmod{3} \text{ for } 1 \leq i \leq n - 1 \\
1, & \text{if } i \equiv 0 \pmod{3}
\end{cases}
\]

\[
f(u_i v_{i+1}) = \begin{cases} 
1, & \text{if } i \equiv 1 \pmod{3} \\
2, & \text{if } i \equiv 2 \pmod{3} \text{ for } 1 \leq i \leq n - 1 \\
3, & \text{if } i \equiv 0 \pmod{3}
\end{cases}
\]

It is clear that the above rule of total coloring, the graph \(T(P_n)\) is properly total colored with 5 colors. Hence the total chromatic number of the total graph of path graph \(T(P_n)\), \(\chi''(T(P_n)) = 5\).

Illustration 4.2: Consider the total graph of a path \(P_6\)

![Figure 2. Total graph of path \(P_6\)](image)

By applying the above rule of total coloring pattern as given in the theorem 4.1, the colors \{1, 2, 3, 4, 5\} to the vertices and edges are received as shown in Fig 2. Thus the total chromatic number of total graph of path is 5.

5. TOTAL COLORING OF MIDDLE GRAPH OF SUNLET GRAPH

Theorem 5.1. Let \(M(S_n)\) be the middle graph of sun let graph \(S_n\). Then
The total coloring is obtained by coloring these vertices and edges as follows:

**Case (i):** When \( n \) is even

\[
\begin{align*}
    f(u_i) &= 7; & \quad f(u'_i) &= 6 \quad \text{for } 1 \leq i \leq n \\
    f(v_i) &= \begin{cases} 
1, & \text{if } i \equiv 1 \pmod{2} \\
3, & \text{if } i \equiv 0 \pmod{2}
\end{cases} \quad \text{for } 1 \leq i \leq n \\
    f(v'_i) &= \begin{cases} 
2, & \text{if } i \equiv 1 \pmod{2} \\
4, & \text{if } i \equiv 0 \pmod{2}
\end{cases} \quad \text{for } 1 \leq i \leq n \\
    f(v_i v'_i) &= \begin{cases} 
4, & \text{if } i \equiv 1 \pmod{2} \\
2, & \text{if } i \equiv 0 \pmod{2}
\end{cases} \quad \text{for } 1 \leq i \leq n \\
    f(v'_i v_{i+1}) &= \begin{cases} 
1, & \text{if } i \equiv 1 \pmod{2} \\
3, & \text{if } i \equiv 0 \pmod{2}
\end{cases} \quad \text{for } 1 \leq i \leq n - 1 \\
    f(v_{i+1} v'_i) &= 3; \\
    f(u'_i u'_j) &= \begin{cases} 
3, & \text{if } i \equiv 1 \pmod{2} \\
1, & \text{if } i \equiv 0 \pmod{2}
\end{cases} \quad \text{for } 1 \leq i \leq n \\
    f(u'_i u'_{i+1}) &= 7; & \quad f(v'_i u'_i) &= 7; \quad \text{for } 1 \leq i \leq n - 1 \\
    f(v'_i v'_{i+1}) &= \begin{cases} 
5, & \text{if } i \equiv 1 \pmod{2} \\
6, & \text{if } i \equiv 0 \pmod{2}
\end{cases} \quad \text{for } 1 \leq i \leq n - 1 \\
    f(v_{i+1} u'_i) &= \begin{cases} 
2, & \text{if } i \equiv 1 \pmod{2} \\
4, & \text{if } i \equiv 0 \pmod{2}
\end{cases} \quad \text{for } 1 \leq i \leq n \\
    f(v'_i v'_{i+1}) &= 6; & \quad f(u_i u'_i) &= 5 \quad \text{for } 1 \leq i \leq n
\end{align*}
\]

It is clear that the above rule of total coloring, the graph \( M(S_n) \) is properly total colored with 7 colors. Hence the total chromatic number of the middle graph of sunlet graph \( S_n \), \( \chi''(M(S_n)) = 7 \) for \( n \) is even.

**Case (ii):** When \( n \) is odd

\[
\begin{align*}
    f(v_i) &= 7; & \quad f(u'_i) &= 4; & \quad f(u_i) &= 8 \quad \text{for } 1 \leq i \leq n \\
    f(v'_i) &= \begin{cases} 
1, & \text{if } i \equiv 1 \pmod{2} \\
2, & \text{if } i \equiv 0 \pmod{2}
\end{cases} \quad \text{for } 1 \leq i \leq n - 1 \\
    f(v'_i) &= 3 \\
    f(v'_i v'_{i+1}) &= \begin{cases} 
6, & \text{if } i \equiv 1 \pmod{2} \\
7, & \text{if } i \equiv 1 \pmod{2}
\end{cases} \quad \text{for } 1 \leq i \leq n - 1
\end{align*}
\]
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\[ f(v_i v_i') = 8; \quad f(v_i v_1') = 4 \quad \text{for} \quad 1 \leq i \leq n \]
\[ f(v_i v_{i+1}) = 5; \quad f(v_n v_1) = 5 \quad \text{for} \quad 1 \leq i \leq n - 1 \]
\[ f(u_i u_i') = \begin{cases} 2, & \text{if} \quad i \equiv 1 \pmod{2} \\ 1, & \text{if} \quad i \equiv 1 \pmod{2} \end{cases} \quad \text{for} \quad 1 \leq i \leq n \]
\[ f(v_i u_{i+1}) = 3 \quad \text{for} \quad 1 \leq i \leq n - 1 \]
\[ f(u_i u_i') = 5; \quad f(v_i u_i') = 8 \quad \text{for} \quad 1 \leq i \leq n, f(v_n v_1') = 1; \]

It is clear that the above rule of total coloring, the graph \( M(S_n) \) is properly total colored with 8 colors. Hence the total chromatic number of the middle graph of sunlet graph \( M(S_n) \), \( \chi''(M(S_n)) = 8 \), for \( n \) is odd.

**Illustration 5.2:** Consider the middle graph of a sunlet graph \( S_6 \)

![Figure 3. Middle graph of a sun let graph \( S_6 \)](image)

By applying the above rule of total coloring pattern as given in the theorem 5.1, of case (i) the colors \( \{1, 2, 3, 4, 5, 6, 7\} \) to these vertices and edges are received as shown in Fig 3. Thus the total chromatic number of middle graph of sunlet graph is 7 for \( n \) is even.

**Illustration 5.3:** Consider the middle graph of a sun let graph \( S_5 \)

![Figure 4. Middle graph of a sun let graph \( S_5 \)](image)
6. TOTAL COLORING OF TOTAL GRAPH OF SUNLET GRAPH

Theorem 6.1. Let \( T(S_n) \) be the total graph of sunlet graph \( S_n \). Then

\[
\chi''(T(S_n)) = \begin{cases} 7, & \text{if } n \text{ is even} \\ 8, & \text{if } n \text{ is odd} \end{cases}
\]

Proof: Let \( \{u_1, u_2, u_3, \ldots, u_n\} \) and \( \{v_1, v_2, v_3, \ldots, v_n\} \) be the vertices of the sunlet graph and let \( \{e_1, e_2, e_3, \ldots, e_n\} \) and \( \{e_1', e_2', e_3', \ldots, e_n'\} \) be the edges of the sunlet graph, where \( e_i = v_iv_{i+1} : 1 \leq i \leq n - 1 \) be the edges, \( e_n \) is the edge \( v_nv_1 \) and \( e_i' \) is the edges \( v_iu_i \). By the definition of total graph, each edges \( e_i = v_iv_{i+1} : 1 \leq i \leq n - 1 \) and \( v_nv_1 \) in \( T(S_n) \) are subdivided by the vertices \( \{v_i' : 1 \leq i \leq n\} \) and \( \{u_i' : 1 \leq i \leq n\} \) in \( T(S_n) \). Therefore, the vertex set and the edge set is given by

\[
V(T(S_n)) = \{v_1, v_2, \ldots, v_n\} \cup \{u_1, u_2, \ldots, u_n\} \cup \{v_i' : 1 \leq i \leq n\} \cup \{u_i' : 1 \leq i \leq n\},
\]

where \( u_i' \) and \( v_i' \) represents the edges \( e_i \) and \( e_i' \) respectively.

\[
E(T(S_n)) = \begin{cases} (v_i', v_i': 1 \leq i \leq n) & \cup (u_i', u_i': 1 \leq i \leq n) \cup (v_i', v_{i+1} : 1 \leq i \leq n - 1) \cup (v_i', u_i' : 1 \leq i \leq n) \cup (v_i', u_i') \cup (v_i', v_i') \cup (u_i', u_i': 1 \leq i \leq n) \cup (v_i', v_{i+1} : 1 \leq i \leq n - 1) \cup (u_i', u_i' : 1 \leq i \leq n) \end{cases}
\]

We define total coloring \( f \), such that \( f : S \to C \) as follows, where \( S = V(T(S_n)) \cup E(T(S_n)) \) and \( C = \{1, 2, 3, 4, 5, 6, 7, 8\} \).

Case (i): When \( n \) is even

\[
f(u_i) = \begin{cases} 1, & \text{if } i \equiv 1 \pmod{2} \\ 3, & \text{if } i \equiv 0 \pmod{2} \end{cases} \text{ for } 1 \leq i \leq n
\]

\[
f(v_i) = \begin{cases} 2, & \text{if } i \equiv 1 \pmod{2} \\ 4, & \text{if } i \equiv 0 \pmod{2} \end{cases} \text{ for } 1 \leq i \leq n
\]

\[
f(v_i') = \begin{cases} 3, & \text{if } i \equiv 1 \pmod{2} \\ 1, & \text{if } i \equiv 0 \pmod{2} \end{cases} \text{ for } 1 \leq i \leq n
\]

\[
f(v_i', v_{i+1}) = \begin{cases} 4, & \text{if } i \equiv 1 \pmod{2} \\ 2, & \text{if } i \equiv 0 \pmod{2} \end{cases} \text{ for } 1 \leq i \leq n - 1
\]

\[
f(v_i', v_i') = \begin{cases} 5, & \text{if } i \equiv 1 \pmod{2} \\ 6, & \text{if } i \equiv 0 \pmod{2} \end{cases} \text{ for } 1 \leq i \leq n - 1
\]

\[
f(v_i', u_i') = \begin{cases} 2, & \text{if } i \equiv 1 \pmod{2} \\ 4, & \text{if } i \equiv 0 \pmod{2} \end{cases} \text{ for } 1 \leq i \leq n
\]

\[
f(v_i', u_i') = \begin{cases} 6, & \text{if } i \equiv 1 \pmod{2} \\ 5, & \text{if } i \equiv 0 \pmod{2} \end{cases} \text{ for } 1 \leq i \leq n
\]
It is clear that the above rule of total coloring, the graph $T(S_n)$ is properly total colored with 7 colors. Hence the total chromatic number of the total graph of sunlet graph $T(S_n)$, $\chi''(T(S_n)) = 7$, for $n$ is even.

Case (ii): When $n$ is odd

$$f(v_i) = 7; \quad f(u'_i) = 4; \quad f(u_i) = 8 \text{ for } 1 \leq i \leq n$$

$$f(v'_i) = \begin{cases} 1, & \text{if } i \equiv 1 \pmod{2} \\ 2, & \text{if } i \equiv 0 \pmod{2} \end{cases} \text{ for } 1 \leq i \leq n - 1$$

$$f(v'_n) = 3; \quad f(u_i v_i) = 6 \text{ for } 1 \leq i \leq n$$

$$f(v'_i v'_i) = \begin{cases} 6, & \text{if } i \equiv 1 \pmod{2} \\ 7, & \text{if } i \equiv 0 \pmod{2} \end{cases} \text{ for } 1 \leq i \leq n - 1$$

$$f(v'_n v'_i) = 8; \quad f(v_i v'_i) = 4 \text{ for } 1 \leq i \leq n$$

$$f(v'_i v'_i+1) = 5 \text{ for } 1 \leq i \leq n - 1; \quad f(v'_n v'_1) = 5$$

$$f(u'_i v'_i) = \begin{cases} 2, & \text{if } i \equiv 1 \pmod{2} \\ 1, & \text{if } i \equiv 0 \pmod{2} \end{cases} \text{ for } 1 \leq i \leq n$$

$$f(u'_i u'_i) = 3 \text{ for } 1 \leq i \leq n - 1$$

$$f(u_i u'_i) = 5; \quad f(u'_n u'_1) = 1; \quad f(v_i u'_i) = 8 \text{ for } 1 \leq i \leq n - 1$$

It is clear that the above rule of total coloring, the graph $T(S_n)$ is properly total colored with 8 colors. Hence the total chromatic number of the total graph of sunlet graph $T(S_n)$, $\chi''(T(S_n)) = 8$, for $n$ is odd.

**Illustration 6.2:** Consider the total graph of a sunlet graph $T(S_5)$

By applying the above method of total coloring pattern as given in the theorem 6.1 of case (i) the colors $\{1, 2, 3, 4, 5, 6, 7\}$ to these vertices and edges are received as shown in Fig 5. Thus the total chromatic number of total graph of sunlet graph is 7 for $n$ is odd.

**Illustration 6.3:** Consider the total graph of a sunlet graph $T(S_5)$

![Figure 5. Total graph of a sunlet graph $T(S_5)$](image-url)
By applying the above method of total coloring pattern as given in the theorem 6.1, of case (ii) the colors\{1, 2, 3, 4, 5, 6, 7, 8\} to these vertices and edges are received as shown in Fig 6. Thus the total chromatic number of total graph of sunlet graph is 8 for n is odd.

7. CONCLUSION

The total coloring of the middle and total graph of path and sunlet graph are discussed in this paper and found the total chromatic numbers to be

1. \(\chi''(M(P_n)) = 5, n \geq 4.\)
2. \(\chi''(T(P_n)) = 5, n \geq 3.\)
3. \(\chi''(M(S_n)) = \begin{cases} 7, & n \text{ is even} \\ 8, & n \text{ is odd} \end{cases} \)
4. \(\chi''(T(S_n)) = \begin{cases} 7, & n \text{ is even} \\ 8, & n \text{ is odd} \end{cases} \)

REFERENCES

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Citation: D. Muthuramakrishnan, G. Jayaraman, “Total Chromatic Number of Middle and Total Graph of Path and Sunlet Graph”, International Journal of Scientific and Innovative Mathematical Research, vol. 6, no. 4, p. 1-9, 2018., http://dx.doi.org/10.20431/2347-3142.0604001

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