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Abstract: We analyzed the combined influence of dissipation, variable viscosity effect, thermo-diffusion and Hall current on the hydromagnetic nonlinear convective heat and mass transfer flow past a stretching surface. The governing equations have been solved by employing fifth-order Runge-Kutta-Fehlberg method along with shooting technique. The effects of various parameters on the velocity, temperature and concentration as well as on the local skin-friction coefficient, local Nusselt number and local Sherwood number are presented graphically and discussed.

Keywords: Nonlinear Thermal Radiation, Stretching Sheet, Chemical Reaction, Thermo Diffusion and Dissipation

1. INTRODUCTION

With the fuel crisis deepening all over the world, there is a great concern to utilize the enormous power beneath the earth’s crust in the geothermal region. Liquid in the geothermal region is an electrically conducting liquid because of high temperature. Hence the study of interaction of the geomagnetic field with the fluid in the geothermal region is of great interest, thus leading to interest in the study of MHD convection flows through porous medium. The influence of a uniform transverse magnetic field on the motion of an electrically conducting fluid past a stretching sheet was investigated by Pavlov [57]. The effect of chemical reaction on free-convective flow and mass transfer of a viscous, incompressible and electrically conducting fluid over a stretching sheet was investigated by Afify [5] in the presence of transverse magnetic field. Das et al. [23] investigated MHD Boundary layer slip flow and heat transfer of nanofluid past a vertical stretching sheet with non-uniform heat generation/absorption. Ibrahim and Makinde [33] studied the Double-diffusive mixed convection and MHD Stagnation point flow of nanofluid over a stretching sheet. Rudraswamy et al. [64] considered the effects of magnetic field and chemical reaction on stagnation-point flow and heat transfer of a nanofluid over an inclined stretching sheet. Bharathi et al [12] have discussed Non Darcy Hydromagnetic Mixed convective Heat and Mass Transfer flow of a viscous fluid in a vertical channel with Heat generating sources. Non-Darcy Hydromagnetic convective Heat and Mass transfer through a porous medium in a cylindrical annulus with Soret effect, radiation and dissipation. Balasubramanyam et al [13] have discussed Non Darcy viscous electrically conductive Heat and Mass transfer flow through a porous medium in a vertical channel in the presence of heat generating sources. The combined effect of thermal and mass diffusion in channel flows has been studied in the recent times by a few authors Das et al[24], Gnaneswara Reddy et al[26], Nelson and Wood [52], Raptis [61,62].

Hall currents are important and they have a marked effect on the magnitude and direction of the current density and consequently on the magnetic force term. The problem of MHD free convection flow with Hall currents has many important engineering applications such as in power generators, MHD accelerators, refrigeration coils, transmission lines, electric transformers, heating elements etc., Watanabe and Pop [79], Abo-Eldahab and Salem [3], Rana et al. [60], Singh and Gorla [70], Sht [69] among others have advanced studies on Hall effect on MHD past stretching sheet. Shateyi et al.

Most of the previous studies are concerning the constant thermo-physical properties of the fluid. It is well known that the fluid property most sensitive to temperature rise is viscosity. For many liquids, among them water, petroleum oils, glycerin, glycols, silicone fluids and some molten salts, the viscosity shows a rather pronounced variation with temperature. To accurately predict the flow and heat transfer rates, it is necessary to take into account the temperature-dependent viscosity of the fluid. For example, when the temperature increases from 10 °C (μ=0.00131g/cm) to 50 °C (μ=0.00548g/cm), the viscosity of the water decreases by 240%. To predict the heat transfer rate accurately, it is necessary to take the variation of viscosity with temperature into consideration. The variation of viscosity in thermal boundary layer is large. There exist several applications of this problem, for example, in the processes of hot rolling, wire drawing, glass fiber production, paper production, gluing of labels on hot bodies, drawing of plastic films and the study of spilling pollutant crude oils over the surface of seawater. Many researchers have studied the flows with temperature dependent viscosity in different geometries and under various flow conditions (El-Aziz [11], Rahman and Eitayab [65]). The effect of temperature-dependent viscosity on heat and mass transfer laminar boundary layer flow has been discussed by many authors (Mukhopadhyay et al. [50], Mukhopadhyay and Layek [51], Ali [9], Makinde [40], Prasad et al. [59], Alam et al. [7]) in various situations. They showed that when this effect was included, the flow characteristics might change substantially compared with the constant viscosity assumption. Salem [66] investigated variable viscosity and thermal conductivity effects on MHD flow and heat transfer in viscoelastic fluid over a stretching sheet. Anjali Devi and Ganga [10] have considered the viscous dissipation effects on MHD flows past stretching porous surfaces in porous media. Mohamed El-Aziz [45] analyzed unsteady mixed convection heat transfer along a vertical stretching surface with variable viscosity and viscous dissipation. Xi-Yan Tian et al. [80] investigated the 2D boundary layer flow and heat transfer in variable viscosity MHD flow over a stretching plate.

If the temperature of the surrounding fluid becomes high then the thermal radiation effect play a vital role in the case of space technology. Steady heat flow on a moving continuous flat surface was first considered by Sakiadis [67] who developed a numerical solution using a similarity transformation. Crane [15] motivated by the processes of polymer extrusion, in which the extrudate emerges from a narrow slit, examined semi-infinite fluid flow driven by a linearly stretching surface. Many researchers and academicians have advanced their studies relating to problems involving MHD in stretching sheet considering various parameters. (Sharidan [68], Gupta and Gupta [27], Ishak et al. [34-36], Aziz et al. [11], Abel et al. [2], Mukhopadhyay and Mondal [48], Krishnendu [31]), Swati Mukhopadhyay et al. [49], Prasad et al. [58], Wang [78] among others). Rashad et al [63] considered MHD free convective heat and mass transfer of a chemically-reacting fluid from radiate stretching surface embedded in a saturated porous medium. Theuri et al [22] studied unsteady double diffusive magneto hydrodynamic boundary layer flow of a chemically reacting fluid over a flat permeable surface. Vajravelu and Hadjinicolaou [77] studied the heat transfer characteristics in the laminar boundary layer of a viscous fluid over a stretching sheet with viscous dissipation or frictional heating and internal heat generation. Hosain et al [29] studied the problem of natural convection flow along a vertical wavy surface with uniform surface temperature in the presence of heat generation or absorption. Alam et al [8] studied the problem of free convection heat and mass transfer flow past an inclined semi-infinite heated surface of an electrically conducting and steady viscous incompressible fluid in the presence of a magnetic field and heat generation. Chamkha [19] investigated unsteady convective heat and mass transfer past a semi-infinite porous moving plate with heat absorption.

Some recent interesting contributions pertaining to heat transfer aspects of viscous dissipation and Joule heating are cited in Aziz et al. [11], Nandeppanavar et al. [1], Pal et al. [55], Gebhart and Mollendorf [25] have shown that viscous dissipation heat in the natural convective flow is important when the fluid is of extreme size or is at extremely low temperature or in high gravitational field. On the other hand, Zanchini [81] pointed out that relevant effects of viscous dissipation on the temperature profiles and the Nusselt number may occur in the fully developed convection in tubes. In view of this, several authors, notably, Sreevani [76] have studied the effect of viscous dissipation on the convective flows past an infinite vertical plate and through vertical channels and ducts.
The thermal-diffusion effect, for instance, has been utilized for isotope separation and in mixture between gases with very light molecular weight (Hydrogen-Helium) and of medium molecular weight (Nitrogen-air) the diffusion-thermo effect was found to be of a magnitude such that it cannot be neglected Kafoussias and Williams [32]. Alam et al. [6] studied Dufour and Soret effects on steady free convection and mass transfer flow past a semi-infinite vertical porous plate in a porous medium. Makinde [42] studied numerically the influence of a magnetic field on heat and mass transfer by mixed convection from vertical surfaces in porous media considering Soret and Dufour effects. Rashidi et al [63] analyzed heat and mass transfer for MHD viscoelastic fluid flow over a vertical stretching sheet considering Soret and Dufour effects. Sreedevi et al [75] have discussed the effect of radiation absorption and variable viscosity on hydromagnetic convective heat and mass transfer flow past a stretching sheet.

In this paper, we investigate the effect of hall currents, dissipation, chemical reaction, on nonlinear convective heat and mass transfer flow past a stretching sheet. The governing equations have been solved by Runge-Kutta-Fehlberg method along with shooting technique. The effect of various governing parameter in the problem on all the flow characteristics have been represented graphically.

2. FORMULATION OF THE PROBLEM

We consider the steady free-convective flow, heat and mass transfer of an incompressible, viscous and electrically conducting fluid past a stretching sheet and the sheet is stretched with a velocity proportional to the distance from a fixed origin O (Fig. 1).

A uniform strong magnetic field of strength $B_0$ is imposed along the $y$-axis and the effect of Hall current is taken into account. Taking Hall effects into account the generalized Ohm’s law provided in the following form

$$
\bar{J} = \frac{\sigma}{1 + m^*}(\bar{E} + \bar{V} \times \bar{B} - \frac{1}{en_r} \bar{f} \times \bar{B})
$$

(1)

Where $m^* = \frac{cB_e}{en_r}$ is defined as the Hall current parameter. A very interesting fact that the effect of Hall current gives rise to a force in the $z$-direction which in turn produces a cross-flow velocity in this direction and then the flow becomes three-dimensional. The temperature and the species concentration are maintained at prescribed constant values $T_w$, $C_w$ at the sheet and $T_\infty$ and $C_\infty$ are the fixed values far away from the sheet.

The fluid viscosity $\mu$ is assumed to vary as a reciprocal of a linear function of temperature given by

$$
\frac{1}{\mu} = \frac{1}{\mu_0}[1 + \gamma_0(T - T_e)]
$$

(2)

$$
\frac{1}{\mu_e} = a(T - T_e)
$$

(3)

Where $a = \frac{\gamma_e}{\mu_e}$ and $T_e = T_\infty - \frac{1}{V_0}$

In the above equation both $a$ and $T_e$ are constants, and their values depend on the thermal property of the fluid, i.e., $g_0$. In general $a > 0$ represent for liquids, whereas for gases $a < 0$. 

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Owing to the above mentioned assumptions, the boundary layer free-convection flow with mass transfer and generalized Ohm’s law with Hall current effect are governed by the following system of equations:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]

\[
\rho \left( \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = \frac{\partial}{\partial y} \left( \mu \frac{\partial u}{\partial y} \right) + \rho \alpha \beta \left(T - T_w\right)
\]

\[
+ \rho \alpha \beta \left( C - C_w \right) \frac{\partial b}{\partial y} \left( u + m w \right)
\]

\[
\rho \left( \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} \right) = \frac{\partial}{\partial y} \left( \mu \frac{\partial w}{\partial y} \right) + \rho \beta \gamma \left( u + m w \right)
\]

\[
\rho C_p \left( \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k \left( \frac{\partial^2 T}{\partial y^2} + \mu (u^2 + w^2) - \frac{\partial (q_r)}{\partial y} \right)
\]

\[
u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_m \frac{\partial^2 C}{\partial y^2} - k_0 \left(C - C_w\right) + \frac{D_m K_T}{T_w} \frac{\partial^2 T}{\partial y^2}
\]

where \((u,v,w)\) are the velocity components along the \((x,y,z)\) directions respectively. \(D_m\) is the solution diffusivity of the medium, \(K_T\) is the thermal diffusion ratio, \(C_s\) is the concentration susceptibility, \(C_p\) is the specific heat at constant pressure, \(T_w\) is the mean fluid temperature., \(q_r\) is the radiative heat flux.

The boundary conditions for the present problem can be written as

\[
u = bx + L \frac{\partial u}{\partial y}, v = w = 0, T = T_w, C = C_w \quad \text{at} \quad y = 0
\]

\[
u \to 0, w \to 0, T \to T_w, C \to C_w \quad \text{at} \quad y \to \infty
\]

where \(b > 0\) being stretching rate of the sheet. The boundary conditions on velocity in Eq. (9) are the no-slip condition at the surface \(y = 0\), while the boundary conditions on velocity at \(y \to \infty\) follow from the fact that there is no flow far away from the stretching surface.

We consider the solution of equation(1) as:

\[
w = -w_0
\]

The radiation heat term (Brewester(47)) by using The Rosseland approximation is given by

\[
q_r = -\frac{4\sigma T^4}{\beta_w \gamma}
\]

\[
T^4 \equiv 4TT_w^3 - 3T_w^4
\]

\[
\frac{\partial q_r}{\partial z} = \frac{16\sigma T_w^3 \frac{\partial T}{\partial z}}{3\beta_w \gamma}
\]

The non-dimensional temperature \(\theta(\eta) = \frac{T - T_w}{T_w - T_c}\) can be simplified as

\[
T = T_w(1 + (\theta_w - 1)\theta)
\]

Where \(\theta = \frac{T_w}{T_c}\) is the temperature parameter.

To examine the flow regime adjacent to the sheet, the following transformations are invoked

\[
u = bxf'(\eta); v = -\sqrt{b} y f(\eta); w = bxg(\eta);
\]

\[
\eta = \frac{b}{V} y; \theta(\eta) = \frac{T - T_w}{T_w - T_c}; \phi = \frac{C - C_w}{C_w - C_c}
\]
where $f$ is a dimensionless stream function, $h$ is the similarity space variable, $\Theta$ and $\phi$ are the dimensionless temperature and concentration respectively. Clearly, the continuity Eq. (4) is satisfied by $u$ and $v$ defined in Eq. (15), Substituting Eq. (15) the Eqs. (5)-(8) reduce to

\[ \frac{\partial}{\partial \xi} (f'' - f) + \frac{1}{m^2} \left( \frac{\partial}{\partial \xi} f' \right) = 0 \]

\[ \frac{\partial}{\partial \xi} \left( f^2 - g^2 \right) = \frac{\partial}{\partial \xi} \left( \frac{\partial}{\partial \xi} \right) = 0 \]

\[ \frac{\partial}{\partial \xi} \left( f^2 - g^2 \right) = \frac{\partial}{\partial \xi} \left( \frac{\partial}{\partial \xi} \right) = 0 \]

\[ \phi' - Sc (f \phi' - \gamma \phi) = -ScSr \phi' \]

Similarly, the transformed boundary conditions are given by

\[ f(0) = 1 + A11f''(0), f(0) = 0, g(0) = 0, \Theta(0) = 1, \phi(0) = 1 \text{ at } \eta = 0 \]

\[ f'(\eta) \rightarrow 0, \ g(\eta) \rightarrow 0, \ 0(\eta) \rightarrow 0, \ \phi(\eta) \rightarrow 0 \text{ at } \eta \rightarrow \infty \]

where a prime denotes the differentiation with respect to $\eta$ only and the dimensionless parameters appearing in the Eqs. (16)-(21) are respectively defined as $\Theta = \frac{T_{G} - T_{w}}{T_{w} - T_{o}} = \left[ \frac{1}{\gamma_{0}(T_{w} - T_{o})} \right]$ the viscosity parameter, $M = \frac{\sigma_{b}^2}{r_{0}^{2}}$ the magnetic parameter, $P_{r} = \frac{\rho C_{p} v}{k}$ the Prandtl number, $\gamma = \frac{k_{0}}{b}(C_{w} - C_{e})$ the non-dimensional chemical reaction parameter, $G = \frac{\beta_{r}(T_{w} - T_{o})}{b^{2} \sigma_{k}}$ the local Grashof number, $N = \frac{\beta_{r}(C_{w} - C_{e})}{b_{r}(T_{w} - T_{o})}$ the local Biot number, $Rd = \frac{kk_{o}}{4T_{w} \sigma^{2}}$ is the thermal radiation parameter, $Sr = \frac{D_{r} k_{b}}{C_{v} \sigma_{k}}$ the Soret parameter, $Sc = \frac{\mu}{\rho c_{p} D}$ the Schmidt number, and $Ec = \frac{\mu^{2}}{(T_{w} - T_{o})C_{v} b}$ is the Eckert number.

3. METHOD OF SOLUTION

The set of non-linear coupled differential eqs. (16 – 18) with appropriate boundary conditions given in eq(19) constitute a two-point boundary value problem. The equations are highly non-linear and so, cannot be solved analytically. Therefore, those equations are solved numerically using the symbolic computer algebra software Maple 17. This software uses a Runge-Kutta-Fehlberg method as the default to solve boundary value problems numerically. The asymptotic boundary conditions in (19) at $\eta \rightarrow \infty$ are replaced by those at $\eta = \eta_{w}$ as is usually the standard practice in the boundary layer analysis. The inner iteration is counted until the nonlinear solution converges with a convergence criterion of $10^{-6}$ in all cases.

4. SKIN FRICTION, NUSSELT NUMBER AND SHERWOOD NUMBER

The local skin-friction coefficient $C_{f}$, the local Nusselt number $Nu$ and the local Sherwood number $Sh$ defined by

\[ C_{f} = \frac{\tau_{w}}{\mu b \sqrt{f''(0)}} = f''(0) \]

\[ \tau_{w} = \mu \left( \frac{\partial f}{\partial y} \right)_{y=0} = \mu b \sqrt{f''(0)}, \ Nu = \frac{q_{w}}{k \sqrt{\frac{2}{T_{w} - T_{o}}}} \]

where $q_{w} = -k \frac{\partial f}{\partial y} |_{y=0} = -k \sqrt{\frac{2}{T_{w} - T_{o}}} \phi''(0)$ and
\[ Sh = \frac{m_w}{D \sqrt{\gamma} (C_w - C_r)} = -\phi'(0) \]  

(24)

where \[ m_w = -D \left( \frac{\partial C}{\partial y} \right)_{y=0} = -D \sqrt{\gamma} (C_w - C_r) \phi'(0) \]

**Comparison**

The results of this paper are compared with the results of previous published paper of Shit et al. [73] and Rahman et al. [59a] as shown in Table 1 and Table 2 and the outcomes are in good concurrence.

**Table 1.** Comparison of Nu and Sh at η=0 with Shit et al. (73) with Sr=0, Ec=0, \( \theta_r = 1 \)

<table>
<thead>
<tr>
<th>M</th>
<th>Rd</th>
<th>( \gamma )</th>
<th>0r</th>
<th>Shit et al.[73]Results</th>
<th>Present Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>1</td>
<td>0.5</td>
<td>-2</td>
<td>-0.6912</td>
<td>0.6265</td>
</tr>
<tr>
<td>1.5</td>
<td>1</td>
<td>0.5</td>
<td>-2</td>
<td>-0.6977</td>
<td>0.6543</td>
</tr>
<tr>
<td>0.5</td>
<td>3</td>
<td>0.5</td>
<td>-2</td>
<td>-12.3751</td>
<td>0.9278</td>
</tr>
<tr>
<td>0.5</td>
<td>1</td>
<td>1.5</td>
<td>-2</td>
<td>-0.6956</td>
<td>1.0959</td>
</tr>
<tr>
<td>0.5</td>
<td>1</td>
<td>-0.5</td>
<td>-2</td>
<td>-0.6966</td>
<td>0.4898</td>
</tr>
<tr>
<td>0.5</td>
<td>1</td>
<td>-1.5</td>
<td>-2</td>
<td>-0.6968</td>
<td>0.4245</td>
</tr>
<tr>
<td>0.5</td>
<td>1</td>
<td>0.5</td>
<td>-4</td>
<td>-0.6969</td>
<td>0.6253</td>
</tr>
</tbody>
</table>

**Table 2.** Comparison of Nu and Sh at η=0 with Rahman et al. [59a] with Pr=0.71, m= Sr= N= \( \theta_r = Ec=0 \), \( \theta_r = 1 \) for different values of \( \theta_r \)

<table>
<thead>
<tr>
<th>Different Values of ( \theta_r )</th>
<th>Rahman et al. [59a] Results</th>
<th>Present Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nu(0)</td>
<td>Sh(0)</td>
<td>Nu(0)</td>
</tr>
<tr>
<td>-2</td>
<td>-0.37865</td>
<td>4.803562</td>
</tr>
<tr>
<td>-4</td>
<td>-0.30536</td>
<td>4.54266</td>
</tr>
<tr>
<td>2</td>
<td>0.248266</td>
<td>4.485639</td>
</tr>
<tr>
<td>4</td>
<td>0.238773</td>
<td>4.33562</td>
</tr>
</tbody>
</table>

If we consider \( M = m = 0, \theta_r = 1 \), \( \theta_r \rightarrow \infty \) and \( Sr=Sc=\theta_r=N=0 \), the present flow problem becomes hydrodynamic boundary-layer flow past a stretching sheet whose analytical solution put forwarded by Crane (2) as follows:

\[ f'(\eta) = 1 - e^{-\eta} \text{ i.e., } f'(\eta) = e^\eta \]

An attempt has been made to validate our result for the axial velocity \( f'(\eta) \). We compared our results with this analytical solution and found to be in good agreement.

**5. RESULTS AND DISCUSSION**

Figs. 2-8 represent the axial velocity \( f' \), g distribution of the \( z' \) component of velocity which is induced due to the presence of the Hall effects and rotation. All these figures show that for any particular values of the physical parameters \( g(\eta) \) reaches a maximum value at a certain high \( \eta \) above the sheet and beyond which \( g(\eta) \) decreases gradually in asymptotic nature for different velocities of \( m \), Rd, Sr, \( \gamma \), A.A11 and \( 0r \). Figs. 2(a) shows that the axial velocity \( f'(\eta) \) increases with increase in Hall parameter. This may be due to the fact that as \( m \) increases the Lorentz force which opposes the flow and leads to the degeneration of the fluid motion. The anomalous behavior of \( \theta \) with variation of \( m \) is observed due to the presence of the Hall Current and there by induces a cross flow velocity component \( g(\eta) \). For an increase in the Hall parameter \( m \) we noticed an enhancement in cross flow velocity. From Figs.2(c & d), we find that an increase in the Hall parameter \( m \), results in a depreciation in the temperature and concentration. Figs. 6(a & b) represent \( f'(\eta) \) and \( g(\eta) \) with chemical reaction parameter \( \gamma \). It can be seen from the profiles that the axial and cross flow velocity components enhance in the degenerating chemical reaction parameter \( \gamma > 0 \) and reduces in generating chemical reaction \( \gamma < 0 \). The temperature enhances and reduces the concentration in the degeneration chemical reaction case and in the generating case, the temperature reduces and concentration enhances in the boundary layer. Figs. 3(a & b) depicts the axial velocity \( f'(\eta) \) and cross flow velocity \( g(\eta) \) with \( \theta_r \). It can be seen from the profiles that the velocities decrease with the increase of viscosity parameter \( 0r \). This observation leads to the increase of thermal boundary layer thickness. Figs. 3(c & d) represent...

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temperature and concentration with viscosity parameter $\theta r$. It is found that an increase in $\theta r(<0)$, enhances the temperature and concentration in the boundary layer. An increase in temperature parameter ($\theta$) enhances the axial velocity and temperature and reduces the cross flow velocity, concentration in the boundary layer (figs. 4a-4d). Figs. 5a-5d represent the variation of the variation of $f'(\eta)$ and $g(\eta)$ with slip parameter $A$. It can be seen from the profiles that higher the slip parameter $A$ larger the axial and cross flow velocity components. The temperature increases and the concentration reduces with increase in $A$ (figs. 5c & d).

The effect of thermal radiation parameter $R d$, on $f'(\eta)$ and $g(\eta)$ is shown in Fig. 8(a & b). It is found that higher the radiative heat flux larger the axial and cross flow velocity. It is due to the fact that an increase in $R d$, the thickness of the boundary layer increases. The temperature enhances and the concentration reduces with increase in the radiation parameter.

The variation of Skin friction with different parameters is exhibited in table 3. It is found that An increase the Hall parameter (m) enhances $\tau x$ and $\tau y$ at the wall. Also higher the Coriolice force smaller the skin friction components at the wall. The variation of skin friction with radiation parameter $R d$ shows that both the components increases with increase in $R d$. With respect to the chemical reaction parameter $\gamma$, we find that the skin friction components enhance in the degenerating chemical reaction case and in the generating chemical reaction case, they experience a depreciation at the wall. An increase in the viscosity parameter $\theta r$ leads to a reduction in $\tau x$ and $\tau y$. Higher the slip parameter ($A 11$) smaller the skin friction component $\tau x$ and larger $\tau y$ at the wall. With reference to temperature parameter (A) we find that an increase in A results in a reduction in the skin friction component $\tau x$ and enhancement in $\tau y$.

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**Fig 2a. Effect of m on $f'(\eta)$**

$R d=0.5, S r=0.5, \gamma=0.5, \theta r=-2, A 11=0.2, A=1.01$

**Fig 2b. Effect of m on g(\eta)**

$G Rd=0.5, S r=0.5, \gamma=0.5, \theta r=-2, A 11=0.2, A=1.01$

**Fig 2c. Effect of m on $\theta(\eta)$**

$R d=0.5, S r=0.5, \gamma=0.5, \theta r=-2, A 11=0.2, A=1.01$

**Fig 2d. Effect of m on C(\eta)**

$R d=0.5, S r=0.5, \gamma=0.5, \theta r=-2, A 11=0.2, A=1.01$

**Fig 3a. Effect of $\theta r$ on $f'(\eta)$**

$m=0.5, R d=0.5, S r=0.5, \gamma=0.5, A 11=0.2, A=1.01$

**Fig 3b. Effect of $\theta r$ on g(\eta)**

$m=0.5, R d=0.5, S r=0.5, \gamma=0.5, A 11=0.2, A=1.01$
The rate of heat transfer (Nusselt number) at the wall $\eta=0$ is shown in Tables.3 for different parametric variations. The rate of heat transfer reduces with increase in Hall parameter ($m$). The variation of Nu with Soret parameter (Sr) and viscosity parameter shows that higher the thermodiffusion effects smaller the rate of heat transfer at the wall. The variation of Nu with $A11$ shows that higher the slip parameter ($A11$) smaller the rate of heat transfers. The rate of heat transfer at the wall reduce with increase in the strength of the heat generating source and in the generating heat source, it enhances on the wall. An increase in temperature parameter ($A$) enhances the Nusselt number at the wall.

Fig 3c. Effect of $\theta r$ on $\theta (\eta)$

$m=0.5, Rd=0.5, Sr=0.5, \gamma=0.5, A11=0.2, A=1.01$

Fig 3d. Effect of $\theta r$ on $C (\eta)$

$m=0.5, Rd=0.5, Sr=0.5, \gamma=0.5, A11=0.2, A=1.01$

Fig 4a. Effect of $A11$ on $f (\eta)$

$m=0.5, Rd=0.5, Sr=0.5, \gamma=0.5, \theta r=-2, A=1.01$

Fig 4b. Effect of $A11$ on $g (\eta)$

$m=0.5, Rd=0.5, Sr=0.5, \gamma=0.5, \theta r=-2, A=1.01$

Fig 4c. Effect of $A11$ on $\theta (\eta)$

$m=0.5, Rd=0.5, Sr=0.5, \gamma=0.5, \theta r=-2, A=1.01$

Fig 4d. Effect of $A11$ on $C (\eta)$

$m=0.5, Rd=0.5, Sr=0.5, \gamma=0.5, \theta r=-2, A=1.01$

Fig 5a. Effect of $A$ on $f (\eta)$

$m=0.5, Rd=0.5, Sr=0.5, \gamma=0.5, \theta r=-2, A=1.01$

Fig 5b. Effect of $A$ on $g (\eta)$

$m=0.5, Rd=0.5, Sr=0.5, \gamma=0.5, \theta r=-2, A=1.01$
The rate of mass transfer (Sherwood Number) at the wall is shown in table 3 for different variations of the parameters. Higher the viscosity parameter smaller the rate of mass transfer at the wall. The variation of Sh with $\gamma$ shows that the rate of mass transfer enhances at the wall in degenerating chemical reaction case and reduces in the generating chemical reaction case. An increase in the temperature parameter (A) / slip parameter (A11) enhances the Sherwood number at the wall.

![Fig5c. Effect of A on $\theta(\eta)$](image1)

$m=0.5, Rd=0.5, Sr=0.5, \gamma=0.5, \theta r=-2, A=1.01$

![Fig5d. Effect of A on $C(\eta)$](image2)

$m=0.5, Rd=0.5, Sr=0.5, \gamma=0.5, \theta r=-2, A=1.01$

![Fig6a. Effect of $\gamma$ on $f(\eta)$](image3)

$m=0.5, Rd=0.5, Sr=0.5, \theta r=-2, A11=0.2, A=1.01$

![Fig6b. Effect of $\gamma$ on $g(\eta)$](image4)

$m=0.5, Rd=0.5, Sr=0.5, \theta r=-2, A11=0.2, A=1.01$

![Fig6c. Effect of $\gamma$ on $\theta(\eta)$](image5)

$m=0.5, Rd=0.5, Sr=0.5, \theta r=-2, A11=0.2, A=1.01$

![Fig6d. Effect of $\gamma$ on $C(\eta)$](image6)

$m=0.5, Rd=0.5, Sr=0.5, \theta r=-2, A11=0.2, A=1.01$

![Fig7a. Effect of $Sr$ on $f(\eta)$](image7)

$m=0.5, Rd=0.5, \gamma=0.5, \theta r=-2, A11=0.2, A=1.01$

![Fig7b. Effect of $Sr$ on $g(\eta)$](image8)

$m=0.5, Rd=0.5, \gamma=0.5, \theta r=-2, A11=0.2, A=1.01$

Fig7c. Effect of Sr on $\theta(\eta)$  
$m=0.5$, $Rd=0.5$, $\gamma=0.5$, $\theta_r=-2$, $A1=0.2$, $A=1.01$

Fig7d. Effect of Sr on $C(\eta)$  
$m=0.5$, $Rd=0.5$, $\gamma=0.5$, $\theta_r=-2$, $A1=0.2$, $A=1.01$

Fig8a. Effect of Rd on $f(\eta)$  
$m=0.5$, $Sr=0.5$, $\gamma=0.5$, $\theta_r=-2$, $A1=0.2$, $A=1.01$

Fig8b. Effect of Rd on $g(\eta)$  
$m=0.5$, $Sr=0.5$, $\gamma=0.5$, $\theta_r=-2$, $A1=0.2$, $A=1.01$

Fig8c. Effect of Rd on $\theta(\eta)$  
$m=0.5$, $Sr=0.5$, $\gamma=0.5$, $\theta_r=-2$, $A1=0.2$, $A=1.01$

Fig8d. Effect of Rd on $C(\eta)$  
$m=0.5$, $Sr=0.5$, $\gamma=0.5$, $\theta_r=-2$, $A1=0.2$, $A=1.01$

Table3. Skin friction ($\tau$) and Nusselt Number (Nu) at $\eta = 0$.  

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$\tau_x(0)$</th>
<th>$\tau_z(0)$</th>
<th>Nu(0)</th>
<th>Sh(0)</th>
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<td>0.246249</td>
<td>0.244089</td>
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<tr>
<td>Rd</td>
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<tr>
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<td>0.244089</td>
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The problem of combined influence of rotation, variable viscosity effect, Hall current, nonlinear thermal radiation on MHD free-convective flow and heat and mass transfer over a stretching sheet in the presence of heat generation/absorption has been analyzed. The fluid viscosity is assumed to vary as an inverse linear function of temperature. The numerical results were obtained and compared with previously reported cases available in the literature and they were found to be in good agreement. Graphical results for various parametric conditions were presented and discussed for different values. The main findings are summarized below:

- In the presence of Hall current, both axial and cross flow velocity distribution enhances. Increase in Hall parameter $m$ results into a depreciation in the temperature and concentration. With increase in $m$, skin friction parameter $\tau_x$ and $\tau_z$ enhance at $\eta=0$. The rate of heat transfer reduces with increase in Hall parameter, whereas the rate of mass transfer enhances with increase in Hall parameter.

- The presence of degenerating chemical reaction cases, the axial velocity and cross flow velocity enhances and in the generating chemical reaction case, $f'$ & $g$ reduce. The temperature enhances and concentration reduces in the degeneration chemical reaction case and in the generation chemical reaction case, the temperature reduces and concentration enhances. The skin friction parameter reduces with chemical reaction parameter $\gamma$ in the degenerating case and enhances with the generating case. The rate of heat transfer reduces with lower values of $\gamma$ and enhances with higher values of $\gamma$. The rate of mass transfer enhances with lower values of $\gamma$ and reduces with higher values of $\gamma$.

- An increase in thermo-diffusion $Sr$, the axial velocities, temperature enhance and reduces the concentration. The skin friction parameter $\tau_x$ increases with increase in $Sr$ at $\eta=0$. The skin friction parameter $\tau_z$ reduces with increase in $Sr$ at $\eta=0$. The rate of heat transfer reduces with increase in thermo-diffusion $Sr$. Thus higher the thermo-diffusion $Sr$, larger the rate of mass transfer at the sheet.

- An increase in $Rd$ enhances the velocities, temperature and reduces the concentration. The skin friction components, Sherwood number enhances while Nusselt number at the wall reduces with increase in $Rd$.

- The axial and cross flow velocities increase in the boundary layer with the increase in the viscosity parameter $\theta_r$. With increase in the viscosity parameter the temperature of the fluid flow also increases and reduces the concentration of the boundary layer. The skin friction components, the rate of heat and mass transfer at the wall reduces with increase in $\theta_r$.

- An increase in temperature parameter $A$ enhances the velocities, temperature and reduces the concentration. $\tau_x$ reduces, $\tau_y$, $Nu$ and $Sh$ enhance at the wall with increase in $A$.
REFERENCES


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