Weakly 2-Absorbing Ideals of So-Rings

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1. Introduction
Partially defined infinitary operations occur in the contexts ranging from integration theory to programming language semantics. The general cardinal algebras studied by Tarski in 1949, Hausdorff commutative groups studied by Bourbaki in 1966, sum-structures studied by Higgs in 1980, sum-ordered partial monoids and sum-ordered partial semirings (so-rings) studied by Arbib, Manes and Benson [2], [4], and Streenstrup [10] are some of the algebraic structures of the above type.

G.V.S. Acharyulu [8] and P.V. Srinivasarao [6] developed the ideal theory for the sum-ordered partial semirings (so-rings). Continuing this study, in [9] & [5] we introduced the notion of 2-absorbing ideals in so-rings and obtained their characteristics in a commutative so-ring. In this paper, we introduce the notion of weakly 2-absorbing ideals in so-rings and obtain its characteristics in so-rings.

2. Preliminaries

Abstract: A partial semiring is a structure possessing an infinitary partial addition and a binary multiplication, subject to a set of axioms. In this paper we introduce the notion of weakly 2-absorbing ideals in so-rings and study the conditions under which a weakly 2-absorbing ideal is a 2-absorbing ideal. Also, we obtain various equivalent conditions on the weakly 2-absorbing ideals of Cartesian product of so-rings.

Keywords: Ideal, Prime ideal, 2-absorbing ideal, weakly 2-absorbing ideal, commutative so-ring.

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G.V.S. Acharyulu [8] and P.V. Srinivasarao [6] developed the ideal theory for the sum-ordered partial semirings (so-rings). Continuing this study, in [9] & [5] we introduced the notion of 2-absorbing ideals in so-rings and obtained their characteristics in a commutative so-ring. In this paper, we introduce the notion of weakly 2-absorbing ideals in so-rings and obtain its characteristics in so-rings.

2. PRELIMINARIES

In this section we collect some important definitions and results for our use in this paper.

2.1. Definition. [4] A partial Monoid is a pair (M, \(\sum\)) where M is a non-empty set and \(\sum\) is a partial addition defined on some, but not necessarily all families \((x_i : i \in I)\) in M subject to the following axioms:

(i) Unary Sum Axiom. If \((x_i : i \in I)\) is a one element family in M and \(I = \{j\}\), then \(\sum(x_i : i \in I)\) is defined and equals \(x_j\).

(ii) Partition-Associativity Axiom. If \((x_i : i \in I)\) is a family in M and \((I_j : j \in J)\) is a partition of I, then \(\sum(x_i : i \in I)\) is summable if and only if \(\sum(x_i : i \in I_j)\) is summable for every \(j \in J\) and \(\sum(x_i : i \in I_j) : j \in J\) is summable. We write \(\sum(x_i : i \in I) = \sum(\sum(x_i : i \in I_j) : j \in J)\).

2.2. Definition. [4] A Partial Semiring is a quadruple \((R, \sum, 1)\), where \((R, \sum)\) is a partial monoid, \((R, 1)\) is a monoid with multiplicative operation \(\cdot\) and unit 1, and the additive and multiplicative
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structures obey the following distributive laws: If $\sum(x_i : i \in I)$ is defined in $R$, then for all $y$ in $R$, $\sum(y.x_i : i \in I)$ and $\sum(x_i.y : i \in I)$ are defined and $y.\sum(x_i : i \in I) = \sum(y.x_i : i \in I)$.

2.3. Definition. [4] A partial semiring $(R, \Sigma, 1)$ is said to be commutative if $xy = yx \forall x, y \in R$.

2.4. Definition. [4] The sum ordering $\leq$ on a partial monoid $(M, \Sigma)$ is the binary relation $\leq$ such that $x \leq y$ if and only if there exists a ‘$h$’ in $M$ such that $y = x + h$ for $x, y \in M$.

2.5. Definition. [4] A sum-ordered partial semiring or so-ring, for short, is a partial semiring in which the sum ordering is a partial order.

2.6. Example. [4] Let $D$ be a set and let the set of all partial functions from $D$ to $D$ be denoted by $Pfn(D, D)$. A family $(x_i : i \in I)$ is summable if and only if for $i, j$ in $I$, and $i \neq j$, $dom(x_i) \cap dom(x_j) = \phi$. If $(x_i : i \in I)$ is summable then for any $d$ in $D$

$$d(\sum, x_i) = \begin{cases} dx_i, & \text{if } d \in dom(x_i) \text{ for some (unique) } i \in I; \\ \text{undefined, otherwise.} & \end{cases}$$

And ‘.’ defined as the usual functional composition, the ordering as the extension of functions and unit defined as the identity defined on $D$. Then $(Pfn(D, D), \Sigma, 1)$ is a so-ring.

2.7. Example. [4] Let $D$ be a set. A multi-function $x : D \to D$ maps each element in $D$ to an arbitrary subset of $D$. Such multi-functions correspond bijectively to the relation $r \subseteq D \times D$, where $(d, e) \in r$ if and only if $e \in dx$. The set of all multi-functions from $D$ to $D$, denoted by $Mfn(D, D)$, together with $\Sigma$ defined such that $d$ in $D$, $d(\sum, x_i) = \bigcup_i (dx_i)$, and ‘.’ defined as the usual relational composition. That is, for each $d$ in $D$ and for $x, y$ in $Mfn(D, D)$, $d(\sum, x) = \bigcup (ey : e \in dx)$, and $d1 = \{d\}$. Then $(Mfn(D, D), \Sigma, 1)$ is a so-ring.

2.8. Definition. [8] Let $R$ be a so-ring. A subset $N$ of $R$ is said to be an ideal of $R$ if the following are satisfied:

$(I_1)$. If $(x_i : i \in I)$ is a summable family in $R$ and $x_i \in N \forall i \in I$ then $\sum(x_i : i \in I) \in N$,

$(I_2)$. If $x \leq y$ and $y \in N$ then $x \in N$,

$(I_3)$. If $x \in N$ and $r \in R$ then $xr, rx \in N$.

2.9. Definition. [7] A proper ideal $P$ of a so-ring $R$ is said to be weakly prime if for any $a, b$ of $R$, $0 \neq ab \in P$ imply $a \in P$ or $b \in P$.

2.10. Definition. [9] A proper ideal $I$ of a so-ring $R$ is said to be $2$-absorbing if for any $a, b, c \in R$, $abc \in I$ implies $ab \in I$ or $bc \in I$ or $ac \in I$.

2.11. Remark. [9] Every prime ideal of a so-ring $R$ is a $2$-absorbing ideal of $R$.

The following is an example of a so-ring $R$ in which a $2$-absorbing ideal need not be a prime ideal of $R$.

2.12. Example. [9] Consider the so-ring $R = \{0, u, v, x, y, 1\}$ with $\Sigma$ defined on $R$ by $\sum(x_i : i \in I) = \begin{cases} x_i, & \text{if } x_i = 0 \forall i \neq j \text{ for some } j, \\ \text{undefined, otherwise.} & \end{cases}$

And ‘.’ defined by the following table:
Then the ideal \( I = \{0, u, x\} \) is a 2-absorbing ideal, but \( I \) is not a prime ideal. Since \( v.y = z \in I \), but \( v \notin I \) and \( y \notin I \).

Throughout this paper, \( R \) denotes a commutative so-ring.

3. WEAKLY 2-ABSORBING IDEALS

We introduce the notion of weakly 2-absorbing ideals in so-rings as follows:

3.1. Definition. An ideal \( I \) of a so-ring \( R \) is said to be weakly 2-absorbing if for some \( a, b, c \in R \) and \( 0 \neq abc \in I \), then \( ab \in I \) or \( bc \in I \) or \( ac \in I \).

3.2. Remark. Every 2-absorbing ideal \( I \) of a so-ring \( R \) is a weakly 2-absorbing ideal of \( R \).

The following is an example of a so-ring \( R \) in which a weakly 2-absorbing ideal is not a 2-absorbing ideal of \( R \).

3.3. Example. Consider the so-ring \( Z_8 \times Z_8 \). Take \( R := Z_8 \times Z_8 \). Then \( R \) is a commutative so-ring with respect to \( \times \) operation. Take \( I := \{(0,0),(0,4)\} \). Then it can be verified that \( I \) is a weakly 2-absorbing ideal of \( R \). Since \( (2,0)(2,0)(2,0) = (0,0) \in I \) and \( (2,0)(2,0) = (4,0) \notin I \), \( I \) is not a 2-absorbing ideal of \( R \).

3.4. Theorem. Let \( R \) be a so-ring, \( I \) be an ideal of \( R \) and \( a \in R \). Then the following statements are hold in \( R \):

(i) Suppose \( (0 : a) \subseteq Ra \), then the ideal \( Ra \) is 2-absorbing if and only if it is weakly 2-absorbing.

(ii) Suppose \( (0 : a) \subseteq Ia \), then the ideal \( Ia \) is 2-absorbing if and only if it is weakly 2-absorbing.

Proof. Let \( a \in R \).

(i) Assume that \( (0 : a) \subseteq Ra \). Suppose \( Ra \) is a weakly 2-absorbing ideal of \( R \). Let \( r, s, t \in R \) such that \( rst \in Ra \). Suppose \( rst \neq 0 \). Since \( Ra \) is weakly 2-absorbing, we have \( rs \in Ra \) or \( st \in Ra \) or \( rt \in Ra \). Suppose \( rst = 0 \). Then \( r(s+a)t = rst + rat = rat = rta \in Ra \). Therefore \( r(s+a)t \in Ra \). If \( r(s+a)t \neq 0 \). Then \( r(s+a) \in Ra \) or \( (s+a)t \in Ra \) or \( rt \in Ra \) (Since \( Ra \) is weakly 2-absorbing). That implies \( rs \in Ra \) or \( st \in Ra \) or \( rt \in Ra \). If \( r(s+a)t = 0 \). Then \( rst + rat = 0 \). That implies \( rat = 0 \). That implies \( rta = 0 \). That implies \( rt \notin (0 : a) \subseteq Ra \). That implies \( rt \in Ra \). Hence \( Ra \) is a 2-absorbing ideal of \( R \). By Remark 3.2., if \( Ra \) is a 2-absorbing ideal of \( R \) then \( Ra \) is a weakly 2-absorbing ideal of \( R \).

(ii) Assume that \( (0 : a) \subseteq Ia \). Suppose \( Ia \) is a weakly 2-absorbing ideal of \( R \). Let \( r, s, t \in R \) such that \( rst \in Ia \). Suppose \( rst \neq 0 \). Since \( Ia \) is weakly 2-absorbing, we have \( rs \in Ia \) or \( st \in Ia \) or \( rt \in Ia \). Suppose \( rst = 0 \). Then \( 0 r(s+a)t = rst + rat = rat = rta \in Ia \). Therefore \( r(s+a)t \in Ia \). If \( r(s+a)t \neq 0 \). Then \( r(s+a) \in Ia \) or \( (s+a)t \in Ia \) or \( rt \in Ia \) (Since \( Ia \) is weakly 2-absorbing). That implies \( rs \in Ia \) or \( st \in Ia \) or \( rt \in Ia \). If \( r(s+a)t = 0 \). Then \( rst + rat = 0 \). That implies \( rat = 0 \). That implies \( rta = 0 \). That implies \( rt \in (0 : a) \subseteq Ia \). That implies \( rt \notin Ia \). Hence \( Ia \) is a 2-absorbing ideal of \( R \). By Remark 3.2., if \( Ia \) is a 2-absorbing ideal of \( R \) then \( Ia \) is a weakly 2-absorbing ideal of \( R \).
3.5. Theorem. If $I$ and $J$ are weakly prime ideals of a so-ring $R$, then $I \cap J$ is a weakly 2-absorbing ideal of $R$.

**Proof.** Let $0 \neq abc \in I \cap J$ for some $a,b,c \in R$. i.e., $0 \neq abc \in I$ & $0 \neq abc \in J$. Since $I,J$ are weakly prime ideals of $R$, we have either $a \in I$ or $bc \in I$ & either $a \in J$ or $bc \in J$. Suppose $bc = 0$ then $bc \in I \cap J$, there is nothing to prove (Since $0 \in I \cap J$). So assume that $bc \neq 0$. Then either $a \in I$ or $0 \neq bc \in I$ & either $a \in J$ or $0 \neq bc \in J$. That implies $a \in I$ or $b \in I$ or $c \in I$ & $a \in J$ or $b \in J$ or $c \in J$ (Since, $I,J$ are weakly prime ideals). That implies $ab \in I \cap J$ or $bc \in I \cap J$ or $ac \in I \cap J$. Hence $I \cap J$ is a weakly 2-absorbing ideal of $R$.

3.6. Definition. Let $I$ be a weakly 2-absorbing ideal of a so-ring $R$ and $a,b,c \in R$. We say that $(a,b,c)$ is a triple-zero of $I$ for some $a,b,c \in R$. Then

(i) $abI = bcI = acI = \{0\}$

(ii) $aI^2 = bI^2 = cI^2 = \{0\}$.

**Proof.** (i) In a contrary way suppose that $abI \neq \{0\}$. i.e., $abI \neq \{0\}$ for some $i \in I$. That implies $ab(c+i) \neq 0$. Since $ab \notin I$ and $ab(c+i) \neq 0$, we have $a(c+i) \in I$ or $b(c+i) \in I$ (Since $I$ is weakly 2-absorbing). i.e., $ac \in I$ or $bc \in I$, a contradiction to the fact that $(a,b,c)$ is a triple-zero. So our assumption is wrong. Hence $abI = \{0\}$. Similarly we can prove that $bI = \{0\}$, $acI = \{0\}$.

(ii) In a contrary way suppose that $aI^2 \neq \{0\}$. i.e., $a_iI_2 \neq 0$ for some $i_1,i_2 \in I$. That implies $a(b+i_1)(c+i_2) = a_iI_2 \neq 0 \in I$ (Since by (i), $abI = bcI = acI = \{0\}$). Since $I$ is a weakly 2-absorbing ideal of $R$, either $a(b+i_1) \in I$ or $a(c+i_2) \in I$ or $(b+i_1)(c+i_2) \in I$. i.e., either $ab \in I$ or $bc \in I$ or $ac \in I$, a contradiction to the fact that $(a,b,c)$ is a triple-zero. So our assumption is wrong. Hence $aI^2 = \{0\}$. Similarly we can prove that $bI^2 = cI^2 = \{0\}$.

4. WEAKLY 2-ABSORBING IDEALS IN CARTESIAN PRODUCTS

4.1. Theorem. Let $R_1$ and $R_2$ be so-rings and $I$ be a proper ideal of $R_1$. Then the following conditions are equivalent:

(i) $I \times R_2$ is a weakly 2-absorbing ideal of $R = R_1 \times R_2$,

(ii) $I \times R_2$ is a weakly 2-absorbing ideal of $R = R_1 \times R_2$.

**Proof.** (i) $\Rightarrow$ (ii): Suppose $I$ is a weakly 2-absorbing ideal of $R$. Let $(a_1,a_2),(b_1,b_2),(c_1,c_2) \in R = R_1 \times R_2$ such that $0 \neq (a_1,a_2)(b_1,b_2)(c_1,c_2) \in I \times R_2$. Then $0 \neq (a_1b_1,a_2b_2,c_1c_2) \in I \times R_2$. Therefore $0 \neq a_1b_1c_1 \in I$. Since $I$ is a weakly 2-absorbing ideal of $R$, $a_1b_1 \in I$ or $b_1c_1 \in I$ or $a_1c_1 \in I$. If $a_1b_1 \in I$ then $(a_1,a_2)(b_1,b_2) \in I \times R_2$. If $b_1c_1 \in I$ then $(b_1,b_2)(c_1,c_2) \in I \times R_2$. If $a_1c_1 \in I$ then $(a_1,a_2)(c_1,c_2) \in I \times R_2$. Hence $I \times R_2$ is a weakly 2-absorbing ideal of $R = R_1 \times R_2$.

(ii) $\Rightarrow$ (i): Suppose $I \times R_2$ is a weakly 2-absorbing ideal of $R = R_1 \times R_2$. Let $0 \neq abc \in I$ for some $a,b,c \in R$. Then for each $0 \neq r \in R_2$, we have $0 \neq (a,1)(b,1)(c,r) \in I \times R_2$. Since $I \times R_2$, is a weakly 2-absorbing ideal of $R = R_1 \times R_2$, $(a,1)(b,1) \in I \times R_2$ or $(b,1)(c,r) \in I \times R_2$ or $(a,1)(c,r) \in I \times R_2$. That implies $ab \in I$ or $bc \in I$ or $ac \in I$. Hence $I$ is a weakly 2-absorbing ideal of $R$. 
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4.2. Theorem. Let $R = R_1 \times R_2$ where $R_1$ and $R_2$ are so-rings. Let I be a proper ideal of $R$, and J be a proper ideal of $R_2$. Then the following statements are equivalent:

(i) $I \times J$ is a weakly 2-absorbing ideal of $R$,

(ii) ($J = R_2$ and I is a weakly 2-absorbing ideal of $R_1$) or (J is a prime ideal of $R_2$ and I is a prime ideal of $R_1$).

Proof. (i) $\Rightarrow$ (ii): Suppose $I \times J$ is a weakly 2-absorbing ideal of $R$. If $J = R_2$ then $I \times R_2$ is a weakly 2-absorbing ideal of $R$. Then by theorem 4.2., I is a weakly 2-absorbing ideal of $R_1$. Suppose $J \neq R_2$. We have to prove that J is a prime ideal of $R_2$ and I is a prime ideal of $R_1$. Let $a, b \in R_2$ such that $ab \in J$, and let $0 \neq i \in I$. Then $0 \neq (i, l)(1, a)(1, b) = (i, ab) \in I \times J$. Now $(1, a)(1, b) = (1, ab) \notin I \times J$ (Since $I \notin J$). Since $I \times J$ is a weakly 2-absorbing ideal of $R$, either $(i, l)(1, a) = (i, a) \in I \times J$ or $(i, l)(1, b) = (i, b) \in I \times J$. That implies either $a \in J$ or $b \in J$. Hence J is a prime ideal of $R_2$. Similarly let $c, d \in R_1$ such that $cd \in I$, and let $0 \neq j \in J$. Then $0 \neq (c, l)(d, l)(1, j) = (cd, j) \in I \times J$. Now $(c, l)(d, l) = (cd, l) \notin I \times J$ (Since $I \notin J$). Since $I \times J$ is a weakly 2-absorbing ideal of $R$, either $(c, l)(1, j) = (c, j) \in I \times J$ or $(d, l)(1, j) = (d, j) \in I \times J$. That implies either $c \in I$ or $d \in I$. Hence I is a prime ideal of $R_1$.

(ii) $\Rightarrow$ (i): Suppose $J = R_2$ and I is a weakly 2-absorbing ideal of $R_1$. If $J$ is a prime ideal of $R_2$, then I is a weakly 2-absorbing ideal of $R$. Suppose $J = R_2 \& I$ is a weakly 2-absorbing ideal of $R_1$, by theorem 4.2., $I \times R_2$ is a weakly 2-absorbing ideal of $R$. i.e., $I \times J$ is a weakly 2-absorbing ideal of $R$. Suppose I is a prime ideal of $R_2$ & I is a prime ideal of $R_1$. Let $0 \neq (a_1, b_1)(a_2, b_2)(a_3, b_3) \in I \times J$ for some $a_1, a_2, a_3 \in R_1$ and $b_1, b_2, b_3 \in R_2$. Then $a_1 \in I$ or $a_2 \in I$ or $a_3 \in I$ and $b_1 \in J$ or $b_2 \in J$ or $b_3 \in J$. Thus $(a_1, b_1)(a_2, b_2) \in I \times J$ or $(a_2, b_2)(a_3, b_3) \in I \times J$ or $(a_1, b_1)(a_3, b_3) \in I \times J$. Hence $I \times J$ is a weakly 2-absorbing ideal of $R$.

4.3. Theorem. Let $R_1, R_2$ be a so-rings such that $R_2$ has no nonzero divisors. Let I be a proper ideal of $R_1$ and J be an ideal of $R_2$. Then the following statements are equivalent:

(i) $I \times J$ is a weakly 2-absorbing ideal of $R = R_1 \times R_2$,

(ii) I is a weakly prime ideal of $R_1$ and $J = \{0\}$ is a prime ideal of $R_2$.

Proof. (i) $\Rightarrow$ (ii): Suppose $I \times J$ is a weakly 2-absorbing ideal of $R$. Suppose $J = \{0\}$. We have to prove that $J = \{0\}$ is a prime ideal of $R_2$. Let $ab \in J = \{0\}$ for some $a, b \in R_2$. Let $0 \neq i \in I$, we have $0 \neq (i, l)(1, a)(1, b) = (i, ab) \in I \times J$. Also we have $(1, a)(1, b) = (1, ab) \notin I \times J$ (Since $1 \notin J$). Since $I \times J$ is a weakly 2-absorbing ideal of $R$, either $(i, l)(1, a) = (i, a) \in I \times J$ or $(i, l)(1, b) = (i, b) \in I \times J$. That implies either $a \in J$ or $b \in J$. Hence $J = \{0\}$ is a prime ideal of $R_2$. Now we have to prove that I is a weakly prime ideal of $R$, that is not a prime ideal. Suppose $0 \neq ab \in I$ for some $a, b \in R_1$. We have $0 \neq (a, l)(b, l)(1, 0) = (ab, 0) \in I \times \{0\}$. Since $(a, l)(b, l) = (ab, l) \notin I \times \{0\}$ & $I \times \{0\}$ is a weakly 2-absorbing ideal of $R$, either $(a, l)(1, 0) = (a, 0) \in I \times \{0\}$ or $(b, l)(1, 0) = (b, 0) \in I \times \{0\}$. That implies either $a \in I$ or $b \in I$. Hence I is a weakly prime ideal of $R_1$.

(ii) $\Rightarrow$ (i): Suppose I is a weakly prime ideal of $R_1$ that is not a prime ideal & $J = \{0\}$ is a prime ideal of $R_2$. We have to prove that $I \times \{0\}$ is a weakly 2-absorbing ideal of $R$. Let $0 \neq (a, b)(c, d)(e, f) = (ace, bdf) \in I \times \{0\}$. Since I is a weakly prime ideal of $R_1$, we may assume

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that $a \in I$. Since $R_2$ has no nonzero divisors, we may assume that $d = 0$. Therefore $(a,b)(c,d) = (a,b)(c,0) = (ac,0) \in I \times \{0\}$. Hence $I \times \{0\}$ is a weakly 2-absorbing ideal of $R$.

4.4. Theorem. Let $R = R_1 \times R_2 \times R_3$ where $R_1, R_2, R_3$ are so-rings. Let $I_1$ be a proper ideal of $R_1$, $I_2$ be an ideal of $R_2$, and $I_3$ be an ideal of $R_3$ such that $I = I_1 \times I_2 \times I_3 \neq \{(0,0,0)\}$. Then the following statements are equivalent:

(i) $I = I_1 \times I_2 \times I_3$ is a weakly 2-absorbing ideal of $R$,

(ii) $I = I_1 \times R_2 \times R_3$ and $I_1$ is a weakly 2-absorbing ideal of $R_1$ or $I = I_1 \times I_2 \times R_3$ such that $I_2$ is a prime ideal of $R_2$ and $I_3$ is a prime ideal of $R_3$ or $I = I_1 \times R_2 \times I_3$ such that $I_1$ is a prime ideal of $R_1$ and $I_3$ is a prime ideal of $R_3$.

Proof. (i) $\Rightarrow$ (ii): Suppose $I = I_1 \times I_2 \times I_3$ is a weakly 2-absorbing ideal of $R$. Since $I$ is a weakly 2-absorbing ideal of $R$, $I_1$ is a weakly 2-absorbing ideal of $R_1$. If $I_2 = R_2$ and $I_3 = R_3$, then $I = I_1 \times R_2 \times R_3$. Suppose $I_2 \neq R_2$ & $I_3 = R_3$. i.e., $I = I_1 \times I_2 \times R_3$. Now we have to prove that $I_1$ is a prime ideal of $R_1$ and $I_2$ is a prime ideal of $R_2$. Let $a,b \in R_1$ such that $ab \in I_1$ and $c,d \in R_2$ such that $cd \in I_2$. Then $0 \neq (a,l,l)(1,cd,l)(b,l,l) = (ab,cd,l) \in I$. Now $(a,l,l)(b,l,l) = (ab,l,l) \in I$ (Since $I = I_1 \times I_2 \times R_3$ and $1 \notin I_2$). Since $I$ is a weakly 2-absorbing ideal of $R$, we have either $(a,l,l)(1,cd,l) = (ac,cd,l) \in I$ or $(1,cd,l)(b,l,l) = (bc,cd,l) \in I$. That implies either $a \in I_1$ or $b \in I_1$. Hence $I_1$ is a prime ideal of $R_1$. Similarly $0 \neq (ab,l,l)(1,c,d,l) = (ab,cd,l) \in I$. Now $(1,c,d,l)(1,d,l) \notin I$. Since $I = I_1 \times I_2 \times R_3$ & $1 \notin I_1$. Since $I$ is a weakly 2-absorbing ideal of $R$, we have either $(ab,l,l)(1,c,d,l) = (abc,c,d,l) \in I$ or $(ab,l,l)(1,d,l) = (ab,d,l) \in I$. That implies either $c \in I_2$ or $d \in I_2$. Hence $I_2$ is a prime ideal of $R_2$. Finally assume that $I_2 = R_2$ and $I_3 \neq R_3$ (i.e., $I = I_1 \times R_2 \times I_3$). By applying the above arguement, we conclude that $I_1$ is a prime ideal of $R_1$ and $I_3$ is a prime ideal of $R_3$.

(ii) $\Rightarrow$ (i): Suppose $I$ is one of the given three forms. Then by theorem 4.2., $I = I_1 \times I_2 \times I_3$ is a weakly 2-absorbing ideal of $R$.

5. CONCLUSION

In this paper we introduced the notion of weakly 2-absorbing ideals in so-rings and provided a counter example that proves the class of weakly 2-absorbing ideals is strictly wider than the class of all 2-absorbing ideals. Also we obtained the conditions under which a weakly 2-absorbing ideal is a 2-absorbing ideal. We considered this notation of weakly 2-absorbing ideals in the Cartesian product of so-rings and obtained various equivalent conditions on the weakly 2-absorbing ideals of Cartesian product of so-rings.

REFERENCES


Weakly 2-Absorbing Ideals of So-Rings


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