L-Cyclic Magma versus R-Cyclic Magma

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Abstract: A mapping \( \ast : X \times X \rightarrow X \) is a (binary) operation, and the pair \((X, \ast)\) is named as a Magma [1]. Magma with the property: \(x \ast(y \ast z) = z \ast(x \ast y) = y \ast(z \ast x)\) for all \(x,y,z\) in Magma is named as L-cyclic magma or with the property \((x \ast y) \ast z = (z \ast x) \ast y = (y \ast z) \ast x\) for all \(x,y,z\) in Magma is named as R-cyclic magma. In this paper, every result show only identical goal, that is, to prove when a L-cyclic magma becomes R-cyclic magma and vice versa.

Keywords: Magma, L-cyclic magma, R-cyclic magma, L-R-cyclic magma, L-identity, R-identity and cross cancellation law.

1. INTRODUCTION

The algebraic objects encountered in this chapter are sets with a binary operation defined on them. Andreas[1] introduced a term “magma” in his Ph.D., theses with entitle “Classification and Enumeration of finite semigroups”. Magma nothing but an algebraic structure with one binary operation on a nonempty set. Throughout this paper, we consider the magma with atleast any one of the property

i. \(x \ast(y \ast z) = z \ast(x \ast y) = y \ast(z \ast x)\) for all \(x,y,z\) in Magma

or

ii. \((x \ast y) \ast z = (z \ast x) \ast y = (y \ast z) \ast x\) for all \(x,y,z\) in Magma

The magma with first property is named as L-cyclic magma, with second property is named as R-cyclic magma. If it has both properties, then it is named as L-R-cyclic magma.

This paper contains two sections: In section 1, it contains the introduction and in section 2 shows the results when a L-cyclic magma becomes R-cyclic magma and vice versa.

2. L-CYCLIC MAGMA BECOMES R-CYCLIC MAGMA AND VICE VERSA

In this section contains all necessary and sufficient conditions of L-cyclic magma becomes R-cyclic magma by using additional property: “commutative, left cancellation, right cancellation right identity, left identity or idempotent” on it.

Result 2.1: Let \((S, \ast)\) be a commutative magma. Then \((S, \ast)\) is L-cyclic magma if and only if it is R-cyclic magma.

Proof:

Let \((S, \ast)\) be a L-cyclic magma \((S, \ast)\).

Consider, \((x \ast y) \ast z\)

By using commutative property on \((x \ast y) \ast z\), we get \((x \ast y) \ast z = z \ast(x \ast y)\)

By using cyclic property on \(z \ast(x \ast y)\), we get \(z \ast(x \ast y) = y \ast(z \ast x)\)

By using commutative property on \(y \ast(z \ast x)\), so \(y \ast(z \ast x) = (z \ast x) \ast y\)

Once again by using cyclic property on \(*((z \ast x))\), so \(*((z \ast x)) = x \ast(y \ast z)\)
By using commutative property on \( x \ast (y \ast z) \), so \( x \ast (y \ast z) = (y \ast z) \ast x \)

Thus \( (x \ast y) \ast z = (z \ast x) \ast y = (y \ast z) \ast x \) for all \( x, y, z \) in \( S \).

Conversely, Consider \( x \ast (y \ast z) \) in R-cyclic magma \((S,\ast)\).

Similarly by applying commutative and R-cyclic properties on \( x \ast (y \ast z) \), we have
\[
x \ast (y \ast z) = z \ast (x \ast y) = y \ast (z \ast x)
\]
for any \( x, y, z \) in \( S \).

**Result 2.2:** let \((S,\ast)\) be a magma with L-cancellation property. Then \((S,\ast)\) is L-cyclic magma if and only if it is R-cyclic magma.

**Proof:**

Since magma \((S,\ast)\) is L-cyclic magma, so \( x \ast (x \ast y) = y \ast (x \ast x) = x \ast (y \ast x) \), for any \( x, y \) in \( S \).

That is \( x \ast (x \ast y) = x \ast (y \ast x) \)

Since magma \((S,\ast)\) has L-cancellation property, so \( x \ast (x \ast y) = x \ast (y \ast x) \Rightarrow x \ast y = y \ast x \)

Thus, \( x \ast y = y \ast x \) for any \( x, y \) in \( S \).

Hence \((S,\ast)\) is a commutative magma.

By using result 2.1, \((S,\ast)\) is R-cyclic magma.

Conversely,

Since magma \((S,\ast)\) is R-cyclic magma,

for any \( x, y \) in \( S \), \( (x \ast x) \ast (x \ast y) = ((x \ast y) \ast x) \ast x \).

\[
= ((x \ast x) \ast y) \ast x \\
= ((y \ast x) \ast x) \ast x \\
= (x \ast (y \ast x)) \ast x \\
= (x \ast x) \ast (y \ast x)
\]

Since magma \((S,\ast)\) has L-cancellation property,

so \( (x \ast x) \ast (x \ast y) = (x \ast x) \ast (y \ast x) \Rightarrow x \ast y = y \ast x \)

Thus, \( x \ast y = y \ast x \) for any \( x, y \) in \( S \).

Hence \((S,\ast)\) is a commutative magma.

By using result 2.1, \((S,\ast)\) is R-cyclic magma.

**Result 2.3:** let \((S,\ast)\) be a magma with R-cancellation property. Then \((S,\ast)\) is L-cyclic magma if and only if it is R-cyclic magma.

**Proof:**

**Necessary Condition:**

Since magma \((S,\ast)\) is L-cyclic magma, for any \( x, y \) in \( S \),

\[
(x \ast y) \ast (x \ast x) = x \ast ((x \ast y) \ast x)
= x \ast (x \ast (x \ast y))
= x \ast (y \ast (x \ast x))
= x \ast (x \ast (y \ast x))
= (y \ast x) \ast (x \ast x)
\]

Since magma \((S,\ast)\) has R-cancellation property,

So \( (x \ast y) \ast (x \ast x) = (y \ast x) \ast (x \ast x) \Rightarrow x \ast y = y \ast x \)
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Thus, \( x \ast y = y \ast x \) for any \( x, y \) in \( S \).

Hence \((S,\ast)\) is a commutative magma.

By using result 2.1, \((S,\ast)\) is R-cyclic magma.

**Sufficient condition:**

Since magma \((S,\ast)\) is R-cyclic magma,

for any \( x, y \) in \( S \), \((x \ast y) \ast x = (x \ast x) \ast y = (y \ast x) \ast x \)

Since magma \((S,\ast)\) has R-cancellation property,

so \((x \ast y) \ast x = (y \ast x) \ast x \Rightarrow x \ast y = y \ast x \)

Thus, \( x \ast y = y \ast x \) for any \( x, y \) in \( S \).

Hence \((S,\ast)\) is a commutative magma.

By using result 2.1, \((S,\ast)\) is L-cyclic magma.

**Note:** From above two results, it is understood that every L-cyclic magma is R-cyclic magma and vice versa if magma with cancellation property.

**Result 2.4:** let \((S,\ast)\) be a magma with R-identity. Then \((S,\ast)\) is L-cyclic magma if and only if it is R-cyclic magma.

**Proof:**

Since \((S,\ast)\) magma has R-identity, so there exist an element \( e \) in \( S \), such that \( x \ast e = x \) for all \( x \) in \( S \).

**Necessary Condition:**

Since the magma \((S,\ast)\) has L-cyclic magma,

so it has L-cyclic property \( x \ast (y \ast z) = z \ast (x \ast y) = y \ast (z \ast x) \) for all \( x, y, z \) in \( S \).

Thus \( x \ast y = (x \ast e) \ast (y \ast e) \)

\[ = e \ast ((x \ast e) \ast y) \]
\[ = y \ast (e \ast (x \ast e)) \]
\[ = y \ast (y \ast (e \ast x)) \]
\[ = y \ast (x \ast (e \ast e)) \]
\[ = y \ast (x \ast e) \]
\[ = y \ast x \]

Thus the magma \((S,\ast)\) is commutative magma.

By using result 2.1, \((S,\ast)\) is R-cyclic magma.

**Sufficient Condition:**

Since the magma \((S,\ast)\) has R-cyclic magma,

so it has \((x \ast y) \ast z = (z \ast x) \ast y = (y \ast z) \ast x \) for all \( x, y, z \) in \( S \).

Thus \( x \ast y = (x \ast e) \ast (y \ast e) \)

\[ = ((y \ast e) \ast x) \ast e \]
\[ = (y \ast x) \ast e \]
\[ = y \ast x \]

Thus the magma \((S,\ast)\) is commutative magma.

By using result 2.1, \((S,\ast)\) is L-cyclic magma.
Result 2.5: let \((S, *)\) be a magma with \(L\)-identity. Then \((S, *)\) is \(L\)-cyclic magma if and only if it is \(R\)-cyclic magma.

Proof:
Since \((S, *)\) magma has \(L\)-identity, so there exist an element \(e\) in \(S\), such that \(e * x = x\) for all \(x\) in \(S\).

Necessary Condition:
Since the magma \((S, *)\) has \(L\)-cyclic magma, so it has \(L\)-cyclic property \(x * (y * z) = z * (x * y) = y * (z * x)\) for all \(x, y, z\) in \(S\).
Thus \(x * y = (e * x) * (e * y)\)
\[\begin{align*}
  & = y * ((e * x) * e) \\
  & = y * ((x * e)) \\
  & = e * (y * x) \\
  & = y * x
\end{align*}\]
Thus the magma \((S, *)\) is commutative magma.
By using result 2.1, \((S, *)\) is \(R\)-cyclic magma.

Sufficient Condition:
Since the magma \((S, *)\) has \(R\)-cyclic magma, so it has \(R\)-cyclic property \((x * y) * z = (z * x) * y = (y * z) * x\) for all \(x, y, z\) in \(S\).
Thus \(x * y = (e * x) * (e * y)\)
\[\begin{align*}
  & = ((e * y) * e) * x \\
  & = (y * e) * x \\
  & = (y * x) * e \\
  & = (e * y) * x \\
  & = y * x
\end{align*}\]
Thus the magma \((S, *)\) is commutative magma.
By using result 2.1, \((S, *)\) is \(L\)-cyclic magma.

Note: from above two results, it is easily show that \(L\)-cyclic magma is \(R\)-cyclic magma and vice versa if magma with identity.

Result 2.6: A cross cancelation magma \((S, *)\) with idempotent element \(e\). Then \((S, *)\) is \(L\)-cyclic magma if and only if it is \(R\)-Cyclic magma.

Proof:
Since the magma \((S, *)\) has a cross cancelation property, so \(x * y = y * z \Rightarrow x = z\) for any \(x, y, z\) in \(S\).

Necessary Condition:
Let \(e\) be an idempotent element of magma \((S, *)\).
So, \(e * e = e\)

Case 1:
since \(S\) has a cyclic property, so \(x * (e * e) = x * e\) for all \(x\) in \(S\).
\[\Rightarrow e * (x * e) = x * e\]
\[\Rightarrow\text{ since } S\text{ has a cross cancelation property, } s o x * e = x , \text{ for all } x \text{ in } S.\]
Thus idempotent element \(e\) is a right identity in magma \((S, *)\).
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**Case 2:**
Consider an element $e \ast x$
By using left identity of case 1, this is equal to $e \ast (e \ast x)$
By using L-cyclic property on $e \ast (e \ast x)$, we have $e \ast (e \ast x) = x \ast (e \ast e)$
Using idempotent property of $e$ on $x \ast (e \ast e)$, $x \ast (e \ast e) = x \ast e$
Thus, $e \ast (e \ast x) = x \ast e$
⇒ since $S$ has a cross cancelation property, so $e \ast x = x$, for all $x$ in $S$.
Thus idempotent element $e$ is a left identity in magma $(S, \ast)$.
Hence, the idempotent element $e$ is identity in $(S, \ast)$.
Next to show that commutative property of $(S, \ast)$.
Let $x, y$ are any two elements in magma $(S, \ast)$.
$x \ast y = (e \ast x) \ast (e \ast y)$
$= y \ast ((e \ast x) \ast e)$
$= e \ast (y \ast (e \ast x))$
$= e \ast (y \ast x)$ (since $e$ is an left identity of $S$)
$= y \ast x$ (since $e$ is an left identity of $S$)
Thus the magma $(S, \ast)$ is commutative magma
By using result 2.1, $(S, \ast)$ is R-cyclic magma.

**Sufficient Condition:**
Next to show that it is L-cyclic magma.
Consider $x \ast (y \ast z)$
By using commutative property on $x \ast (y \ast z)$, so we have $x \ast (y \ast z) = (y \ast z) \ast x$ Let $e$ be an idempotent element of magma $(S, \ast)$.
So, $e \ast e = e$

**Case 1:**
since $S$ has a cyclic property, so $(e \ast e) \ast x = e \ast x$ for all $x$ in $S$.
⇒ $(x \ast e) \ast e = e \ast x$
⇒ $(e \ast x) \ast e = e \ast x$, for all $x$ in.
⇒ since $S$ has a cross cancelation property, so $(e \ast x) = x$, for all $x$ in $S$.
Thus idempotent element $e$ is a left identity in magma $(S, \ast)$.

**Case 2:**
Consider and element $x \ast e$
By using left identity of case 1, this is equal to $(e \ast x) \ast e$.
By using R-cyclic property on $(e \ast x) \ast e$, we have $(e \ast x) \ast e = (e \ast e) \ast x$
Again twice by using left identity of case 1, we have $(e \ast e) \ast x = e \ast x$ & $e \ast x = x$
Thus, $x \ast e = x$
Next to show that $(S, \ast)$ is commutative magma.

**Commutative property:**
Let $x, y$ are any two elements in magma $(S, \ast)$.
\[
x * y = (e * x) * (e * y) = ((e * y) * e) * x = ((e * e) * y) * x = (e * y) * x = y * x
\]

Thus the magma \((S, *)\) is commutative magma.

By using result 2.1, \((S, *)\) is L-cyclic magma.

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