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Some New Classes of Heronian Mean Graphs

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Abstract: *In this paper, We discuss Heronian Mean labeling behavior of Caterpillar, Shadow graph, Middle graph, Total graph, Splitting graph and Duplication of a vertex. Also We investigate Heronian Mean labeling of Wheel Graph.*

Keywords: *Heronian Mean labeling, Shadow graph, Middle graph, Total graph, Splitting graph, Duplication, Wheel Graph.*

AMS Subject Classification: 05C78

1. INTRODUCTION

The graph considered here will be simple, finite and undirected graph without loops or parallel edges. For all detailed survey of graph labeling, we refer to J.A.Gallian [1]. Terms not defined here are used in the sense of Harary [2].

We shall make frequent references to the following definitions

Definition 1.1:

A graph **G=(V,E)** with p vertices and q edges is said to be a **Heronian Mean graph** if it is possible to label the vertices $\mathbf{x \in V}$ with distinct labels $\mathbf{f(x)}$ from **1,2,…,q+1** in such a way that when each edge $\mathbf{e = uv}$ is labeled with,

$$\mathbf{f(e = uv)} = \left\lceil \frac{\mathbf{f(u)} + \sqrt{\mathbf{f(u)f(v)}} + \mathbf{f(v)}}{\mathbf{3}} \right\rceil \mathbf{(OR)} \left\lfloor \frac{\mathbf{f(u)} + \sqrt{\mathbf{f(u)f(v)}} + \mathbf{f(v)}}{\mathbf{3}} \right\rfloor$$

then the edge labels are distinct. In this case **f** is called a **Heronian Mean labeling** of G.

Definition 1.2:

A tree which yields a path when its pendant vertices are removed is called a **Caterpillar**

Definition 1.3:

Let G be a connected graph and G' be the copy of G. Then Shadow graph $D_2(G)$ is obtained by joining each vertex u in G to the neighbours of the corresponding vertex u' in G'.

Definition 1.4:

The Middle graph M(G) of a graph G whose vertex set is V(G)∪E(G) and in which two vertices are adjacent if and only if either they are adjacent edges of G (or) one is a vertex of G and the other is an edge incident on it.

Definition 1.5:

The Total graph T(G) of graph G is the graph whose vertex set is V(G)∪E(G) and two vertices are adjacent whenever they are either adjacent (or) incident in G.

Definition 1.6:

The Splitting graph $S^{'}(G)$ is obtained by adding new vertex $v^{'}$ corresponding to each vertex v of G such that N(v)=N($v^{'}$), Where N(v) and N($v^{'}$) are the neighbourhood sets of v and $v^{'}$ resp.

Definition 1.7:

Duplication of a vertex v_k by a new edge $e = v_k{}^{'} v_k{}^{''}$ in a graph G produces a new graph $G^{'}$ such that N($v_k{}^{'}$)∩ N($v_k{}^{''}$)= v_k.

The notion of Heronian Mean labeling was introduced by S.S.Sandhya, E.Ebin Raja Merly and S.D.Deepa and the Heronian Mean labeling behavior of Path P_n, Cycle C_n, Star $K_{1,n}$, Triangular Snake T_n, Quadrilateral Snake Q_n , Comb , ladder $\boldsymbol{L_n}$**,** Step Ladder, Crown, Complete graph has been investigated in [5] and [6]. In this paper we contribute some new results for Heronian Mean labeling of graphs.

2. MAIN RESULTS

Theorem: 2.1

Let G be a graph obtained by attaching pendent edges to both sides of each vertex of a path P_n. Then G is a Heronian mean graph.

Proof:

Consider a graph G which is obtained by attaching pendant edges to both sides of each vertex of a path P_n. Let P_n be a path $u_1, u_2, u_3, \ldots\ldots u_n$. Let v_i and w_i be the pendant vertices adjacent to $u_i, 1 \le i \le n$.

Define a function $f: V(G) \rightarrow \{1,2,3, \ldots, q+1\}$ by $f(u_i) = 3i - 2, 1 \le i \le n.$

$$f(v_i) = 3i - 1, \qquad 1 \le i \le n.$$

$$f(w_i) = 3i, 1 \le i \le n.$$

Edges are labeled with, $f(u_i u_{i+1}) = 3i,\ 1 \le i \le n - 1$

$$f(u_i v_i) = 3i, 1 \le i \le n$$

$$f(u_i w_i) = 3i - 1, 1 \le i \le n$$

Hence G is a Heronian mean graph.

Example 2.2: A Heronian mean labeling of G with 15 vertices and 14 edges is given below

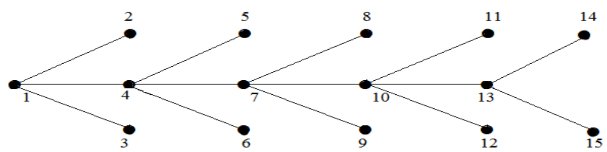

Figure:1

Theorem: 2.3

Let G be a graph obtained by attaching paths of length 0, 1, 2,……., $n - 1$ on both sides of each vertex of P_n ,then G is a Heronian mean graph.

Proof:

Let G be a graph obtained by attaching paths of length 0,1,2,……..$n - 1$ on both sides of each vertex of P_n. Let $u_{11} u_{22} \ldots \ldots u_{nn}$ are the vertices of P_n.

Define a function , $f: V(\,G) \rightarrow \{1,2,3, \ldots\ldots, q+1\}$ by

$$f(u_{ij}) = (i-1)^2 + j, 1 \le i \le n, 1 \le j \le 2i - 1$$

From the above labeling pattern ,we get distinct edge labels.

Thus f provides a Heronian mean labeling.

Example 2.4: A Heronian mean labeling of $\boldsymbol{P_6(P_1, 2P_2, 2P_3, 2P_4, 2P_5, 2P_6)}$ is given below.

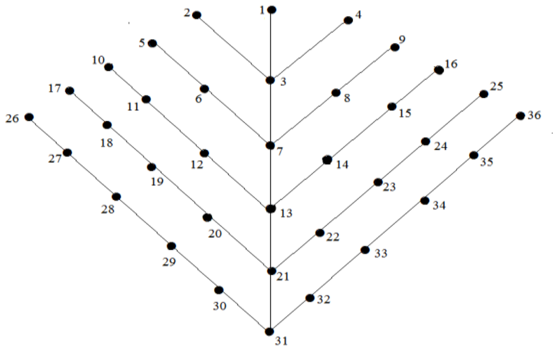

Figure:2

Theorem: 2.5

The graph $D_2(P_n)$ is a Heronian mean graph.

Proof:

Let $u_1u_2 \ldots \ldots u_n$ be the vertices of a Path P_n and $v_1v_2 \ldots \ldots v_n$ be the newly added vertices corresponding to the vertices $u_1u_2 \ldots \ldots u_n$ in order to obtain $D_2(P_n)$. Denoting $G = D_2(P_n)$, then $|V(G)| = 2n$ and $|E(G)| = 4(n-1)$.

Define a function, $f: V(\boldsymbol{G}) \to \{1,2, \ldots.. q+1\}$ by $f(u_1) = 1$, $f(u_2) = 3$,

$$f(u_i) = 4(i-1), 3 \le i \le n,$$

$$f(v_i) = \begin{cases} 4i-2, & 1 \le i \le n-1, \\ 4i-3, & i = n. \end{cases}$$

Edges are labeled with $f(u_iu_{i+1}) = 4i-3,\ 1 \le i \le n-1$,

$$f(v_iv_{i+1}) = 4i,\ 1 \le i \le n-1,$$

$$f(u_iv_{i+1}) = \begin{cases} 4i-1, & i = 1 \\ 4i-2, & 2 \le i \le n-1, \end{cases}$$

$$f(v_iu_{i+1}) = \begin{cases} 4i-2, & i = 1 \\ 4i-1, & 2 \le i \le n-1. \end{cases}$$

Hence $D_2(P_n)$ is a Heronian Mean graph.

Example 2.6: Shadow graph of Path P_5 and its Heronian mean labeling is shown in figure:3.

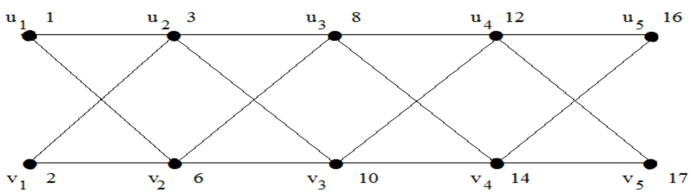

Figure:3

Theorem: 2.7

Middle **graph** of Path P_n **is** a Heronian mean graph.

Proof:

Let $u_1u_2 \ldots \ldots u_n$ be the vertices and $e_1e_2 \ldots \ldots e_{n-1}$ be the edges of Path P_n and $G = M(P_n)$ be the Middle graph of Path P_n. Here $|V(G)| = 2n-1$ and $|E(G)| = 3n-4$.

Define a function $f: V(G) \to \{1,2,3, \ldots, q+1\}$ by, $(u_1) = 1$, $f(u_i) = 3i-3, 2 \le i \le n$.

$$f(e_i) = 3i-1, 1 \le i \le n-1.$$

Edges are labeled with $f(u_i e_i) = 3i - 2,\ 1 \le i \le n-1,$

$f(u_i e_{i-1}) = 3i - 1, 2 \le i \le n-1,$

$f(e_i e_{i+1}) = 3i, 1 \le i \le n-2.$

Hence $M(P_n)$ is a Heronian Mean graph.

Example 2.8: Middle graph of Path P_5 and its Heronian mean labeling is shown in figure:4.

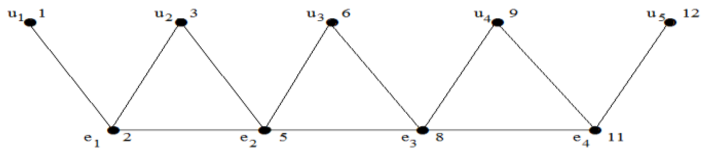

Figure:4

Theorem: 2.9

Total **graph** of Path P_n **is** a Heronian mean graph.

Proof:

Let $u_1 u_2 \dots \dots u_n$ be the vertices and $e_1 e_2 \dots \dots e_{n-1}$ be the edges of Path P_n and $G = T(P_n)$ be the Total graph of Path P_n. Here $|V(G)| = 2n - 1$ and $|E(G)| = 4n - 5$.

Define a function $f: V(G) \to \{1,2,3,\dots,q+1\}$ by $f(u_1) = 1$, $f(u_i) = 4i - 4,\ 2 \le i \le n.$

$f(e_1) = 3,\ f(e_i) = 4i - 2, 2 \le i \le n-1.$

Edges are labeled with $f(u_i u_{i+1}) = 4i - 2,\ 1 \le i \le n-1$

$f(u_i e_i) = 4i - 3,\ 1 \le i \le n-1$

$f(u_i e_{i-1}) = 4i - 1, 2 \le i \le n,$

$f(e_i e_{i+1}) = 4i, 1 \le i \le n-2.$

Hence $T(P_n)$ is a Heronian Mean graph.

Example 2.10: Total graph of Path P_6 and its Heronian mean labeling is shown in figure:5.

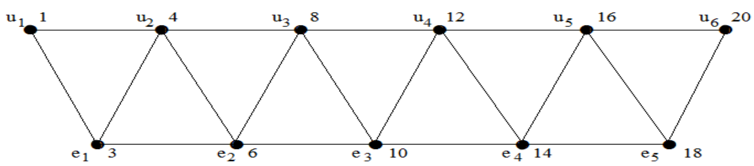

Figure:5

Theorem: 2.11

Splitting **graph** of Path P_n **is** a Heronian mean graph.

Proof:

Let $u_1 u_2 \dots \dots u_n$ be the vertices and $e_1 e_2 \dots \dots e_{n-1}$ be the edges of Path P_n . Let $v_1 v_2 \dots \dots v_n$ be the newly added vertices to form the Splitting graph of Path P_n . Here $|V(G)| = 2n$ and $|E(G)| = 3n - 3$.

Define a function $f: V(G) \to \{1,2,3,\dots,q+1\}$ by $(u_1) = 2$, $f(u_i) = 3i - 3,\ 2 \le i \le n.$

$f(v_i) = 3i - 2, 1 \le i \le n.$

Edges are labeled with, $f(v_i u_{i+1}) = \begin{cases} 3i - 2,\ i = 1 \\ 3i - 1,\ 2 \le i \le n-1, \end{cases}$

$$f(v_i v_{i+1}) = \begin{cases} 3i-1, & i=1 \\ 3i, & 2 \le i \le n-1, \end{cases}$$

$$f(v_i u_{i-1}) = \begin{cases} 3i-3, & i=2 \\ 3i-5, & 3 \le i \le n-1, \end{cases}$$

Hence $S^{'}(P_n)$ is a Heronian Mean graph.

Example 2.12: Splitting graph of Path P_6 and its Heronian mean labeling is shown in figure:6.

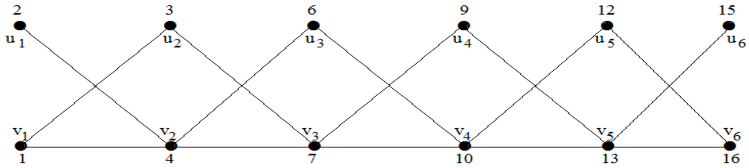

Figure:6

Theorem: 2.13

Duplicating each vertex by an edge in Path P_n **is** a Heronian mean graph.

Proof:

Let $u_1 u_2 \dots\dots u_n$ be the vertices of a Path P_n. Let G be the graph obtained by duplicating each vertex v_i of P_n by an edge $v_i^{'}, v_i^{''}$ at a time, $1 \le i \le n$. Here $|V(G)| = 3n$ and $|E(G)| = 4n-1$.

Define a function $f: V(G) \rightarrow \{1,2,3,\dots,q+1\}$ by $f(u_i) = 4i-2,\ 1 \le i \le n,$

$f(v_i^{'}) = 4i-3, 1 \le i \le n,$

$f(v_i^{''}) = 4i-1,\ 1 \le i \le n,$

Edges are labeled with $f(u_i u_{i+1}) = 4i\ ,\ 1 \le i \le n-1,$

$f(u_i v_i^{'}) = 4i-3\ ,\ 1 \le i \le n,$

$f(u_i v_i^{''}) = 4i-1,\ 1 \le i \le n,$

$f(v_i^{'}\ v_i^{''}) = 4i-2,\ 1 \le i \le n,$

Hence duplicating each vertex by an edge in Path P_n is a Heronian Mean graph.

Example 2.14: Duplicating each vertex by an edge in Path P_4 and its Heronian mean labeling is shown in figure:7.

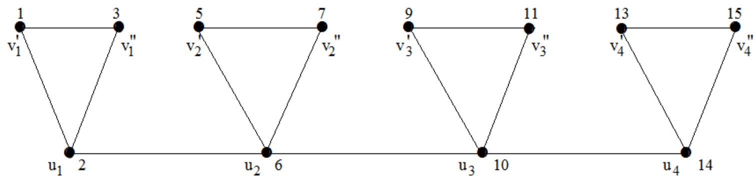

Figure:7

Theorem: 2.15

For $\mathbf{n} > 4$, Wheel Graph W_n is not a Heronian mean graph.

Proof:

For $n = 3$ and $n = 4$ clearly W_3 and W_4 are Heronian mean graphs. The labeling pattern of W_3 and W_4 is shown below.

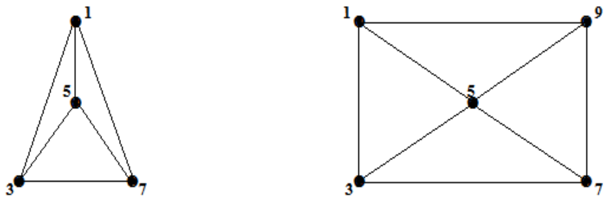

Figure:8

If $n > 4$, We have repetition of edge labels, which is not possible. Hence to get the edge label 1, We need a vertex u with label 1. There are four more vertices incident with u. This is not possible by remark 1.3[5].

Hence $W_n, n > 4$ is not a Heronian mean graph.

3. Conclusion

All graphs are not Heronian mean graphs. In this paper, we proved some more graphs which admits Heronian Mean Graphs. It is very interesting to investigate graphs which admits Heronian Mean Labeling. The derived results are demonstrated by means of sufficient illustrations which provide better understanding. It is possible to investigate more and more graphs and find results which admits Heronian Mean Labeling.

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