Square Multiplicative Labeling for Some Graphs

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Abstract: Let G be (p,q) graph. G is said to be a Square multiplicative labeling if there exists a bijection f:V(G) → {1,2,...,p} such that the induced function f*: E(G) → N given by f*(uv) = f(u)^2. f(v)^2 for every uv ∈ E(G) are all distinct. A graph which admits Square multiplicative labeling is called Square multiplicative graph.

Keywords: Strongly multiplicative graphs, Square multiplicative graphs, Double triangular snake, Chord, Two chords, Bistar Bn,n.

1. INTRODUCTION

Graph labelings were first introduced in the mid sixties. A labeling of a graph G is an assignment of labels to vertices or edges or both following certain rules [4]. In this paper we deal only finite, simple, connected and undirected graphs obtained through graph operations. Labeling of graphs plays an important role in application of graph theory in Neural Networks, Coding theory, Circuit Analysis etc. A useful survey on graph labeling by J.A.Gallian (2015) can be found in [1]. In most applications labels are positive or nonnegative integers. Beineke and Hedge [3] call a graph with p vertices strongly multiplicative if the vertices of G can be labelled with distinct integers 1,2,...,p such that the labels induced on the edges by the product of the end vertices are distinct. Triangular snakes [8] was proved by Murugesan, J.Shiama in Square Difference 3-Equitable Labeling of Some Graphs. Quadrilateral snakes [2] was proved by J.Shiama in Square Difference Labeling for Some Graphs. G is said to be a Square multiplicative labeling if there exists a bijection f: V(G) → {1,2,...,p} such that the induced function f*: E(G) → N given by f*(uv) = f(u)^2. f(v)^2 for every uv ∈ E(G) are all distinct.

Definition.1.1: G is said to be a Square multiplicative labeling if there exists a bijection f: V(G) → {1,2,...,p} such that the induced function f*: E(G) → N given by f*(uv) = f(u)^2. f(v)^2 for every uv ∈ E(G) are all distinct. A graph which admits Square multiplicative labeling is called Square multiplicative graph.

Definition.1.2: A double triangular snake [5] is a graph formed by two triangular snakes have a same path.

Definition.1.3: A chord [6] of a cycle Cn is an edge joining two non-adjacent vertices of cycle Cn.

Definition.1.4: Two chords of a cycle are twin chords (consecutive chords) if they form a triangle with an edge of the cycle Cn.

Definition.1.5: Bistar Bn,n [7] is the graph obtained by joining the center (apex) vertices of two copies of K1,n by an edge.

2. SQUARE MULTIPLICATIVE LABELING

Theorem.2.1
Every Cycle with one chord is a Square multiplicative graph.
Proof:
Let $G$ be the cycle $C_n$ with one chord and $v_1, v_2, \ldots, v_n$ be the consecutive of $C_n$ arranged in the clockwise direction. Let $e = v_1v_i$ be the chord vertices of cycle.

Let us define labeling $f: V(G) \rightarrow \{1, 2, \ldots, n\}$ as follows

- $f(v_1) = 1$;
- $f(v_i) = p_1$; Where $p_1$ is the highest prime number such that $p_1 \leq n$.
- $f(v_n) = p_2$; Where $p_2$ is the second highest prime number such that $1 < p_2 < p_1 \leq n$.

Now label the remaining vertices starting from $v_2$ consecutively in clockwise direction from the set $\{1, 2, \ldots, n\}$ except 1. $p_1$ & $p_2$ as these numbers are already used as labels.

Theorem 2.2
Every Cycle with twin chords are square multiplicative graphs.

Proof: Let $G$ be the cycle $C_n$ with twin chords. Let $v_1, v_2, \ldots, v_n$ be the consecutive vertices of $C_n$ arranged in the clockwise direction. Let $e_1 = v_1v_i$, $e_2 = v_1v_{i+1}$ be two chords of $C_n$.

Let us define labeling $f: V(G) \rightarrow \{1, 2, \ldots, n\}$ as follows

- $f(v_1) = 1$;
- $f(v_i) = p_1$; Where $p_1$ is the highest prime number such that $p_1 \leq n$.
- $f(v_{i+1}) = p_2$; Where $p_2$ is the second highest prime number such that $1 < p_2 < p_1 \leq n$.
- $f(v_n) = p_3$; Where $p_3$ is the third highest prime number such that $1 < p_3 < p_2 < p_1 \leq n$.

Now label the remaining vertices starting from $v_2$ consecutively in clock wise direction from the set $\{1, 2, \ldots, n\}-\{1, p_1, p_2, p_3\}$.

Theorem 2.3
Quadrilateral snakes are square multiplicative graphs.

Proof:
Consider $S_{4,n}$ with vertices labeled.

We define a function $f: V(G) \rightarrow \{1, 2, 3, \ldots, 3n + 1\}$ by
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\[ f(u_i) = 3i - 2; \ i = \{1,2,3,\ldots,n + 1\} \]
\[ f(v_i) = 3i; \ i = \{1,2,\ldots,n\} \]
\[ f(w_i) = 3i - 1; \ i = \{1,2,\ldots,n\} \]

And the induced function \( f^*: E(G) \rightarrow N \) defined by \( f^*(uv) = f(u)^2 f(v)^2 \) for every \( uv \in E(G) \) are all distinct. Hence \( S_{4,n} \) are square multiplicative graphs.

\[ \text{Figure 3} \]

**Theorem 2.4**

Triangular snakes are square multiplicative graph.

**Proof:**

Consider \( T_2 \) with vertices labeled.

We define a function \( f: V(G) \rightarrow \{1,2,\ldots,2n + 1\} \) by

\[ sf(u_i) = 2i - 1; \ i = \{1,2,\ldots,n + 1\} \]
\[ f(v_i) = 2i; \ i = \{1,2,3,\ldots,n\} \]

And the induced function \( f^*: E(G) \rightarrow N \) defined by \( f^*(uv) = f(u)^2 f(v)^2 \) for every \( uv \in E(G) \) are all distinct. Hence \( T_2 \) are square multiplicative graphs.

\[ \text{Figure 4} \]

**Theorem 2.5**

All double triangular snake (\( 2\Delta_k - \text{snake} \)) are square multiplicative graphs.

**Proof:**

Let \( G \) denote the \( 2\Delta_k - \text{snake} \) with \( p \) vertices. Let the vertices of \( G \) be

\[ V(G) = \{u_i / 1 \leq i \leq k\} \cup \{v'_i / 1 \leq i \leq k, 1 \leq j \leq 2\} \] and
\[ E(G) = \{u_i u_{i+1} / 1 \leq i \leq k\} \cup \{u_i v'_i / 1 \leq i \leq k, 1 \leq j \leq 2\} \cup \{u_{i+1} v'_i / 1 \leq i \leq k, 1 \leq j \leq 2\} \]

Here \( |V(G)| = 3k + 1 \).

Define \( f: V(G) \rightarrow \{1,2,\ldots,3k + 1\} \) by

\[ f(u_1) = 1, \]
\[ f(u_i) = f(u_{i-1}) + 3; \ 2 \leq i \leq k + 1 \]
\[ f(v'_i) = f(u_i) + j; \ 1 \leq i \leq k, 1 \leq j \leq 2 \]
One can easily verify that $f$ so defined is injective. With the above defined vertex label, induced edge labels can be arranged in increasing order.

Theorem 2.6

$2m\Delta_k - \text{snake}$ with $k$-blocks are square multiplicative graphs.

Proof:

Let $G$ denote the $2m\Delta_k - \text{snake}$ with $p$ vertices. Let the vertices of $G$ be $V(G) = \{u_i / 1 \leq i \leq k + 1\} \cup \{v'_i / 1 \leq i \leq k, 1 \leq j \leq 2m\}$ and $E(G) = k(2m + 1) + 1$.

Define $f: V(G) \to \{1, 2, ..., k(2m + 1) + 1\}$ by

- $f(u_1) = 1,$
- $f(u_i) = f(u_{i-1}) + 2m + 1;\ 2 \leq i \leq k + 1$
- $f(v'_i) = f(u_i) + j; 1 \leq i \leq k, 1 \leq j \leq 2m$

One can easily verify that $f$ so defined is injective. With the above defined vertex label, induced edge labels can be arranged in increasing order.

Theorem 2.7

Bistar $B_{n,n}$ is a square multiplicative graph.

Proof:

Consider $B_{n,n}$ with vertex set $\{u, v, u_i, v_i, 1 \leq i \leq n\}$ where $u_i, v_i$ pendant vertices are. If $G = B_{n,n}$ then $|V(G)| = 2n + 2$ and $|E(G)| = 2n + 1$. We define vertex labeling $f: V(G) \to \{1, 2, ..., 2n + 2\}$ as follows $f(u) = 1,$
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\[ f(v) = 2n + 1, \]
\[ f(u_i) = 1 + i; \ 1 \leq i \leq n \]
\[ f(v_i) = n + 1 + i; \ 1 \leq i \leq n - 1 \]
\[ f(v_n) = 2n + 2 \]

The induced function \( f^*: E(G) \rightarrow N \) defined by \( f^*(uv) = f(u)^2 \cdot f(v)^2 \) for every \( uv \in E(G) \) are all distinct. Hence Bistar \( B_{n,n} \) is a square multiplicative graph.

Square multiplicative labelling of the graph \( B_{5,5} \) is shown in the Figure 7.

![Figure 7](image)

3. CONCLUSION

Square multiplicative labeling for cycle with one chord, cycle with twin chords, quadrilateral triangles, triangular snakes, \( 2m\Delta \) snake, double triangular snakes, Bistar \( B_{n,n} \) have been discussed in this paper.

REFERENCES


