On Almost Supra N-continuous Function

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Abstract: In this paper, we introduce the concept of almost supra N-continuous function and investigated the relationship of this functions with other functions. Also we have defined mildly supra N-normal space.

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1. INTRODUCTION


In this paper, we bring out the concept of almost supra N-continuous function and investigated the relationship with other functions in supra topological spaces. Also a new type of normal space called mildly supra N-normal space is also defined and its properties are investigated.

2. PRELIMINARIES

Definition 2.1[3]
A subfamily $\mu$ of $X$ is said to be supra topology on $X$ if
i) $X, \varphi \in \mu$

ii) If $A_i \in \mu$, $i \in J$ then $\cup A_i \in \mu$

$(X, \mu)$ is called supra topological space.

The element of $\mu$ are called supra open sets in $(X, \mu)$ and the complement of supra open set is called supra closed sets and it is denoted by $\mu^c$.

Definition 2.2[3]
The supra closure of a set $A$ is denoted by $cl^H(A)$, and is defined as supra $cl(A) = \cap\{B : B \text{ is supra closed and } A \subseteq B\}$.

The supra interior of a set $A$ is denoted by $int^H(A)$, and is defined as supra $int(A) = \cup\{B : B \text{ is supra open and } A \supseteq B\}$.

Definition 2.3[3]
Let $(X, \tau)$ be a topological space and $\mu$ be a supra topology on $X$. We call $\mu$ be a supra topology associated with $\tau$, if $\tau \subseteq \mu$.

Definition 2.4
A subset \( A \) of a space \( X \) is called

(i) supra semi-open set[2], if \( A \not\subseteq \text{cl}^H(\text{int}^H(A)) \).

(ii) supra \( \alpha \)-open set[1], if \( A \not\subseteq \text{int}^H(\text{cl}^H(\text{int}^H(A))) \).

(iii) supra \( \Omega \)-closed set[5], if \( \text{sc}^H(A) \not\subseteq \text{int}^H(U) \). whenever \( A \not\subseteq U, \) \( U \) is supra open set.

(iv) supra \( N \)-closed set[7], if \( \Omega \text{cl}^H(A) \not\subseteq U, \) whenever \( A \not\subseteq U, \) \( U \) is supra \( \alpha \)-open set.

(v) supra regular open[10], if \( A=\text{int}^H\text{cl}^H(A) \)

The complement of the above mentioned sets are their respective open and closed sets and vice-versa.

**Definition 2.5** A map \( f:(X, \tau) \to (Y, \sigma) \) is said to be

(i) supra \( N \)-continuous [8] if \( f^{-1}(V) \) is supra \( N \)-closed in \( (X, \tau) \) for every supra closed set \( V \) of \( (Y, \sigma) \).

(ii) supra \( N \)- irresolute[8] if \( f^{-1}(V) \) is supra \( N \)-closed in \( (X, \tau) \) for every supra \( N \)-closed set \( V \) of \( (Y, \sigma) \).

(iii) perfectly supra \( N \)-continuous[10] if \( f^{-1}(V) \) is supra clopen in \( (X, \tau) \) for every supra \( N \)-closed set \( V \) of \( (Y, \sigma) \).

(iv) Strongly supra \( N \)-continuous[10] if \( f^{-1}(V) \) is supra closed in \( (X, \tau) \) for every supra \( N \)-closed set \( V \) of \( (Y, \sigma) \).

(v) perfectly contra supra \( N \)- irresolute[9] if \( f^{-1}(V) \) is supra \( N \)-closed and supra \( N \)-open in \( (X, \tau) \) for every supra \( N \)-open set \( V \) of \( (Y, \sigma) \).

(vi) Contra supra \( N \)- irresolute[9], if \( f^{-1}(V) \) is supra \( N \)-closed in \( (X, \tau) \) for every supra \( N \)-open set \( V \) of \( (Y, \sigma) \).

(vii) Almost contra supra \( N \)- continuous[9], if \( f^{-1}(V) \) is supra \( N \)-closed in \( (X, \tau) \) for every supra regular open set \( V \) of \( (Y, \sigma) \).

**Definition 2.6[11]** A Space \( (X, \tau) \) is said to be

(i) supra \( N \)-normal if for any pair of disjoint supra closed sets \( A \) and \( B \), there exist disjoint supra \( N \)-open sets \( U \) and \( V \) such that \( A \subset U \) and \( B \subset V \).

(ii) weakly supra \( N \)-normal if for any pair of disjoint supra \( N \)-closed sets \( A \) and \( B \), there exist disjoint supra open sets \( U \) and \( V \) such that \( A \subset U \) and \( B \subset V \).

3. **ALMOST SUPRA N-CONTINUOUS FUNCTION**

**Definition 3.1** A map \( f:(X, \tau) \to (Y, \sigma) \) is called Almost supra continuous function if \( f^{-1}(V) \) is supra open set in \( (X, \tau) \) for every supra regular open set \( V \) of \( (Y, \sigma) \).

**Definition 3.2** A map \( f:(X, \tau) \to (Y, \sigma) \) is called Almost supra \( N \)-continuous function if \( f^{-1}(V) \) is supra \( N \)-open in \( (X, \tau) \) for every supra regular open set \( V \) of \( (Y, \sigma) \).

**Theorem 3.3** For a function \( f:(X, \tau) \to (Y, \sigma) \), the following are equivalent:

i) \( f \) is almost supra \( N \)-continuous.

ii) \( f^{-1}(V) \) is supra \( N \)-closed in \( X \) for every supra regular closed set \( V \) of \( Y \).

iii) \( f^{-1}(\text{cl}^H\text{int}^H(V)) \) is supra \( N \)-closed in \( X \), for every supra closed set \( V \) of \( Y \).
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iv) \( f^{-1}(\text{int}^{\mu} \text{cl}^{\mu}(V)) \) is supra N-open in X, for every supra open set V of Y.

Proof

(i) \( \Rightarrow \) (ii) Let V be supra regular closed set in Y. Then Y-V is supra regular open set in Y. Since f is almost supra N-continuous, \( f^{-1}(Y-V) = X-f^{-1}(V) \) is supra N-open in X. Hence \( f^{-1}(V) \) is supra N-closed in X.

(ii) \( \Rightarrow \) (iii) Let V be supra closed set in Y. Then V=\( \text{cl}^{\mu} \text{int}^{\mu}(V) \) is supra regular closed set in Y, then by hypothesis, \( f^{-1}(\text{cl}^{\mu} \text{int}^{\mu}(V)) \) is supra N-closed in X.

(iii) \( \Rightarrow \) (iv) Let V be supra open set in Y. Then V=\( \text{int}^{\mu} \text{cl}^{\mu}(V) \) is supra regular open set in Y. Then Y-\( \text{int}^{\mu} \text{cl}^{\mu}(V) \) is supra regular closed set in Y. Then by hypothesis, \( f^{-1}(Y-\text{int}^{\mu} \text{cl}^{\mu}(V)) = X-f^{-1}(\text{int}^{\mu} \text{cl}^{\mu}(V)) \) is supra N-closed in X. Hence \( f^{-1}(\text{int}^{\mu} \text{cl}^{\mu}(V)) \) is supra N-open in X.

(iv) \( \Rightarrow \) (i) Let V be supra open set in Y. Then V=\( \text{int}^{\mu} \text{cl}^{\mu}(V) \) is supra regular open set and every regular open set is open set in Y. Then by hypothesis, \( f^{-1}(\text{int}^{\mu} \text{cl}^{\mu}(V)) = f^{-1}(V) \) is supra N-open in X. Hence f is almost supra N-continuous.

Theorem 3.4 Every supra N-continuous function is almost supra N-continuous function.

Proof Let f:(X, \( \tau \)) \( \rightarrow \) (Y, \( \sigma \)) be a supra N-continuous function. Let V be supra regular open set in (Y,\( \sigma \)). Then V is supra open set in (Y,\( \sigma \)), since every supra regular open set is supra open set. Since f is supra N-continuous function \( f^{-1}(V) \) is both supra N-open in (X, \( \tau \)). Therefore f is almost supra N-continuous function. The converse of the above theorem need not be true. It is shown by the following example.

Example 3.5 Let X=Y={a, b, c} and \( \tau = \{ X, \varphi, \{a \}, \{a, b \}\} \), \( \sigma = \{ Y, \varphi, \{a \}, \{b \}, \{a, b \}, \{b, c \}\} \). N-open set in (X, \( \tau \)) are \{X, \varphi, \{a \}, \{b \}, \{a, b \}, \{b, c \}\}. N-open set in (Y,\( \sigma \)) are \{Y, \varphi, \{a \}, \{b \}, \{a, b \}, \{b, c \}\}. f:(X, \( \tau \)) \( \rightarrow \) (Y, \( \sigma \)) be the function defined by f(a)=b, f(b)=c, f(c)=a. Here f is almost supra N-continuous but not supra N-continuous, since V={a, b} is supra open in (Y, \( \sigma \)) but \( f^{-1}(\{a, b\}) = \{b, c\} \) is not supra N-open set in (X, \( \tau \)).

Theorem 3.6 Every strongly supra N-continuous function is almost supra N-continuous function.

Proof Let f:(X, \( \tau \)) \( \rightarrow \) (Y, \( \sigma \)) be a strongly supra N-continuous function. Let V be supra regular open set in (Y,\( \sigma \)), then V is supra N-open set in (Y,\( \sigma \)), since every supra regular open set is supra open set and every supra open set is supra N-open set. Since f is strongly supra N-continuous function, then \( f^{-1}(V) \) is supra N-open in (X, \( \tau \)). Therefore f is Almost supra N-continuous function.

The converse of the above theorem need not be true. It is shown by the following example.

Example 3.7 Let X=Y={a, b, c} and \( \tau = \{ X, \varphi, \{a \}, \{a, b \}\} \), \( \sigma = \{ Y, \varphi, \{a \}, \{b \}, \{a, b \}, \{b, c \}\} \). N-open set in (X, \( \tau \)) are \{X, \varphi, \{a \}, \{b \}, \{a, b \}, \{b, c \}\}. N-open set in (Y,\( \sigma \)) are \{Y, \varphi, \{a \}, \{b \}, \{a, b \}, \{b, c \}\}. f:(X, \( \tau \)) \( \rightarrow \) (Y, \( \sigma \)) be the function defined by f(a)=b, f(b)=c, f(c)=a. Here f is almost supra N-continuous but not strongly supra N-continuous, since V={a, b} is supra N-open in (Y, \( \sigma \)) but \( f^{-1}(\{a, b\}) = \{b, c\} \) is not supra open set in (X, \( \tau \)).

Theorem 3.8 Every perfectly supra N-continuous function is almost supra N-continuous function.

Proof Let f:(X, \( \tau \)) \( \rightarrow \) (Y, \( \sigma \)) be a perfectly supra N-continuous function. Let V be supra regular open set in (Y,\( \sigma \)), then V is supra N-open set in (Y,\( \sigma \)), since every supra regular open set is supra open set and every supra open set is supra N-open set. Since f is perfectly supra N-
continuous function, then $f^{-1}(V)$ is supra clopen in $(X,\tau)$, then $f^{-1}(V)$ is supra N-clopen in $(X,\tau)$, implies $f^{-1}(V)$ is supra N-open in $(X,\tau)$. Therefore $f$ is Almost supra N-continuous function.

The converse of the above theorem need not be true. It is shown by the following example.

**Example 3.9** Let $X=Y=\{a,b,c\}$ and $\tau=\{X,\varphi,\{a\},\{a,b\}\}, \sigma=\{Y,\varphi,\{a\},\{b\},\{a,b\}\}$. N-open set in $(X,\tau)$ are $\{X,\varphi,\{a\},\{a,b\}\}$. N-open set in $(X,\sigma)$ are $\{Y,\varphi,\{a\},\{b\}\}$. f $(X,\tau)\rightarrow (Y,\sigma)$ be the function defined by $f(a)=b, f(b)=c, f(c)=a$. Here $f$ is almost supra N-continuous but not perfectly supra N-continuous, since $V=\{a,b\}$ is supra N-open in $(Y,\sigma)$ but $f^{-1}(\{a,b\})=\{b,c\}$ is not supra clopen set in $(X,\tau)$.

**Theorem 3.10** Every almost supra continuous function is almost supra N-continuous function.

**Proof** Let $f:(X,\tau)\rightarrow (Y,\sigma)$ be a almost supra continuous function. Let $V$ be supra regular open set in $(Y,\tau)$. Since $f$ is almost supra continuous function, then $f^{-1}(V)$ is supra open in $(X,\tau)$, implies $f^{-1}(V)$ is supra N-open in $(X,\tau)$. Therefore $f$ is almost supra N-continuous function.

The converse of the above theorem need not be true. It is shown by the following example.

**Example 3.11** Let $X=Y=\{a,b,c\}$ and $\tau=\{X,\varphi,\{a\},\{a,b\}\}, \sigma=\{Y,\varphi,\{a\},\{b\},\{a,b\}\}$.
N-open set in $(X,\tau)$ are $\{X,\varphi,\{a\},\{a,b\}\}$.
N-open set in $(Y,\sigma)$ are $\{Y,\varphi,\{a\},\{b\}\}$.
$f:(X,\tau)\rightarrow (Y,\sigma)$ be the function defined by $f(a)=b, f(b)=c, f(c)=a$. Here $f$ is almost supra N-continuous but not almost supra continuous, since $V=\{a\}$ is supra regular open in $(Y,\sigma)$ but $f^{-1}(\{a\})=\{c\}$ is not supra open set in $(X,\tau)$.

**Theorem 3.12** If $f:(X,\tau)\rightarrow (Y,\sigma)$ is supra N- irresolute and $g:(Y,\sigma)\rightarrow (Z,\eta)$ is almost supra N-continuous then $g\circ f:(X,\tau)\rightarrow (Z,\eta)$ is almost supra N-continuous.

**Proof** Let $V$ be supra regular open set in $Z$. Since $g$ is almost supra N-continuous, then $g^{-1}(V)$ is supra N-open set in $Y$.
Since $f$ is supra N- irresolute, then $f^{-1}(g^{-1}(V))$ is supra N-open in $X$. Hence $g\circ f$ is almost supra N-continuous.

**Theorem 3.13** If $f:(X,\tau)\rightarrow (Y,\sigma)$ is strongly supra N-continuous and $g:(Y,\sigma)\rightarrow (Z,\eta)$ is almost supra N-continuous then $g\circ f:(X,\tau)\rightarrow (Z,\eta)$ is almost supra N-continuous.

**Proof** Let $V$ be supra regular open set in $Z$. Since $g$ is almost supra N-continuous, then $g^{-1}(V)$ is supra N-open set in $Y$.
Since $f$ is strongly supra N-continuous, then $f^{-1}(g^{-1}(V))$ is supra open in $X$. Implies $f^{-1}(g^{-1}(V))$ is supra N-open in $X$. Hence $g\circ f$ is almost supra N-continuous.

**Theorem 3.14** If $f:(X,\tau)\rightarrow (Y,\sigma)$ is contra supra N- irresolute and $g:(Y,\sigma)\rightarrow (Z,\eta)$ is almost contra supra N-continuous then $g\circ f:(X,\tau)\rightarrow (Z,\eta)$ is almost supra N-continuous.

**Proof** Let $V$ be supra regular open set in $Z$. Since $g$ is almost contra supra N-continuous, then $g^{-1}(V)$ is supra N-closed set in $Y$.
Since $f$ is contra supra N- irresolute, then $f^{-1}(g^{-1}(V))$ is supra N-open in $X$. Hence $g\circ f$ is almost supra N-continuous.

**Definition 3.15** A space $X$ is said to be mildly supra $N$-normal if for every pair of disjoint supra regular closed sets $A$ and $B$ of $X$, there exist disjoint supra N-open sets $U$ and $V$ such that $A\subset U$ and $B\subset V$.

**Theorem 3.16** Every supra normal space is mildly supra $N$-normal.

**Proof** Let $A$ and $B$ be disjoint supra regular closed sets of $X$, then $A$ and $B$ are disjoint supra closed sets of $X$, since every supra regular closed set is supra closed set. Since $X$ is supra normal, there exist disjoint supra open sets $U$ and $V$ such that $A\subset U$ and $B\subset V$. Since every supra open set is supra N-open set, then $U$ and $V$ are disjoint supra N-open sets. Hence $X$ is mildly supra $N$-normal.

The converse of the above theorem need not be true. It is shown by the following example.
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Example 3.17 Let \( X=\{a, b, c, d\} \) and \( \tau = \{X, \varnothing, \{a\}, \{a, b\}, \{a, b, c\}, \{a, b, c, d\}\} \) supra N-open sets in \( (X, \tau) \) are \( \{X, \varnothing, \{a\}, \{a, b\}, \{a, b, c\}, \{a, b, c, d\}\} \). Here \( (X, \tau) \) is mildly supra N-normal but not supra normal, since \( A=\{a, b\} \) and \( B=\{d\} \) is supra closed in \( (X, \tau) \) but \( A \) and \( B \) is not contained in disjoint supra open sets.

Theorem 3.18 Every supra N-normal space is mildly supra N-normal.

Proof Let \( A \) and \( B \) be disjoint supra regular closed sets of \( X \), then \( A \) and \( B \) are disjoint supra closed sets of \( X \), since every supra regular closed set is supra closed set. Since \( X \) is supra N-normal, there exist disjoint supra N-open sets \( U \) and \( V \) such that \( A \subseteq U \) and \( B \subseteq V \). Hence \( X \) is mildly supra N-normal.

The converse of the above theorem need not be true. It is shown by the following example.

Example 3.19 Let \( X=\{a, b, c, d\} \) and \( \tau = \{X, \varnothing, \{a\}, \{a, b\}, \{a, b, c\}, \{a, b, c, d\}\} \) supra N-open sets in \( (X, \tau) \) are \( \{X, \varnothing, \{a\}, \{a, b\}, \{a, b, c\}, \{a, b, c, d\}\} \). Here \( (X, \tau) \) is mildly supra N-normal but not supra N-normal, since \( A=\{a, b\} \) and \( B=\{d\} \) is supra closed in \( (X, \tau) \) but \( A \) and \( B \) is not contained in disjoint supra N-open sets.

Theorem 3.20 Every weakly supra N-normal space is mildly supra N-normal.

Proof Let \( A \) and \( B \) be disjoint supra regular closed sets of \( X \), then \( A \) and \( B \) are disjoint supra closed sets and hence supra N-closed sets of \( X \), since every supra regular closed set is supra closed set. Since \( X \) is weakly supra N-normal, there exist disjoint supra open sets \( U \) and \( V \) such that \( A \subseteq U \) and \( B \subseteq V \). Since every supra open set is supra N-open set, \( U \) and \( V \) are supra N-open sets. Hence \( X \) is mildly supra N-normal.

The converse of the above theorem need not be true. It is shown by the following example.

Example 3.21 Let \( X=\{a, b, c, d\} \) and \( \tau = \{X, \varnothing, \{a\}, \{a, b\}, \{a, b, c\}, \{a, b, c, d\}\} \) supra N-open sets in \( (X, \tau) \) are \( \{X, \varnothing, \{a\}, \{a, b\}, \{a, b, c\}, \{a, b, c, d\}\} \). Here \( (X, \tau) \) is mildly supra N-normal but not weakly supra N-normal, since \( A=\{a, b\} \) and \( B=\{d\} \) is supra N-closed in \( (X, \tau) \) but \( A \) and \( B \) is not contained in disjoint supra open sets.

Theorem 3.20 If \( f:(X, \tau) \to (Y, \sigma) \) be supra N-open map, almost supra N-continuous surjective, and if \( X \) is weakly supra N-normal, then \( Y \) is mildly supra N-normal.

Proof Let \( A \) and \( B \) be disjoint regular closed set in \( Y \). Since \( f \) is almost supra N-continuous, then \( f^{-1}(A) \) and \( f^{-1}(B) \) are supra N-closed set in \( X \). Since \( X \) is weakly supra N-normal, there exist disjoint supra open set \( U \) and \( V \) in \( X \) such that \( f^{-1}(A) \subseteq U \) and \( f^{-1}(B) \subseteq V \). Since \( f \) is supra N-closed map, \( f(U) \) and \( f(V) \) are disjoint supra N-open set in \( Y \). Hence \( Y \) is mildly supra N-normal.

4. CONCLUSION
We introduced the concept of almost supra N-continuous function on supra topological space and investigated its relationship with other functions. Also a new type of normal space called mildly supra N-normal space was introduced and studied some of its properties.

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