Connected Total Dominating Sets and Connected Total Domination Polynomials of Gem Graphs

A. Vijayan
Associate Professor,
Department of Mathematics,
Nesamony Memorial Christian College
Marthandam, Tamil Nadu, India
dravijayan@gmail.com

T. Anitha Baby
Assistant Professor
Department of Mathematics
V.M.C.S.I.Polytechnic College
Viricode, Marthandam, Tamil Nadu, India.
anithasteve@gmail.com

Abstract: Let G = (V, E) be a simple graph. A set S of vertices in a graph G is said to be a total dominating set if every vertex v ∈ V is adjacent to an element of S. A total dominating set S of G is called a connected total dominating set if the induced subgraph <S> is connected. In this paper, we study the concept of connected total domination polynomials of the Gem graph G_n. The connected total domination polynomial of a graph G of order n is the polynomial

\[ D_{ct}(G, x) = \sum_{i=\gamma_{ct}(G)}^{n} d_{ct}(G, i)x^i \]

where \( d_{ct}(G, i) \) is the number of connected total dominating sets of G of size i and \( \gamma_{ct}(G) \) is the connected total domination number of G. We obtain some properties of \( D_{ct}(G_n, x) \) and their coefficients. Also, we obtain the recursive formula to derive the connected total dominating sets of the Gem graph G_n.

Keywords: Gem graph, connected total dominating set, connected total domination number, connected total domination polynomial.

1. INTRODUCTION

Let G = (V, E) be a simple graph of order \( |V| = n \). A set S of vertices in a graph G is said to be a dominating set if every vertex v ∈ V is either an element of S or is adjacent to an element of S.

A set S of vertices in a graph G is said to be a total dominating set if every vertex v ∈ V is adjacent to an element of S. A total dominating set S of G is called a connected total dominating set if the induced subgraph <S> is connected. The minimum cardinality of a connected total dominating set S of G is called the connected total domination number and is denoted by \( \gamma_{ct}(G) \).

Let \( G_n \) be a Gem graph with \( n + 2 \) vertices. In the next section, we construct the families of the connected total dominating sets of \( G_n \) by recursive method. In section 3, we use the results obtained in section 2 to study the connected total domination polynomials of the Gem graph G_n. As usual, we use \( \binom{n}{i} \) for the combination n to i.

2. CONNECTED TOTAL DOMINATING SETS OF A GEM GRAPH \( G_n \)

Gem graph [5] is a graph obtained by joining an additional vertex u to each vertex of a path \( P_{n+1} \) and is denoted by \( G_n \).
A. Vijayan & T. Anitha Baby

Let $G_n$ be a Gem graph with $n + 2$ vertices. Label the vertices of $G_n$ as $v_1$, $v_2$, $v_3$, ..., $v_{n+1}$, $v_{n+2}$. Then, $V(G_n) = \{v_1, v_2, ..., v_{n+1}, v_{n+2}\}$ and $E(G_n) = \{(v_1, v_2), (v_1, v_3), (v_1, v_4), ..., (v_1, v_{n+1}), (v_1, v_{n+2}), (v_2, v_3), (v_3, v_4), ..., (v_{n+1}, v_{n+2})\}$. Let $\text{d}_{ct}(G_n, i)$ be the number of connected total dominating sets of $G_n$ with cardinality $i$.

**Lemma 2.1**

The following properties hold for all graph $G$ with $|V(G)| = n + 2$ vertices.

(i) $\text{d}_{ct}(G, n + 2) = 1$.
(ii) $\text{d}_{ct}(G, n + 1) = n + 2$.
(iii) $\text{d}_{ct}(G, i) = 0$ if $i > n + 2$.
(iv) $\text{d}_{ct}(G, 1) = 0$.

**Proof:**

Let $G = (V, E)$ be a simple graph of order $n + 2$.

(i) We have $\mathcal{D}_{ct}(G, n + 2) = \{v_1, v_2, ..., v_{n+1}, v_{n+2}\}$.

Therefore, $\text{d}_{ct}(G, n + 2) = 1$.

(ii) Also, $\mathcal{D}_{ct}(G, n + 1) = \{\{v_1, v_2, ..., v_{n+1}, v_{n+2}\} - x \mid x \in \{v_1, v_2, ..., v_{n+1}, v_{n+2}\}\}$.

Therefore, $\text{d}_{ct}(G, n + 1) = n + 2$.

(iii) There does not exist a subgraph $H$ of $G$ such that $|V(H)| > |V(G)|$. Therefore, $\text{d}_{ct}(G, i) = 0$ if $i > n + 2$.

(iv) By the definition of total domination, a single vertex cannot dominate totally.

Therefore, $\text{d}_{ct}(G, 1) = 0$.

**Lemma 2.2**

For all $n \in \mathbb{Z}^+$,

\[
\binom{n}{i} = 0 \text{ if } i > n \text{ or } i < 0.
\]

**Lemma 2.3**

For any path graph $P_n$ with $n$ vertices,

(i) $\text{d}_{ct}(P_n, n) = 1$.

(ii) $\text{d}_{ct}(P_n, n - 1) = 2$. 
(iii) $d_{ct}(P_n, n-2) = 1$.
(iv) $d_{ct}(P_n, i) = 0$ if $i < n - 2$ or $i > n$.

**Theorem 2.4**

For any path graph $P_n$ with $n$ vertices, $D_{ct}(P_n, x) = x^{n-2} + 2x^{n-1} + x^n$.

**Proof:**

The proof is given in [6].

**Theorem 2.5**

Let $S_n$ be a star graph with $n$ vertices, then $d_{ct}(S_n, i) = \binom{n}{i} - \binom{n-1}{i}$ for all $n \geq 3$.

**Proof:**

The proof is given in [7].

**Theorem 2.6**

Let $G_n$ be a Gem graph with $n + 2$ vertices, then $d_{ct}(G_n, i) = d_{ct}(S_{n+2}, i) + d_{ct}(P_{n+1}, i)$ for all $i$.

**Proof:**

Let $G_n$ be a Gem graph with $n + 2$ vertices. Let $V(G_n) = \{v_1, v_2, \ldots, v_{n+1}, v_{n+2}\}$ and $E(G_n) = \{(v_1, v_2), (v_1, v_3), (v_1, v_4), \ldots, (v_1, v_{n+1}), (v_2, v_3), (v_2, v_4), \ldots, (v_{n+1}, v_{n+2})\}$. Let $S_{n+2}$ be the star graph with $n + 2$ vertices and $P_{n+1}$ be the path graph with $n + 1$ vertices. $v_1 \in V(S_{n+2})$ is the vertex adjacent to all the vertices of $P_{n+1}$. We have $S_{n+2}$ is a spanning subgraph of $G_n$ and since $G_n - v_1 = P_{n+1}$, $S_{n+2} \cup P_{n+1} = G_n$. Therefore, the number of connected total dominating sets of the Gem graph $G_n$ with cardinality $i$ is the sum of the connected total dominating sets of the star graph $S_{n+2}$ with cardinality $i$ and the number of connected total dominating sets of the Path graph $P_{n+1}$ with cardinality $i$.

Hence, $d_{ct}(G_n, i) = d_{ct}(S_{n+2}, i) + d_{ct}(P_{n+1}, i)$ for all $i$.

**Theorem 2.7**

Let $G_n$ be a Gem graph with $n + 2$ vertices, then $d_{ct}(G_n, i) = d_{ct}(S_{n+2}, i) + d_{ct}(P_{n+1}, i)$ for all $i$.

**Proof:**

(i) By Theorem 2.6, we have, $d_{ct}(G_n, i) = d_{ct}(S_{n+2}, i) + d_{ct}(P_{n+1}, i)$ for all $i$.

Since, $d_{ct}(P_{n+1}, i) = 0$ for all $i < n - 1$, we have,

$d_{ct}(G_n, i) = d_{ct}(S_{n+2}, i)$ for all $i < n - 1$.

$= \binom{n+2}{i} - \binom{n+1}{i}$ for all $i < n - 1$, by Theorem 2.5.

(ii) Since, $d_{ct}(P_{n+1}, i) = 1$ for $i = n - 1, n + 1$, we have,

$d_{ct}(G_n, i) = \binom{n+2}{i} - \binom{n+1}{i} + 1$, if $i = n - 1, n + 1$.  

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(iii) Since, \( d_{ct}(P_{n+1}, i) = 2 \) if \( i = n \), we have,
\[
d_{ct}(G_n, i) = \left( \frac{n+2}{i} \right) - \left( \frac{n+1}{i} \right) + 2 \text{ if } i = n.
\]

**Corollary 2.8**

Let \( G_n \) be a Gem graph with \( n + 2 \) vertices, then

(i) \( d_{ct}(G_n, i) = \left( \frac{n+1}{i-1} \right) \) for all \( i < n - 1 \), \( n \geq 4 \).

(ii) \( d_{ct}(G_n, i) = \left( \frac{n+1}{i-1} \right) + 1 \) for \( i = n - 1 \), \( n + 1 \).

(iii) \( d_{ct}(G_n, i) = \left( \frac{n+1}{i-1} \right) + 2 \) if \( i = n \).

**Proof:**

Since, \( \left( \frac{n+2}{i} \right) - \left( \frac{n+1}{i} \right) = \left( \frac{n+1}{i-1} \right) \), (i), (ii) and (iii) follows from Theorem 2.7 (i), (ii) and (iii).

**Theorem 2.9**

Let \( G_n \) be a Gem graph with \( n + 2 \) vertices, then

(i) \( d_{ct}(G_n, i) = d_{ct}(G_{n-1}, i) + 2 \) if \( i = 2 \).

(ii) \( d_{ct}(G_n, i) = d_{ct}(G_{n-1}, i) + d_{ct}(G_{n-1}, i-1) \) for all \( 3 \leq i \leq n + 2 \) and \( i \neq n - 2 \), \( n - 1 \), \( n \).

(iii) \( d_{ct}(G_n, i) = d_{ct}(G_{n-1}, i) + d_{ct}(G_{n-1}, i - 1) - 1 \) for \( i = n \), \( n - 2 \).

(iv) \( d_{ct}(G_n, i) = d_{ct}(G_{n-1}, i) + d_{ct}(G_{n-1}, i - 1) \) for \( i = n - 1 \).

**Proof:**

(i) When \( i = 2 \), \( d_{ct}(G_n, 2) = \left( \frac{n+1}{1} \right) \), by Corollary 2.8 (i).

\[
= n + 1.
\]

Consider, \( d_{ct}(G_{n-1}, 2) + 1 = \left( \frac{n}{1} \right) + 1 \).

\[
= n + 1.
\]

\( d_{ct}(G_{n-1}, 2) + 1 = d_{ct}(G_n, 2) \).

Therefore, \( d_{ct}(G_n, i) = d_{ct}(G_{n-1}, i) + 1 \) if \( i = 2 \).

(ii) By Corollary 2.8 (i), we have, \( d_{ct}(G_n, i) = \left( \frac{n+1}{i-1} \right) \) for all \( i < n - 1 \).

Also, \( d_{ct}(G_{n-1}, i) = \left( \frac{n}{i-1} \right) \) and \( d_{ct}(G_{n-1}, i - 1) = \left( \frac{n}{i-2} \right) \).

Consider, \( d_{ct}(G_{n-1}, i) + d_{ct}(G_{n-1}, i - 1) = \left( \frac{n}{i-1} \right) + \left( \frac{n}{i-2} \right) \).
\[
\binom{n+1}{i-1}.
\]

\[= d_{ct}(G_n, i). \]

Therefore, \(d_{ct}(G_n, i) = d_{ct}(G_{n-1}, i) + d_{ct}(G_{n-1}, i - 1)\) for all \(3 \leq i \leq n + 2\) and \(i \neq n - 2, n - 1, n\).

(iii) When \(i = n\), we have, \(d_{ct}(G_n, n) = \binom{n+1}{n-1} + 2\), by Corollary 2.8 (iii).

\[= \binom{n+1}{2} + 2.\]

\(d_{ct}(G_{n-1}, n) = \binom{n}{n-1} + 2\), by Corollary 2.8 (iii).

\[= \binom{n}{1} + 2.\]

\(d_{ct}(G_{n-1}, n - 1) = \binom{n}{n-2} + 1\), by Corollary 2.8 (ii).

\[= \binom{n}{2} + 1.\]

Consider, \(d_{ct}(G_{n-1}, n) + d_{ct}(G_{n-1}, n - 1) = \binom{n}{1} + 2\binom{n}{2} + 1.

\[= \binom{n+1}{2} + 2 + 1.
\]

\[= d_{ct}(G_n, n) + 1.\]

Therefore, \(d_{ct}(G_n, n) = d_{ct}(G_{n-1}, n) + d_{ct}(G_{n-1}, n - 1) - 1\).

Hence, \(d_{ct}(G_n, i) = d_{ct}(G_{n-1}, i) + d_{ct}(G_{n-1}, i - 1) - 1\) if \(i = n\).

When \(i = n - 2\), we have,

\(d_{ct}(G_n, n - 2) = \binom{n+1}{n-3}\), by Corollary 2.8 (i).

\[= \binom{n+1}{4}.\]

\(d_{ct}(G_{n-1}, n - 2) = \binom{n}{n-3} + 1\), by Corollary 2.8 (ii).

\[= \binom{n}{3} + 1.\]

\(d_{ct}(G_{n-1}, n - 3) = \binom{n}{n-4}\), by Corollary 2.8 (i).
Consider, \( d_{ct}(G_{n+1}, n-2) + d_{ct}(G_{n+1}, n-3) = \binom{n}{3} + 1 + \binom{n}{4} \).
\[
= \binom{n+1}{4} + 1.
\]
\[
= d_{ct}(G_n, n-2) + 1.
\]

Therefore, \( d_{ct}(G_n, n-2) = d_{ct}(G_{n+1}, n-2) + d_{ct}(G_{n+1}, n-3) - 1 \).

Hence, \( d_{ct}(G_n, i) = d_{ct}(G_{n+1}, i) + d_{ct}(G_{n+1}, i-1) - 1 \), if \( i = n-2 \).

(iv) When \( i = n-1 \), we have,
\[
d_{ct}(G_n, n-1) = \binom{n+1}{n-2} + 1, \text{ by Corollary 2.8 (ii)}. \]
\[
= \binom{n+1}{3} + 1.
\]
\[
d_{ct}(G_{n+1}, n-1) = \binom{n}{n-2} + 2, \text{ by Corollary 2.8 (iii)}. \]
\[
= \binom{n}{2} + 2.
\]
\[
d_{ct}(G_{n+1}, n-2) = \binom{n}{n-3} + 1, \text{ by Corollary 2.8 (ii)}. \]
\[
= \binom{n}{3} + 1.
\]

Consider, \( d_{ct}(G_{n+1}, n-1) + d_{ct}(G_{n+1}, n-2) = \binom{n}{2} + 2 + \binom{n}{3} + 1. \)
\[
= \binom{n+1}{3} + 1 + 2.
\]
\[
= d_{ct}(G_n, n-1) + 2.
\]

Therefore, \( d_{ct}(G_n, n-1) = d_{ct}(G_{n+1}, n-1) + d_{ct}(G_{n+1}, n-2) - 2 \).

Hence, \( d_{ct}(G_n, i) = d_{ct}(G_{n+1}, i) + d_{ct}(G_{n+1}, i-1) - 2 \), if \( i = n-1 \).

3. CONNECTED TOTAL DOMINATION POLYNOMIALS OF A GEM GRAPH \( G_n \)

**Definition 3.1**

Let \( d_{ct}(G_n, i) \) be the number of connected total dominating sets of a Gem graph \( G_n \) with cardinality \( i \). Then, the connected total domination polynomial of \( G_n \) is defined as,
\[
D_{ct}(G_n, x) = \sum_{i=\gamma_{ct}(G_n)}^{n+2} d_{ct}(G_n, i) x^i.
\]
Remark 3.2
\[ \gamma_{ct}(G_n) = 2 \]

Proof:
Let \( G_n \) be a Gem graph with \( n + 2 \) vertices. Let \( v_1 \in V(G_n) \) and \( v_1 \) is the vertex adjacent to all the vertices \( v_2, v_3, \ldots, v_{n+2} \). The vertex \( v_1 \) and one more vertex from \( \{v_2, v_3, \ldots, v_{n+2}\} \) is enough to cover all the other vertices. Therefore, the minimum cardinality is 2. Hence, \( \gamma_{ct}(G_n) = 2 \).

Theorem 3.3
Let \( S_n \) be a star graph with \( n \) vertices, then \( D_{ct}(S_n, x) = x \left[ (1 + x)^{n+1} - 1 \right] \).

Proof:
The proof is given in [7].

Theorem 3.4
Let \( S_n, n \geq 3 \) be a star graph with \( n \) vertices, then
\[
\begin{align*}
(i) \quad D_{ct}(S_n, x) &= \sum_{i=2}^{n} \binom{n}{i} x^i - \sum_{i=2}^{n} \binom{n-1}{i} x^i. \\
(ii) \quad D_{ct}(S_n, x) &= \sum_{i=2}^{n} \binom{n-1}{i} x^i.
\end{align*}
\]

Proof:
The proof is given in [7].

Theorem 3.5
Let \( G_n \) be a Gem graph with \( n + 2 \) vertices, then \( D_{ct}(G_n, x) = D_{ct}(S_{n+2}, x) + D_{ct}(P_{n+1}, x) \).

Proof:
By the definition of connected total domination polynomial, we have,
\[
D_{ct}(G_n, x) = \sum_{i=2}^{n+2} d_{ct}(G_n, i) x^i.
\]

\[
= \sum_{i=2}^{n+2} [d_{ct}(S_{n+2}, i) + d_{ct}(P_{n+1}, i)] x^i, \text{ by Theorem 2.6.}
\]

\[
= \sum_{i=2}^{n+2} d_{ct}(S_{n+2}, i) x^i + \sum_{i=2}^{n+1} d_{ct}(P_{n+1}, i) x^i.
\]

Therefore, \( D_{ct}(G_n, x) = D_{ct}(S_{n+2}, x) + D_{ct}(P_{n+1}, x) \).

Theorem 3.6
Let \( D_{ct}(G_n, x) \) be the connected total domination polynomial of a Gem graph with \( n + 2 \) vertices, then \( D_{ct}(G_n, x) = x \left[ (1 + x)^{n+1} - 1 \right] + x^{n-1} + 2x^n + x^{n+1} \).

Proof:
By Theorem 3.5, we have, \( D_{ct}(G_n, x) = D_{ct}(S_{n+2}, x) + D_{ct}(P_{n+1}, x) \).

Therefore, \( D_{ct}(G_n, x) = x \left[ (1 + x)^{n+1} - 1 \right] + x^{n-1} + 2x^n + x^{n+1} \), by Theorem 2.4 and Theorem 3.3.

Theorem 3.7
Let $D_{ct}(G_n, x)$ be the connected total domination polynomial of a Gem graph with $n + 2$ vertices, then

(i) $D_{ct}(G_n, x) = \sum_{i=2}^{n+2} \binom{n+2}{i} x^i - \sum_{i=2}^{n+2} \binom{n+1}{i} x^i + x^{n-1} + 2x^n + x^{n+1}$.

(ii) $D_{ct}(G_n, x) = \sum_{i=2}^{n+2} \binom{n+1}{i-1} x^i + x^{n-1} + 2x^n + x^{n+1}$.

Proof:

(i) follows from Theorem 3.5, Theorem 3.4 (i) and Theorem 2.4.

(ii) follows from Theorem 3.5, Theorem 3.4 (ii) and Theorem 2.4.

**Theorem 3.8**

Let $D_{ct}(G_n, x)$ be the connected total domination polynomial of a Gem graph with $n + 2$ vertices, then

$D_{ct}(G_n, x) = (1 + x) D_{ct}(G_{n-1, 1}, x) + x^2 - x^{n-2} - 2x^{n-1} - x^n$.

Proof:

By the definition of connected total domination polynomial, we have,

$$D_{ct}(G_n, x) = \sum_{i=2}^{n+2} d_{ct}(G_n, i) x^i.$$  

$$= \sum_{i=2}^{n+2} [d_{ct}(G_{n-1, 1}, i) + d_{ct}(G_{n-1, 1}, i-1)] x^i, \text{ by Theorem 2.9}.$$  

$$= \sum_{i=2}^{n+2} d_{ct}(G_{n-1, 1}, i) x^i + \sum_{i=2}^{n+2} d_{ct}(G_{n-1, 1}, i-1) x^i.$$  

$$= \sum_{i=2}^{n+2} d_{ct}(G_{n-1, 1}, i) x^i + x \sum_{i=3}^{n+2} d_{ct}(G_{n-1, 1}, i-1) x^{i-1}.$$  

$$= D_{ct}(G_{n-1, 1}, x) + x D_{ct}(G_{n-1, 1}, x).$$

Hence, $D_{ct}(G_n, x) = (1 + x) D_{ct}(G_{n-1, 1}, x).$ (1)

When $i = 2$, $d_{ct}(G_n, 2) x^2 = [d_{ct}(G_{n-1, 1}, 2) + 1] x^2$, by Theorem 2.9 (i).

Hence, $d_{ct}(G_n, 2) x^2 = d_{ct}(G_{n-1, 1}, 2) x^2 + x^2$ (2)

When $i = n - 2$,

$d_{ct}(G_n, n - 2) x^{n-2} = [d_{ct}(G_{n-1, 1}, n - 2) + d_{ct}(G_{n-1, 1}, n - 3) - 1] x^{n-2}$, by Theorem 2.9 (iii).

Hence,

$$d_{ct}(G_n, n - 2) x^{n-2} = d_{ct}(G_{n-1, 1}, n - 2) x^{n-2} + d_{ct}(G_{n-1, 1}, n - 3) x^{n-2} - x^{n-2}$$ (3)

When $i = n - 1$,

$$d_{ct}(G_n, n - 1) x^{n-1} = [d_{ct}(G_{n-1, 1}, n - 1) + d_{ct}(G_{n-1, 1}, n - 2) - 2] x^{n-1}$$. by Theorem 2.9 (iv).

Hence, $d_{ct}(G_n, n - 1) x^{n-1} = d_{ct}(G_{n-1, 1}, n - 1) x^{n-1} + d_{ct}(G_{n-1, 1}, n - 2) x^{n-1} - 2x^{n-1}$ (4)

When $i = n$, 

$$d_{ct}(G_n, n) x^n = [d_{ct}(G_{n-1, 1}, n) + d_{ct}(G_{n-1, 1}, n - 1) - 1] x^n$$.
connected total dominating sets and connected total domination polynomials of gem graphs

d\_ct(G\_n, n) x^n = [d\_ct(G\_n-1, n) + d\_ct(G\_n-1, n-1) - 1] x^n, by Theorem 2.9 (iii).

Hence, d\_ct(G\_n, n) x^n = d\_ct(G\_n-1, n) x^n + d\_ct(G\_n-1, n-1)x^n - x^n

(5)

Combining (1), (2), (3), (4) and (5) we get,

D\_ct(G\_n, x) = (1 + x) D\_ct(G\_n-1, x) + x^2 - x^n - x^{n-1} - x^n

Example 3.9

D\_ct(G\_5, x) = 6x^2 + 15x^3 + 21x^4 + 17x^5 + 7x^6 + x^7.

By Theorem 3.8, we have,

D\_ct(G\_6, x) = (1 + x) (6x^2 + 15x^3 + 21x^4 + 17x^5 + 7x^6 + x^7) + x^2 - x^4 - 2x^5 - x^6.

= 7x^2 + 21x^3 + 35x^4 + 36x^5 + 23x^6 + 8x^7 + x^8.

We obtain d\_ct(G\_n, i) for 1 ≤ n ≤ 15 as shown in Table 1.

Table 1

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In the following Theorem, we obtain some properties of d\_ct(G\_n, i).

Theorem 3.10

The following properties hold for the coefficients of D(G\_n, x) for all n.

(i) d\_ct(G\_n, 2) = n + 1 for all n ≥ 4.
Proof:

Proof of (i), (ii) and (iii) follows from Corollary 2.8.

(iv) From Table 1, We have, \(d_{ct}(G_n, i) = 0\), if \(i < 2\) or \(i > n + 2\).

Proof of (v), (vi), (vii), (viii), (ix) and (x) follows from Corollary 2.8.

4. CONCLUSION

In this paper, the connected total domination polynomials of a Gem graph has been derived by identifying its connected total dominating sets. It also helps us to characterize the connected total dominating sets and to find the number of connected total dominating sets of cardinality \(i\). We can generalize this study to any power of the Gem graph and some interesting properties can be obtained via the roots of the connected total domination polynomial of \(G_n^k\).

REFERENCES


