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Study of Stressed State of Elastic Prismatic Bodies of Arbitrary Section with a Cavity in Problems of Constraint Torsion

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Abstract: *The problems of constraint torsion, algorithm of design of prismatic bodies of arbitrary section are considered in the paper on the basis of Lagrange variation principle, R-function procedure and Bubnov-Galerkin's method. Worked out algorithm is applied to a design of prismatic body with rectangular and arbitrary section and a cavity of different form; numeric convergence of results which correspond to occurring physical process is investigated. Certain calculations of normal and tangential stresses and their comparisons are given.*

Keywords: *Method of R-functions, Bubnov-Galerkin's method, Elasticity, Torsion, Constraint Torsion, Torsion Function, Tension Tensor, Complex Configuration*

1. Introduction

In practice in machine-building constructive elements in the form of prismatic bodies of arbitrary section (star gear, polygons, etc.) with a cavity of different form to reduce the weight without lessening their strength are often used. In design of these constructive elements it is necessary to know the state of these elements – elastic or elastic-plastic one. The elements of any engineering structure independent of their purpose should be lasting, rigid and light and least material-consuming. So, one of the basic problems of design is to study elastic and elastic-plastic state in elements of machines or elements of structure of a given form and to develop on the basis of carried out investigations new more rational constructive forms with given strength. So optimal design of prismatic elements of a structure of arbitrary section in machine-building and investigation of their strength quality are actual problems, which require an application of modern methods of design and programming which allow to take into consideration real conditions of operation, configuration of a given element and properties of material. Classic methods and algorithms in Mechanics of Deformable Rigid Bodies are inadequate for the solution of practical problems. So it is reasonable to work out new algorithms of methods of solution on the basis of existing ones which will allow us to carry out extensive investigations without any difficulties.

2. Statement of the Problem of Constraint Torsion

Consider elastic prismatic body of arbitrary section with a cavity of different form in Cartesian system of coordinates oxyz under given surface load ($z=\ell$), where oz is directed along the generatrix, that is, parallel to side surface (Fig. 1).

If to state the task of elastic balance of the body in its integrity, our problem is to determine displacements u, v, w, which will satisfy (in a given field, occupied by the body) Lame's equation of equilibrium with boundary conditions [1]. Their integration in a given field presents a serious difficulty of both computational and algorithmic character [2] due to their complex contours and multi-coupling of cross section. So we have to refer to different approaches of simplification or technologies of lowering the order of the systems of equations. One of the ways to simplify them is the use of variation principles [1] and utilization of a concrete hypothesis in accordance with stated problem.

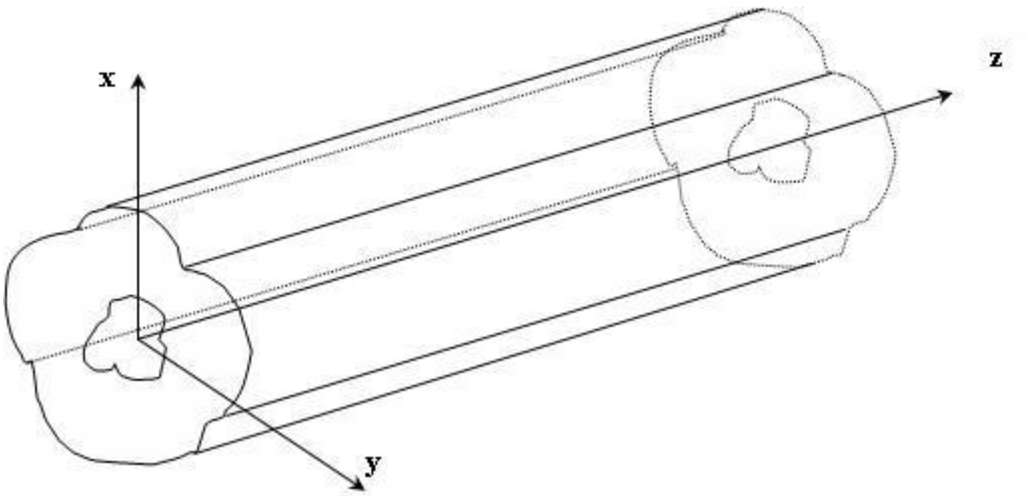


Fig. 1. *Of elastic prismatic bodies of arbitrary section with a cavity*

It is known that the solution of the problems of un-constraint torsion of elastic prismatic rod with the forces applied to its ends, was given by Saint-Venan [3] in the form

$$u=-\tau zy,\ v=\tau zx,\ w=\tau\varphi(x,y), \tag{1}$$

where τ – is a constant angle of torsion per unit of the length of a rod, $\varphi(x,y)$ – is Saint-Venan's function of torsion, determined from equations with boundary conditions:

$$\nabla^2\varphi=0;\ (\varphi_x-y)\Big|_{-a}^{+a}=0;\ (\varphi_y+x)\Big|_{-b}^{+b}=0. \tag{2}$$

where a,b – are sides of cross section of a rod.

Solution (1) corresponds to concrete distribution of loads, applied to the ends of a rod and is equivalent to a given moment. From relationship (1) it is seen that all cross sections are freely turned and do not depend on the length of prismatic body. In a case when the bend of cross sections is complicated and in some cases even impossible, there appear in these sections normal stresses Z_z, directed along axis z. To account these stresses, displacements of arbitrary point in elastic body are presented in the following form

$$u=-\theta(z)y,\ v=\theta(z)x,\ w=\theta^I\varphi(x,y), \tag{3}$$

where θ – is torsion angle, θ^I – relative torsion angle.

In this case deplanation is dependent on the change of relative torsion angle along the axis of a body. The main shortcoming of this solution is a necessity of additional statement of coincidence of the first derivative of torsion angle relative to torsion angle.

In (3), unknown values are θ and φ, respectively, to determine them Lagrange variation principle [1] is used in the form

$$\iint_{\Sigma}\int_{z}\left(Z_z\delta\varepsilon_z+Y_z\delta\varepsilon_{yz}+Z_x\delta\varepsilon_{zx}\right)dzd\Sigma-\iint_{\Sigma}\left(P_{xz}\delta u+P_{yz}\delta v\right)d\Sigma\Big|_{z=\ell}=0; \tag{4}$$

here Z_z, Y_z, Z_x – are components of stress tensors;

$\varepsilon_z, \varepsilon_{yz}, \varepsilon_{zx}$ – components of strain tensors;

P_{xz}, P_{yz} – components of tensor of external load.

Using (3) on the basis of Hooke and Cauchy laws, we determine the components of stress and strain tensors and substituting them into (4), we build the system of resolving equations.

3. Building the Systems of Resolving Equations

Computing the components of stress and strain tensors and using the form of displacements as in (3), then substituting them into Lagrange variation equation (4) and having in mind arbitrary character of variations δθ, δφ, we get differential equations with boundary conditions [4–7]. To do it, we compute

$$\varepsilon_z = \theta^{II}\varphi; \varepsilon_{yz} = \theta^{I}(\varphi_y + x); \varepsilon_{zx} = \theta^{I}(\varphi_x - y)\}; \tag{5}$$

$$\left.\begin{aligned} Z_z &= (\lambda + 2G)\theta^{II}\varphi; \\ Y_z &= G(\varphi_y + x)\theta^{I}; \\ Z_x &= G(\varphi_x - y)\theta^{I}, \end{aligned}\right\} \tag{6}$$

where

$$G = \frac{E}{2(1+\mu)}, \lambda = \frac{E\mu}{(1+\mu)(1-2\mu)};$$

E – is elasticity modulus; G – shear modulus; μ – Poisson's ratio; λ – Lame constant; ^{I} – derivatives on z.

Varying displacements (3) and components of strain tensor (5)

$$\delta u = -y\delta\theta; \delta v = x\delta\theta; \delta w = \varphi\delta\theta^{I} + \theta^{I}\delta\varphi, \tag{7}$$

$$\left.\begin{aligned} &\delta\varepsilon_z = \varphi\delta\theta^{II} + \theta^{II}\delta\varphi; \delta\varepsilon_{yz} = x\delta\theta^{I} + \theta^{I}\delta\varphi_y + \varphi_y\delta\theta^{I}; \\ &\delta\varepsilon_{zx} = -y\delta\theta^{I} + \theta^{I}\delta\varphi_x + \varphi_x\delta\theta^{I}, \end{aligned}\right\} \tag{8}$$

Substituting relations (6) – (8) into (4) and accounting arbitrary character of variations δθ, δφ, we get the following expressions:

$$I_{\varphi\varphi}\theta^{IV} - (I_p + 2I_d + I_k)\theta^{II} = 0; \tag{9}$$

z=0: θ=0; θ^{I}=0;

$$\left.\begin{aligned} z = \ell : I_{\varphi\varphi}\theta^{III} - (I_p + 2I_d + I_k)\theta^{I} + I_{2H} = 0; \\ I_{\varphi\varphi}\theta^{II} = 0; \end{aligned}\right\} \tag{10}$$

$$\psi_3\nabla^2\varphi + \psi_2\varphi = 0; \tag{11}$$

$$\left.\begin{aligned} x = \pm a : (\psi_3\varphi_x + \psi_5 y) = 0; \\ y = \pm b : (\psi_3\varphi_y - \psi_5 x) = 0; \end{aligned}\right\} \tag{12}$$

here

$$I_{\varphi\varphi} = \frac{(\lambda + G)}{G}\iint_{\Sigma} \varphi(x,y)\cdot\varphi(x,y)d\sum; I_p = \iint_{\Sigma}(x^2 + y^2)d\sum; \tag{13}$$

$$I_d = \iint_{\Sigma}(\varphi_y x - \varphi_x y)d\sum; I_k = -\iint_{\Sigma}(\varphi_{xx}\varphi + \varphi_{yy}\varphi)d\sum + \int_y \varphi_x\varphi dy\Big|_x + \int_x \varphi_y\varphi dx\Big|_y;$$

$$\left.\begin{aligned} I_{2H} &= -\frac{M_{KP}}{G}; \;\; \psi_5 = \int_0^{\ell} (\theta^I)^2\,dz; \\ \psi_3 = -\int_0^{\ell} (\theta^I)^2\,dz; &\quad \psi_2 = \frac{(\lambda+2G)}{G}\int_0^{\ell}\left(\theta^{II}\right)^2 dz; \end{aligned}\right\} \tag{14}$$

$$M_{KP} = \iint_{\Sigma} (P_{yz}\cdot x - P_{xz}\cdot y)\,d\sum \quad .$$

To integrate the system of equations (9) – (12) we will reduce them to a more convenient form:

$$\theta^{IV} - r^2\theta^{II} = 0; \tag{15}$$

z=0: $\theta=0$; $\theta^I=0$;

$$z=\ell; \quad \theta^{III} - r^2\theta^{I} + \beta_{1H} = 0; \quad \theta^{II} = 0; \tag{16}$$

$$\nabla^2\varphi + \overline{\psi}_1\varphi = 0; \tag{17}$$

$$\left.\begin{aligned} x = \pm a : \left(\varphi_x - y\right) = 0; \\ y = \pm b : \left(\varphi_y + x\right) = 0, \end{aligned}\right\} \tag{18}$$

where

$$\left. r^2 = \frac{I_p + 2I_d + I_k}{I_{\varphi\varphi}}; \quad \beta_{1H} = \frac{I_{2H}}{I_{\varphi\varphi}}; \quad \overline{\psi}_1 = \frac{\psi_2}{\psi_3}. \right\} \tag{19}$$

So, with hypothesis (3) we get complete set of the systems of equations for the problems of constraint torsion. Knowing concrete geometry of the section and surface loads the problems of constraint torsion may be solved.

4. Development of the Algorithm of Integration of the System of Resolving Equations on the Basis of the Methods of R – Function and Successive Approximations

To integrate the system of resolving equations (15) – (18) of the balance of prismatic bodies of non-classical section the methods of Rvachyev's R – function and successive approximations are used.

The point of this algorithm consists in the following:

1) when assumed that in zero approximation $\overline{\psi}_1 = 0$, we solve equations of the function of torsion (17) with boundary conditions (18);

2) using this solution (φ), we calculate coefficients of equations (15), (16), and then solve them and determine the form of θ;

3) using further solutions (15) and (16), we compute coefficients of equations of the function of torsion and solve (17) and (18) over again;

4) using this solution we compute coefficients of equations (15), (16) and then solve them.

This process goes on till

$$\left|\varphi_{i+1}(x,y) - \varphi_i(x,y)\right| \le \varepsilon.$$

is not fulfilled.

In zero approximation the solution of equation of the function of torsion coincides with Saint-Venan's solution [3], when the section of a given body is rectangular, in opposite case to build them a method of R – function is used.

For prismatic body of rectangular section the form of torsion function is zero and successive approximations has the form:

$$\varphi_c = xy + \sum_{i=1}^{n} \frac{4(-1)^i}{aP_{1i}^3 ch(P_{1i}b)} sh(P_{1i}y)\sin(P_{1i}x); \tag{20}$$

$$\varphi = xy + \sum_{i=1}^{n} \left[q_{1i}y + q_{2i}sh(P_{2i}y)\right]\sin(P_{1i}x), \tag{21}$$

where

$$P_{1i} = \frac{(2i-1)\pi}{2a}; \; P_{2i}^2 = P_{1i}^2 - \overline{\psi}_1; \; q_{1i} = \frac{2\overline{\psi}_1(-1)^{i+1}}{aP_{1i}^2 P_{2i}^2};$$

$$q_{2i} = \frac{2(-1)^i(2P_{2i}^2 + \overline{\psi}_1)}{aP_{1i}^2 P_{2i}^3 ch(P_{2i}b)}.$$

All coefficients of above-mentioned equations are calculated on the basis of Gauss method [8,9] with different number of couplings and weights.

Assuming that coefficients of equations (15) and (16) are calculated, we build their solutions in the form

$$\theta = c_1 + c_2 z + c_3 sh(rz) + c_4 ch(rz), \tag{22}$$

where

$$r^2 = \frac{I_p + 2I_d + I_k}{I_{\varphi\varphi}}. \tag{23}$$

Arbitrary constants c_1, c_2, c_3, c_4 are determined from conditions z=0 and z=ℓ.

So, the solution for prismatic bodies of rectangular section is built. In cases when the section of a given body differs from classical form, construction of the systems of coordinate functions (torsion function) leads to sufficient complications. That is why Rvachyev's method of R - function [10–16,20,31–38] and Bubnov-Galerkin's method are used to build the system of coordinate functions (torsion function).

Now we will proceed to construction of coordinate successions with Rvachyev's method of R - function.

To this effect in boundary conditions (18) we will realize some transformations, that is form Cartesian system of coordinates we will move to curvilinear orthogonal system (n – normal, τ – tangent). In this case they have the form

$$\left.\frac{\partial u}{\partial n}\right|_g = \varphi_0, \tag{24}$$

where

$\varphi_0 = y\dfrac{\partial\omega}{\partial x} - x\dfrac{\partial\omega}{\partial y}$, g – is the border of the area; ω – normalized equation of the border of area.

Structure of solution of boundary problems (17) and (18) has the form [16]:

$$\varphi \equiv \Phi - \omega D_1 \Phi + \varphi_0 \omega; \tag{25}$$

here D_1 – is a differential operator:

$$D_1 = \frac{\partial\Phi}{\partial x}\frac{\partial\omega}{\partial x} + \frac{\partial\Phi}{\partial y}\frac{\partial\omega}{\partial y};$$

Φ – is undefined component of the structure of solution, which is usually presented as in [15]:

$$\Phi = \sum_{i=0}^{n} \sum_{j=0}^{i} C_{ij} X_i(x) Y_j(y);$$

here C_{ij} – are unknown coefficients, subjected to determination; $X_i(x), Y_j(y)$ – complete system of basic polynomials (power ones, trigonometric, Chebyshev's and others).

Power polynomial: $X_i(x)=x^i$, $Y_j(y)=y^j$.

Trigonometric polynomial: $X_i=\cos\frac{i\pi x}{a}$, $Y_i=\cos\frac{i\pi y}{b}$;($a=x_{max}-x_{min}$, $b=y_{max}-y_{min}$).

Chebyshev's polynomial:

$X_i(x) = T_i(x)$,	$X_j(y) = T_j(y)$;
$T_0 = 1$,	$T_0 = 1$;
$T_1 = x$,	$T_1 = y$;
$T_{i+1} = 2xT_i + T_{i-1}$,	$T_{j+1} = 2yT_j + T_{j-1}$.

5. Computing Experiment

Example 1. study of stress-strain state of elastic prismatic bodies of rectangular section with a cavity of different form in problems of constraint torsion. 1.1. Consider elastic prismatic body of rectangular section, with one of sections (z=0) fixed and another one ($z=\ell$) with given values of torque moment; side surfaces are load-free. In this case arbitrary constants in (22) are determined from conditions

$$z=0;\ \theta=0;\ \theta^{I}=0;\ z=\ell;\ \theta^{II}=0;\ \theta^{III}-r^2\theta^{I}=-\beta_{1H},$$

which have the following form:

$$c_1=-\frac{I_{2H}}{rI_1}\operatorname{thr}\ell;\ c_2=\frac{I_{2H}}{I_1};\ c_3=-\frac{I_{2H}}{rI_1};\ \ c_4=\frac{I_{2H}}{rI_1}\operatorname{thr}\ell, \tag{26}$$

where $I_1=(I_p+2I_d+I_k)$.

Substituting the values c_1, c_2, c_3, c_4 into (22), we obtain

$$\theta=\frac{I_{2H}}{rI_1}\left(rz-\operatorname{thr}\ell-\operatorname{shrz}+\operatorname{thr}\ell\operatorname{chrz}\right). \tag{27}$$

In design the following values were used as geometric and mechanical parameters:

a=1sm; b=1cm, 0.5sm, 0.2sm; ℓ =1sm, 2sm, 4sm, 10sm; $\mu=0.3$, $E=2\cdot10^6$ kg/sm^2.

Prismatic bodies of rectangular section were calculated with these parameters by analytical means [17–19], by the method of R – function to build the function of torsion and then using the technology of the method of successive approximations. Here (25) is taken as a structure of solution, and the forms of the borders of a given body are determined by the following way

$$\omega=f_1\wedge_0 f_2,$$

where

$$f_1=\frac{a^2-x^2}{2a};\ f_2=\frac{b^2-y^2}{2b};$$

$\wedge_0$ – R–conjunction.

This statement of the problem is solved when surface load (I_{2H}=0.005) is applied to the surface x=$\pm$a and y=$\pm$b of a given body (I_{2H}/ℓ), results entirely coincide, though the type of solution differs:

$$\theta = \frac{I_{2H}}{rI_1}\left(rz - thr\ell - shrz + thr\ell \cdot chrz - \frac{1}{r\ell \cdot chr\ell} + \frac{chrz}{r\ell \cdot chr\ell} - \frac{rz^2}{r\ell} \right). \tag{28}$$

Solution taken in the form (22), does not satisfy condition of equilibrium:

$$\iint\limits_{\Sigma} \left(xY_z - yX_z \right) d\sum \; = I_{2H}, \tag{29}$$

On the side z=ℓ in boundary conditions there appear an additional term of the type

$$\left(I_d + I_k \right)\theta^{I}(z) - I_{\varphi\varphi}\theta^{III}(z). \tag{30}$$

The character of alteration of this expression along the length of the body qualitatively coincides with alteration of stress component $Z_z \cdot 10^4/G$. At z=0 this expression equals to I_{2H}, and (29) together with (30) satisfy an equilibrium of a given body.

1.2. Consider the same problem, but relative to the body with rectangular cavity with dimensions a_1 = a/10; b_1 = b/10, surfaces of the cavity are load-free. Here (25) is taken as a structure of solution, and the forms of the borders of a given body are determined by the following way:

$$\omega = \omega_1 \wedge_0 \overline{\omega}_2, \; \omega_1 = f_1 \wedge_0 f_2, \quad \overline{\omega}_2 = f_3 \wedge_0 f_4,$$

where

$$f_1 = \frac{a^2 - x^2}{2a}; \quad f_2 = \frac{b - y^2}{2b}; \; f_3 = \frac{a_1^{\;2} - x^2}{2a_1}; \; f_4 = \frac{b_1^{\;2} - y^2}{2b_1}.$$

In this problem in cavity area the following boundary conditions are added for torsion function:

$$x = \pm a_1; \; (\varphi_x - y) = 0; \; y = \pm b_1; \; \left(\varphi_y + x\right) = 0. \tag{31}$$

1.3. Consider the same problem but relative to circular cavity with the following dimension: r_1 = b/10.

Side surfaces of the cavity are load-free. In this problem in cavity area the following boundary conditions are added for torsion function:

$$\left.\frac{\partial u}{\partial n}\right|_g = -xm + y\ell; \tag{32}$$

g – is the border of the area; ℓ, m – directing cosines.

Here (25) is taken as a structure of solution, and the forms of the borders of a given body are determined by the following way:

$$\omega = \omega_1 \wedge_0 \overline{\omega}_2, \; \omega_1 = f_1 \wedge_0 f_2, \; \overline{\omega}_2 = \frac{r_1^{\;2} - x^2 - y^2}{2r_1},$$

where $$f_1 = \frac{a^2 - x^2}{2a}; \quad f_2 = \frac{b^2 - y^2}{2b}.$$

1.4. Consider the problem of prismatic body with elliptic cavity with dimensions a_1 = a/5, b_1 = b/10. Internal surface of a cavity is load-free and has the following boundary condition (32).

In this statement the convergence of results was investigated depending on the length of a given body and area of section. Here (25) was taken as a structure of solution, and the forms of the borders are determined in the form

$$\omega = \omega_1 \wedge_0 \overline{\omega}_2,\ \omega_1 = f_1 \wedge_0 f_2,\ \overline{\omega}_2 = 1 - \frac{x^2}{a_1^{\,2}} - \frac{y^2}{b_1^{\,2}},\ \text{where}\quad f_1 = \frac{a^2 - x^2}{2a};$$

$$f_2 = \frac{b^2 - y^2}{2b}.$$

Table 1 and Figure 2,3 give numeric values of normal $Z_z \cdot 10^4/G$ and tangent $Z_y \cdot 10^4/G$ and $Z_x \cdot 10^4/G$ of stresses depending on alteration of the area of section and cavity in prismatic body (in this Table and Figures the sections of the cavity are numbered in the following order: No 1– with continuous, No 2 – with elliptical, No 3 – with circular, No 4 –with rectangular cavities) under the same external loads M=0.005kg·sm.

Table 1. *Results of components of tensors of stresses in different points of rectangular section with a cavity of different form*

Body section, № / Body dimensions(sm)	coordinates (x,y,z)	№1	№2	№3	№4
1; 1; 4	0.5; 1; 0		a_1=0.2; b_1=0.1	r_1=0.1	a_1=0.1; b_1=0.1
$Z_z \cdot 10^4/G$		-9.345516	-8.935965	-8.900153	-8.859863
1; 0.5; 4	0.5; 0.5; 0		a_1=0.2;b_1=0.05	r_1=0.05	a_1=0.1;b_1=0.05
		-18.836651	-18.729805	-18.621264	-18.533714
1; 0.2; 4	0.5; 0.2; 0		a_1=0.2;b_1=0.02	r_1=0.02	a_1=0.1;b_1=0.02
		-82.397427	-82.325114	-82.235968	-81.482507
1; 1; 4	0.5; 0.5; 4		a_1=0.2; b_1=0.1	r_1=0.1	a_1=0.1; b_1=0.1
$Z_y \cdot 10^4/G$		0.910884	0.910695	0.897783	0.883679
1; 0.5; 4	0.5; 0.25; 4		a_1=0.2;b_1=0.05	r_1=0.05	a_1=0.1;b_1=0.05
		1.469455	1.275869	1.250898	1.396066
1; 0.2; 4	0.5; 0.1; 4		a_1=0.2;b_1=0.02	r_1=0.02	a_1=0.1;b_1=0.02
		2.461752	2.385211	2.391336	2.369444
1; 1; 4	0.5; 0.5; 4		a_1=0.2; b_1=0.1	r_1=0.1	a_1=0.1; b_1=0.1
$Z_x \cdot 10^4/G$		-0.910884	-0.893061	-0.897783	-0.883679
1; 0.5; 4	0.5; 0.25; 4		a_1=0.2;b_1=0.05	r_1=0.05	a_1=0.1;b_1=0.05
		-4.115811	-4.265356	-4.128368	-4.291374
1; 0.2; 4	0.5; 0.1; 4		a_1=0.2;b_1=0.02	r_1=0.02	a_1=0.1;b_1=0.02
		-25.582242	-25.622898	-25.702542	-25.436417

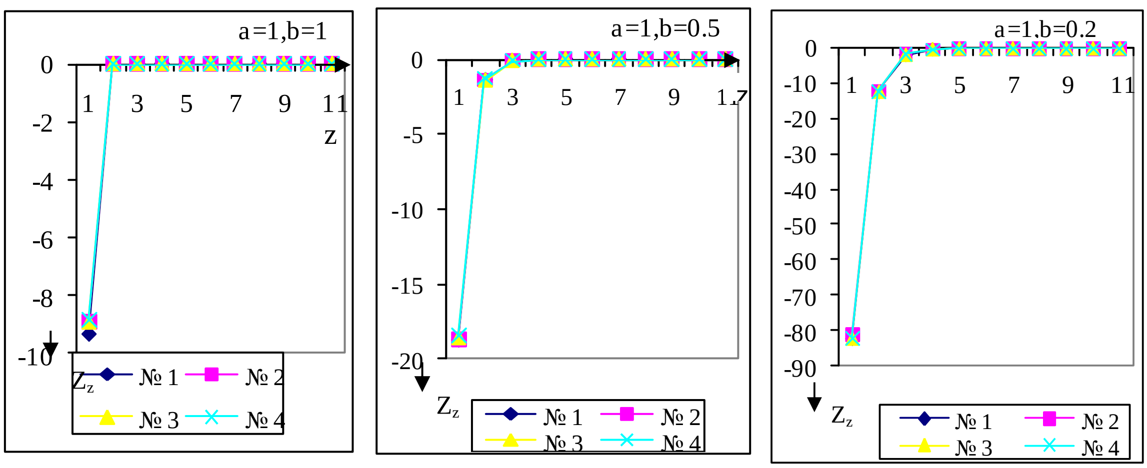


Fig. 2. *The changes of components Z_z of stresses in different points of rectangular section with a cavity of different form*

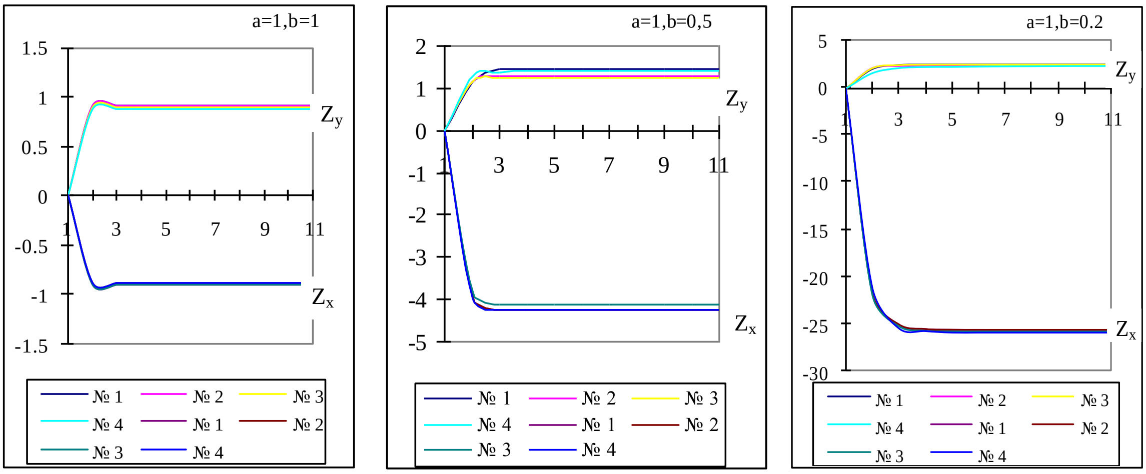


Fig. 3. *The changes of components Z_y and Z_x of stresses in different points of rectangular section with a cavity of different form*

From results given, in Table 1, it is seen that the greatest value $Z_z \cdot 10^4/G$ acquires at continuous section, with the decrease of the section the value is increasing. The least value of $Z_z \cdot 10^4/G$ is observed for prismatic body with rectangular cavity. The character of change of the value of normal stress (successively decreasing or increasing) for prismatic body of rectangular section with/or without a cavity of different form is roughly constant.

The values of tangential stresses $Z_y \cdot 10^4/G$ and $Z_x \cdot 10^4/G$ in prismatic bodies of rectangular section with cavities of different configuration are different values (Table 1).

Fig. 2, 3 show the curves $Z_z \cdot 10^4/G$, $Z_y \cdot 10^4/G$ and $Z_x \cdot 10^4/G$ for three points on the plane at ℓ =10см. In conclusion we should note the following: the values of the components of stress tensors depend on location of a cavity and for each concrete case they should be calculated anew. These curves, beginning from fixed plane in the distance equal approximately to 1 or 1,5 cm of continuous section, come nearer to axis oz in $Z_z \cdot 10^4/G$ and become parallel to changes in tangent stresses $Z_y \cdot 10^4/G$ and $Z_x \cdot 10^4/G$. Projections do not effect quality changes of the values of components of stress tensors [21 – 30, 39].

Combination of Rvachyev's method of R – function and the method of successive approximations in problems of constraint torsion in prismatic bodies of rectangular section with cavities of different form gives satisfactory results.

Example 2. Numeric study of stress-strain state of prismatic bodies of arbitrary section with cavities of different form

2.1. Consider in Cartesian system of coordinates oxyz prismatic body of arbitrary section (Fig. 4), with one section (z=0) fixed (u=0, v=0, w=0), and the second – with given torque moment

$$I_{2H} = -\frac{1}{G}\iint\limits_{\Sigma} P_{xz}\, y\, d\Sigma + \frac{1}{G}\iint\limits_{\Sigma} P_{yz}\, x\, d\Sigma$$

with geometric parameters: a=1sm, b=1sm, 0.5sm, 0.2sm; a_2=a/10sm; b_2=b/10sm; ℓ =1sm, 2sm, 4sm, 10sm; $E=2\cdot 10^6$kg/sm^2; μ=0,3. Side surfaces are free.

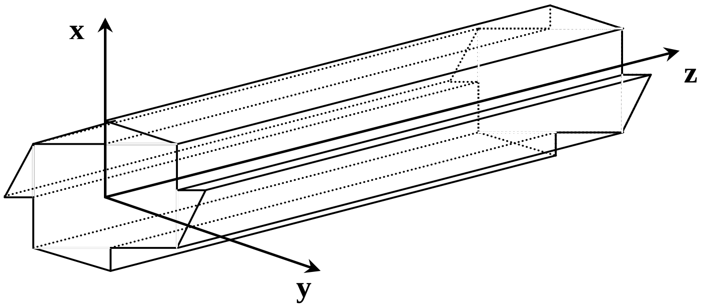


Fig. 4. *Prismatic bodies of arbitrary section*

To build the solution of this problem a combination of methods of R – function and successive approximations was used. Here (25) was taken as a structure of solution, and the equations of the border of area (section) are determined by the following way**:**

$$\omega = ((((\omega_1 \wedge_0 \omega_2) \wedge_0 \omega_3) \wedge_0 \omega_4) \wedge_0 \omega_5),$$

where

$$\omega_1 = f_1 \wedge_0 f_2;\ \omega_2 = f_3 \wedge_0 f_4;\ \omega_3 = f_5 \wedge_0 f_6;\ \omega_4 = f_7 \wedge_0 f_8;\ \omega_5 = f_9 \wedge_0 f_{10};$$

$$f_1 = (bc)^2 - (a_2y+bx)^2;\ f_2 = (ad)^2 - (ay - b_2x)^2;\ f_3 = -x;\ f_4 = b - y;\ f_5 = y;$$

$$f_6 = a - x;\ f_7 = x;\ f_8 = y + b;\ f_9 = -y;\ f_{10} = x + a;\ c = a + a_2;\ d = b + b_2.$$

Solution taken in the form (22), could not satisfy the condition

$$\frac{\iint\limits_F (xY_z - yZ_x)}{I_{2H}} = 1,$$

as a correspondence between tangential stresses and shears is not reached. That is why in building of equation of equilibrium from Lagrange variation principle in the right side of condition there appear additional terms of the form

$$-\frac{I_{\varphi\varphi}}{I_{2H}}\theta^{III} + \frac{I_d + I_k}{I_{2H}}\theta^{I}.$$

The character of changes of this relationship along the length of the body coincides qualitatively with $Z_z \cdot 10^4/G$ and at z=0 equals to one.

2.2. Consider the same problem relative to the body with rectangular cavity with dimensions $a_1 = a/10$; $b_1 = b/10$ and equation of the boundary:

$$\omega = (((((\omega_1 \wedge_0 \omega_2) \wedge_0 \omega_3) \wedge_0 \omega_4) \wedge_0 \omega_5) \wedge_0 \omega_6),$$

where

$$\omega_1 = f_1 \wedge_0 f_2;\ \omega_2 = f_3 \wedge_0 f_4;\ \omega_3 = f_5 \wedge_0 f_6;\ \omega_4 = f_7 \wedge_0 f_8;\ \omega_5 = f_9 \wedge_0 f_{10},$$

$$\omega_6 = \frac{(a^2 - x^2)}{2a} \wedge_0 \frac{\left(b^2 - y^2\right)}{2b};$$

$$f_1 = (bc)^2 - (a_2y + bx)^2;\ f_2 = (ad)^2 - (ay - b_2x)^2;\ f_3 = -x;\ f_4 = b - y;\ f_5 = y;$$

$$f_6 = a - x;\ f_7 = x;\ f_8 = y + b;\ f_9 = -y;\ f_{10} = x + a;\ c = a + a_2;\ d = b + b_2.$$

Inside the cavity the surfaces are load-free. In this problem in the area of a cavity for torsion function an additional boundary conditions are foreseen in the form (31).

Equilibrium of a given body is fulfilled with additional term of the form

$$-\frac{I_{\varphi\varphi}}{I_{2H}}\theta^{III} + \frac{I_d + I_k}{I_{2H}}\theta^{I}.$$

2.3. Consider the same problem relative to circular cavity with the following dimensions: $r_1 = b/10$.

Side surfaces of the cavity are load-free. In this problem in the area of cavity for torsion function an additional boundary conditions are foreseen (32).

Here expression (25) is taken as a structure of solution, and forms of boundary are set in the form

$$\omega = (((((\omega_1 \wedge_0 \omega_2) \wedge_0 \omega_3) \wedge_0 \omega_4) \wedge_0 \omega_5) \wedge_0 \omega_6);$$

expressions $\omega_1 - \omega_5$ were given in previous items, and

$$\omega_6 = \frac{(x^2 + y^2 - r_1^2)}{2r_1}.$$

As in previous items the components of tangent stresses do not satisfy equilibrium (29), but together with (29) and (30) they do satisfy.

2.4. Consider the problem given in 2.1, relative to the body with elliptical cavity with dimensions a_1 = a/5; b_1 = b/10. Internal surface is load-free and has the following additional boundary conditions (32).

Here (25) is taken as a structure of solution, and geometry of the area is described by the following way**:**

$$\omega = (((((\omega_1 \wedge_0 \omega_2) \wedge_0 \omega_3) \wedge_0 \omega_4) \wedge_0 \omega_5) \wedge_0 \omega_6),$$

where $\omega_1 - \omega_5$ were described in previous items, and

$$\omega_6 = \frac{x^2}{a_1^2} + \frac{y^2}{b_1^2} - 1.$$

Table 2 gives numeric values of normal $Z_z \cdot 10^4/G$ and tangent $Z_y \cdot 10^4/G$ and $Z_x \cdot 10^4/G$ stresses depending on the change of area of section and of a cavity of prismatic body (in given Table the sections of the cavity are numbered in the following order: No1– with continuous, No 2 – with elliptical, No 3 – with circular, No 4 – with rectangular cavities) under the same external loads M=0.005kg·sm.

Table 2. *Results of components of tensors of stresses in different points of arbitrary section with a cavity of different form*

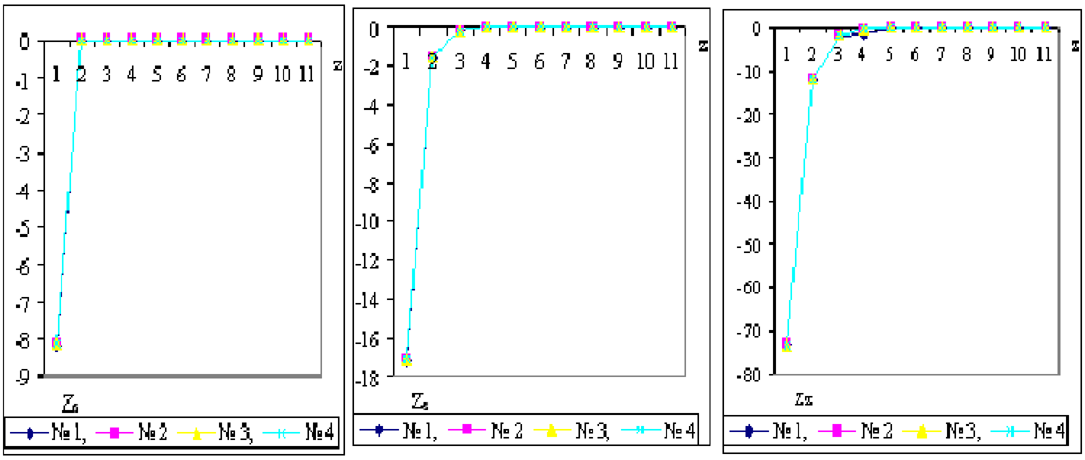
Body section, No — Body dimensions (sm)	Coordinates (x.y.z)	№ 1	№ 2	№ 3	№4
1; 1; 10	0.5; 1; 0	a_2=0.1; b_2=0.1	a_1=0.2; b_1=0.1	r_1=0.1	a_1=0.1; b_1=0.1
$Z_z \cdot 10^4/G$		-8.182672	-8.144766	-8.138466	-8.106696
1; 0.5; 10	0.5; 0.5; 0	a_2=0.1; b_2=0.05	a_1=0.2; b_1=0.05	r_1=0.05	a_1=0.1; b_1=0.05
		-17.104551	-17.051614	-17.093644	-17.046135
1; 0.2; 10	0.5; 0.2; 0	a_2=0.1; b_2=0.02	a_1=0.2; b_1=0.02	r_1=0.02	a_1=0.1; b_1=0.02
		-73.058062	-72.977007	-72.964707	-72.880484
1; 1; 10	0.5; 0.5; 10	a_2=0.1; b_2=0.1	a_1=0.2; b_1=0.1	r_1=0.1	a_1=0.1; b_1=0.1
$Z_y \cdot 10^4/G$		0.830817	0.820099	0.828352	0.821741
1; 0.5; 10	0.5; 0.25; 10	a_2=0.1; b_2=0.05	a_1=0.2; b_1=0.05	r_1=0.05	a_1=0.1; b_1=0.05
		1.179061	1.119732	1.096325	1.065383
1; 0.2; 10	0.5; 0.1; 10	a_2=0.1; b_2=0.02	a_1=0.2; b_1=0.02	r_1=0.02	a_1=0.1; b_1=0.02
		2.108771	2.084743	2.079564	2.047034
1; 1; 10	0.5; 0.5; 10	a_2=0.1; b_2=0.1	a_1=0.2; b_1=0.1	r_1=0.1	a_1=0.1; b_1=0.1
$Z_x \cdot 10^4/G$		-0.845528	-0.832807	-0.834592	-0.831307
1; 0.5; 10	0.5; 0.25; 10	a_2=0.1; b_2=0.05	a_1=0.2; b_1=0.05	r_1=0.05	a_1=0.1; b_1=0.05
		-4.239028	-4.196523	-4.189433	-4.175232
1; 0.2; 10	0.5; 0.1; 10	a_2=0.1; b_2=0.02	a_1=0.2; b_1=0.02	r_1=0.02	a_1=0.1; b_1=0.02
		-23.188684	-23.174199	-23.170607	-23.160899

From results given in Table 2, it is seen that the greatest value $Z_z \cdot 10^4/G$ acquires at continuous section, with a decrease of section the value is increasing. The least value of $Z_z \cdot 10^4/G$ is observed for prismatic body with rectangular cavity. The character of changes of the value of normal stress (successively decreasing or increasing) for prismatic body of arbitrary section with/or without a cavity of different form is approximately constant.

The values of tangential stresses $Z_y \cdot 10^4/G$ and $Z_x \cdot 10^4/G$ in prismatic bodies of arbitrary section and with cavities of different configuration are different values (Table 2).

Figures 5, 6 illustrate the curves $Z_z \cdot 10^4/G$, $Z_y \cdot 10^4/G$ and $Z_x \cdot 10^4/G$ for three points on the plane at ℓ =10. The values of components of stress tensors depend on location of the cavity and for each concrete case of the process, occurring in prismatic body, they should be calculated anew. The change of these curves is observed: beginning from fixed section in the distance approximately 2 – 2,5 sm , the value $Z_z \cdot 10^4/G$ comes nearer to axis oz, and values $Z_y \cdot 10^4/G$ and $Z_x \cdot 10^4/G$ become parallel to this axis. Projections do not effect quality changes of values of components of stress tensors.

Combination of Rvachyev's method of R- function and the method of successive approximations may be used in problems of constraint torsion in prismatic bodies of arbitrary section and a cavity of different configuration. They will always give positive results, corresponding to practice requirements.



cavity of different form

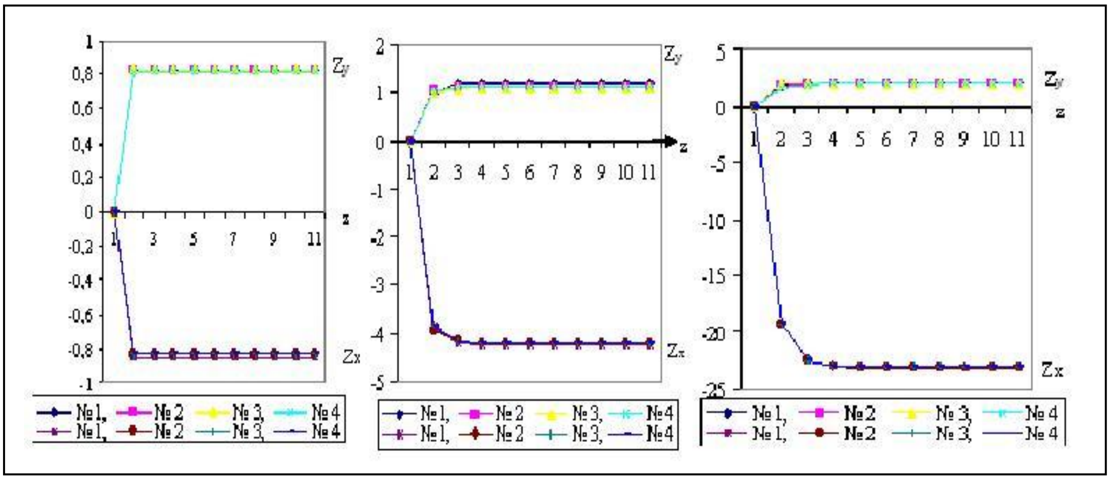


Fig. 6. *The changes of components Z_y and Z_x of stresses in different points of arbitrary section with a cavity of differenr form*

6. Conclusion

According to results obtained we could draw the following conclusions:

1. In solution of torsion problems it was stated that the part of the body should be calculated according to the hypothesis of constraint torsion, and other parts – on the basis of the theory of plane sections.

2. Prismatic body of rectangular section is in equilibrium due to appearance of additional term in boundary conditions (z= ℓ).

3. In practice prismatic elements of the structure of rectangular section with a cavity may be used when their dimensions do not exceed a/10 or b/10.

4. Location of the cavity and its configuration has a great importance, so in each concrete case the calculations should be done anew.

5. In prismatic bodies with arbitrary section and a cavity the distribution of values of tangential stresses is non-uniform. So in each quadrant there are areas working on tension and compression. Calculations of a part of prismatic body may be checked by the hypothesis of constraint torsion, and others – by the theory of plane sections. These prismatic bodies of arbitrary section with cavities of different form are in the state of equilibrium due to additional term in boundary conditions ($z=\ell$).

6. At small section (a/10 or b/10) of a cavity, prismatic bodies with continuous section may be used instead of prismatic ones. For concrete elements of technical structures with inclusions and cavities the studies should be carried out anew, choosing new rigidity of a given prismatic body.

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