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A Series Solution of Ecological Harvested Commensal Model by Homotopy Perturbation Method

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Abstract: *The paper aims to investigate a series solution in an ecological commensalism. The Commensal species is harvested with limited resources. The model equations are constituted by a pair of non linear first order differential equations. Homotopy perturbation method is used as an efficient tool for evaluating series solution. In order to classify the significant interaction between Commensalism and host species, numerical solutions are identified.*

Keywords: *Commensalism, Homotopy Analysis, Stability, Dominance Reversal time.*

1. Introduction

It is very difficult to solve non linear differential and partial differential equations for boundary value problems with the help of analytic methods. Perturbation technique is very useful in the case of non liner problems. This technique has been utilized for getting fruit full results in science & engineering. But perturbation techniques are fully depending upon physical parameters. Hence it is restricted in the beginning stage to use only for the weak non linear problems. Abbasbandy,S [1] used this technique and opened new ideas in the concept of asymptotic techniques .Later Liao[4-7] developed Homotopy Perturbation Method (HPM) in 1992. Some analytic approximation methods with independent physical parameters were invented by Some other Mathematicians [2,3].In the recent years, the HPM methodology with various logic ideas and its applications were systematically developed.

2. Basic Idea of Homotopy Perturbation Method

Step (1): Let us consider nonlinear differential equation:

$$A\,u\,-f\,r\,=0,\ \ r\in\Omega \qquad \text{(I)}$$

With the the boundary condition

$$B\left(u,\frac{\partial u}{\partial n}\right)=0,\qquad r\in\Gamma$$

where A is a general differential operator, B a boundary operator, $f\ r$ is a known analytic function, Γ is the boundary of the domain Ω and $\frac{\partial}{\partial n}$ denotes differentiation along the normal drawn outwards from Ω .

Step (2): In general the operator A, is divided into two parts: a linear part L and a nonlinear part N. Therefore above differential equation(I) is expressed in the form of

$$L(u) - N(u) - f(r) = 0 \tag{II}$$

Step (3):

With the help of Homotopy Perturbation Method (HPM), one can constitute a homotopy $v(r,p):\Omega\times[0,1]\to\mathbb{R}$ which satisfies

$$H(v,p) = (1-p)\left[L(v) - L(u_0)\right] + p\left[A(v) - f(r)\right] = 0,\ p\in[0,1], r\in\Omega \tag{III}$$

It is nothing but

$$H(v,p) = L(v) - L(u_0) + pL(u_0) + p\left[N(v) - f(r)\right] = 0 \tag{IV}$$

where $p\in[0,1]$ is named as an embedding parameter ,and u_0 is an initial approximation of equation(1),which satisfies the boundary conditions.

Step (4): Then equations (III), (IV) follow that

$$H(v,0) = L(v) - L(u_0) = 0$$

and $H(v,1) = A(v) - f(r) = 0$

Thus the changing process of p from zero to unity is just that of $v(r,p)$ from $u_0(r)$ to $u(r)$.

Step (5): According to the HPM, we can first use the imbedding parameter p as a 'small parameter' and assume that the solutions of the equations (III) and (IV) can be written as a power series in p :

$$v=v_0+pv_1+p^2v_2+p^3v_3+p^4v_4+----------$$

The approximate solution of equation (I) can be obtained as

$$u = \underset{p\to 1}{Lt}\, v = v_0 + v_1 + v_2 + v_3 + v_4 + -----------$$

3. Notations Adopted

N_1 (t) : The population rate of the species S_1 at time t

N_2 (t) :The population rate of the species S_2 at time t

a_i : The natural growth rate of S_i, i = 1, 2.

a_{ii} : The rate of decrease of S_i; due to its own insufficient resources ,i=1,2.

a_{12} :The inhibition coefficient of S_1 due to S_2 i.e The Commensal coefficient.

H_1(t) :The replenishment or renewal of S_1 per unit time

H_1 :a_{11} H_1 is the rate of harvest of the Host.

The state variables N_1 and N_2 as well as the model parameters a_1, a_2, a_{11}, a_{22}, K_1, K_2, α , h_1, h_2 are assumed to be non-negative constants.

4. Basic Equations

$$\frac{dN_1}{dt} = a_1N_1 - a_{11}N_1^2 + a_{12}N_1N_2 - a_{11}H_1 \tag{1}$$

$$\frac{dN_2}{dt} = a_2N_2 - a_{22}N_2^2 \quad \text{with initial conditions } N_1(0)=\mathbf{c}_1 \text{ and } N_2(0)=c_2 \tag{2}$$

The following system can be constructed by the concept of homotopy as follows

$$v_1' - N_{10}' + p\left(N_{10}' - a_1v_1 + a_{11}v_1^2 - a_{12}v_1v_2 + a_{11}H_1\right) = 0 \tag{3}$$

$$v_2' - N_{20}' + p\left(N_{20}' - a_2v_2 + a_{22}v_2^2\right) = 0 \tag{4}$$

The initial approximations are considered as

$$v_{1,0}(t) = N_{10}(t) = v_1(0) = c_1 \tag{5}$$

$v_{2,0}(t) = N_{20}(t) = v_2(0) = c_2$ (6)

and $v_1(t) = v_{1,0}(t) + pv_{1,1}(t) + p^2v_{1,2}(t) + p^3v_{1,3}(t) + p^4v_{1,4}(t) + p^5v_{1,5}(t) + \cdots$ (7)

$v_2(t) = v_{2,0}(t) + pv_{2,1}(t) + p^2v_{2,2}(t) + p^3v_{2,3}(t) + p^4v_{2,4}(t) + p^5v_{2,5}(t) + \cdots$ (8)

Where $v_{i,J}(i = 1,2, J = 1,2,3 \ldots)$ *are to be computed by substituting* (5), (6), (7), (8) *in* (3), (4)

We get

$v'_{1,0}(t) + pv'_{1,1}(t) + p^2v'_{1,2}(t) + p^3v'_{1,3}(t) + p^4v'_{1,4}(\mathrm{t}) + p^5v'_{1,5}(\mathrm{t}) + \cdots - N'_{10} +$
$p[N'_{10} - a_1(v_{1,0}(t) + pv_{1,1}(t) + p^2v_{1,2}(t) + p^3v_{1,3}(t) + p^4v_{1,4}(t) + p^5v_{1,5}(t) + \cdots)$
$+a_{11}\big(v_{1,0}(t) + pv_{1,1}(t) + p^2v_{1,2}(t) + p^3v_{1,3}(t) + p^4v_{1,4}(t) + p^5v_{1,5}(t) + \cdots\big)(v_{1,0}(t) + pv_{1,1}(t)$
$+p^2v_{1,2}(t) + p^3v_{1,3}(t) + p^4v_{1,4}(t) + p^5v_{1,5}(t) + \cdots - a_{12}(v_{1,0}(t) + pv_{1,1}(t) + p^2v_{1,2}(t) + p^3v_{1,3}$
$(t) + p^4v_{1,4}(t) + p^5v_{1,5}(t) + \cdots\big)\big(v_{2,0}(t) + pv_{2,1}(t) + p^2v_{2,2}(t) + p^3v_{2,3}(t) + p^4v_{2,4}(t) +$
p5v2,5(t+…)+a11H1]=0 (9)

From equation (4)

$v'_{2,0}(t) + pv'_{2,1}(t) + p^2v'_{2,2}(t) + p^3v'_{2,3}(t) + p^4v'_{2,4}(t) + p^5v'_{2,5}(t) + \cdots - N'_{20}$
$+p[N'_{20} - a_2(v_{2,0}(t) + pv_{2,1}(t) + p^2v_{2,2}(t) + p^3v_{2,3}(t) + p^4v_{2,4}(t) + p^5v_{2,5}(t) + \cdots$
$+a_{22}\big(v_{2,0}(t) + pv_{2,1}(t) + p^2v_{2,2}(t) + p^3v_{2,3}(t) + p^4v_{2,4}(t) + p^5v_{2,5}(t) + \cdots\big)$
$\big(v_{2,0}(t) + pv_{2,1}(t) + p^2v_{2,2}(t) + p^3v_{2,3}(t) + p^4v_{2,4}(t) + p^5v_{2,5}(t) + \cdots\big)]$ =0 (10)

From (9),

$0 + pv'_{1,1}(t) + p^2v'_{1,2}(t) + p^3v'_{1,3}(t) + p^4v'_{1,4}(t) + p^5v'_{1,5}(t) + \cdots - 0$
$+p[0 - a_1v_{1,0}(t) - a_1pv_{1,1}(t) - a_1p^2v_{1,2}(t) - a_1p^3v_{1,3}(t) - a_1p^4v_{1,4}(t) - a_1p^5v_{1,5}(t) - \ldots$
$+a_{11}\,v_{1,0}^2(t) + a_{11}pv_{1,0}(t)v_{1,1}(t) + a_{11}p^2v_{1,0}(t)v_{1,2}(t) + a_{11}p^3v_{1,0}(t)v_{1,3}(t)$
$+a_{11}p^4v_{1,0}(t)v_{1,4}(t) + \cdots + a_{11}pv_{1,0}(t)v_{1,1}(t) + a_{11}p^2v_{1,1}^2(t) + a_{11}p^3v_{1,1}(t)v_{1,2}(t)$
$+a_{11}p^4v_{1,1}(t)v_{1,3}(t) + a_{11}p^5v_{1,1}(t)v_{1,4}(t) + \cdots + a_{11}p^2v_{1,0}(t)v_{1,2}(t) + a_{11}p^3v_{1,1}(t)v_{1,2}(t)$
$+a_{11}p^4v_{1,2}^2(t) + a_{11}p^5v_{1,2}(t)v_{1,3}(t) + \cdots + a_{11}p^3v_{1,0}(t)v_{1,3}(t) + a_{11}p^4v_{1,1}(t)v_{1,3}(t)$
$+a_{11}p^5v_{1,2}(t)v_{1,3}(t) + \cdots + a_{11}p^4v_{1,0}(t)v_{1,4}(t) + a_{11}p^5v_{1,1}(t)v_{1,4}(t) + \cdots$
$+a_{11}p^5v_{1,0}(t)v_{1,5}(t) + \cdots - a_{12}\,v_{1,0}(t)v_{2,0}(t) - a_{12}pv_{1,0}(t)v_{2,1}(t) - a_{12}p^2v_{1,0}(t)v_{2,2}(t)$
$-a_{12}p^3v_{1,0}(t)v_{2,3}(t) - a_{12}p^4v_{10}(t)v_{2,4}(t) - \ldots\ldots - a_{12}\,pv_{1,1}(t)v_{2,0}(t) - a_{12}p^2v_{1,1}(t)v_{2,1}(t)$
$-a_{12}p^3v_{1,1}(t)v_{2,2}(t) - a_{12}p^4v_{1,1}(t)v_{2,3}(t) - \cdots - a_{12}p^2v_{2,0}(t)v_{1,2}(t) - a_{12}p^3v_{1,2}(t)v_{2,1}(t)$
$-a_{12}p^4v_{1,2}(t)v_{2,2}(t) - \cdots - a_{12}p^3v_{1,3}(t)v_{2,0}(t) - a_{12}p^4v_{1,3}(t)v_{2,1}(t) - \cdots - a_{12}p^4v_{1,4}(t)v_{2,0}(t)$
$- \cdots + a_{11}H_1] = 0$ (11)

From (10),

$0 + pv'_{2,1}(t) + p^2v'_{2,2}(t) + p^3v'_{2,3}(t) + p^4v'_{2,4}(t) + p^5v'_{2,5}(t) + \cdots - 0 + p[0 - a_2v_{2,0}(t)$
$-a_2pv_{2,1}(t) - a_2p^2v_{2,2}(t) - a_2p^3v_{2,3}(t) - a_2p^4v_{2,4}(t) - a_2p^5v_{2,5}(t) - \cdots + a_{22}\,v_{2,0}^2(t)$
$+a_{22}pv_{2,0}(t)v_{2,1}(t) + a_{22}p^2v_{2,0}(t)v_{2,2}(t) + a_{22}p^3v_{2,0}(t)v_{2,3}(t) + a_{22}p^4v_{2,0}(t)v_{2,4}(t) + \cdots$
$+a_{22}\,pv_{2,1}(t)v_{2,0}(t) + a_{22}p^2v_{2,1}^2(t) + a_{22}p^3v_{2,1}(t)v_{2,2}(t) + a_{22}p^4v_{2,1}(t)v_{2,3}(t)$
$+a_{22}p^5v_{2,1}(t)v_{2,4}(t) + \cdots + a_{22}p^2v_{2,0}(t)v_{2,2}(t) + a_{22}p^3v_{2,2}(t)v_{2,1}(t) + a_{22}p^4v_{2,2}^2(t)$
$+a_{22}p^5v_{2,2}(t)v_{2,3}(t) + \cdots + a_{22}p^3v_{2,0}(t)v_{2,3}(t) + a_{22}p^4v_{2,1}(t)v_{2,3}(t) + a_{22}$
$p^5v_{2,2}(t)v_{2,3}(t) + \cdots + a_{22}p^4v_{2,0}(t)v_{2,4}(t) + a_{22}p^5v_{2,1}(t)v_{2,4}(t) + \cdots$

$+a_{22}p^5 v_{2,0}(t)v_{2,5}(t)+\cdots\ ]=0$ (12)

Now comparing the coefficient of various powers of p in (11)&(12),we obtain

The co efficient of P^1:

$$v_{1,1}^{'}(t)-a_1 v_{1,0}(t)+a_{11}v_{1,0}^2(t)-a_{12}v_{1,0}(t)v_{2,0}(t)+a_{11}H_1=0$$

$$v_{2,1}^{'}(t)-a_2 v_{2,0}(t)+a_{22}v_{2,0}^2(t)=0$$

The co efficient of P^2:

$$v_{1,2}^{'}(t)-a_1 v_{1,1}(t)+a_{11}v_{1,0}(t)v_{1,1}(t)+a_{11}v_{1,0}(t)v_{1,1}(t)-a_{12}v_{1,0}(t)v_{2,1}(t)$$
$$-a_{12}v_{1,1}(t)v_{2,0}(t)=0$$

$$v_{2,2}^{'}(t)-a_2 v_{2,1}(t)+a_{22}v_{2,0}(t)v_{2,1}(t)+a_{22}v_{2,0}(t)v_{2,1}(t)=0$$

The co efficient of P^3:

$$v_{1,3}^{'}(t)-a_1 v_{1,2}(t)+a_{11}v_{1,0}(t)v_{1,2}(t)+a_{11}v_{1,1}^2(t)+a_{11}v_{1,0}(t)v_{1,2}(t)-a_{12}v_{1,0}(t)v_{2,2}(t)-$$
$$a_{12}v_{1,1}(t)v_{2,1}(t)-a_{12}v_{2,0}(t)v_{1,2}(t)=0$$

$$v_{2,3}^{'}(t)-a_2 v_{2,2}(t)+a_{22}v_{2,0}(t)v_{2,2}(t)+a_{22}v_{2,1}^2(t)+a_{22}v_{2,0}(t)v_{2,2}(t)=0$$

The co efficient of P^4:

$$v_{1,4}^{'}(t)-a_1 v_{1,3}(t)+a_{11}v_{1,0}(t)v_{1,3}(t)+a_{11}v_{1,1}(t)v_{1,2}(t)+a_{11}v_{1,1}(t)v_{1,2}(t)$$
$$+\ a_{11}v_{1,0}(t)v_{1,3}(t)-a_{12}v_{1,0}(t)v_{2,3}(t)-a_{12}v_{1,1}(t)v_{2,2}(t)-a_{12}v_{2,1}(t)v_{1,2}(t)$$
$$-a_{12}v_{2,0}(t)v_{1,3}(t)=0$$

$$v_{2,4}^{'}(t)-a_2 v_{2,3}(t)+a_{22}v_{2,0}(t)v_{2,3}(t)+a_{22}v_{2,1}(t)v_{2,2}(t)+a_{22}v_{2,1}(t)v_{2,2}(t)$$
$$+a_{22}v_{2,0}(t)v_{2,3}(t)=0$$

Now $v_1(0)=c_1, v_2(0)=c_2$

$$v_{1,1}(t)=a_1\int_0^t v_{1,0}(t)dt-a_{11}\int_0^t v_{1,0}^2(t)dt+a_{12}\int_0^t v_{1,0}(t)v_{2,0}(t)dt-a_{11}H_1\int_0^t dt$$
$$=c_1a_1t-a_{11}c_1^2t+a_{12}c_1c_2t-a_{11}H_1t$$
$$\therefore v_{1,1}(t)=(a_1-a_{11}c_1+a_{12}c_2-\frac{a_{11}H_1}{c_1})c_1t$$

$$v_{2,1}(t)=a_2\int_0^t v_{2,0}(t)dt-a_{22}\int_0^t v_{2,0}^2(t)dt$$
$$=a_2c_2t-a_{22}c_2^2t$$
$$\therefore v_{2,1}(t)=(a_2-a_{22}c_2)c_2t$$

$$v_{1,2}(t)=a_1\int_0^t v_{1,1}(t)dt-2a_{11}\int_0^t v_{1,0}(t)v_{1,1}(t)dt+a_{12}\int_0^t v_{1,0}(t)v_{2,1}(t)dt$$
$$+a_{12}\int_0^t v_{1,1}(t)v_{2,0}(t)dt$$
$$=a_1(a_1-a_{11}c_1+a_{12}c_2-\frac{a_{11}H_1}{c_1})c_1\frac{t^2}{2}-2a_{11}c_1(a_1-a_{11}c_1+a_{12}c_2-\frac{a_{11}H_1}{c_1})c_1\frac{t^2}{2}+a_{12}c_1(a_2$$

$$-a_{22}c_2)c_2\frac{t^2}{2}+a_{12}c_2(a_1-a_{11}c_1+a_{12}c_2-\frac{a_{11}H_1}{c_1})c_1\frac{t^2}{2}$$

$$\therefore v_{1,2}(t)=[(a_1-2a_{11}c_1+a_{12}c_2)\left(a_1-a_{11}c_1+a_{12}c_2-\frac{a_{11}H_1}{c_1}\right)c_1+a_{12}c_1(a_2-a_{22}c_2)c_2]\frac{t^2}{2}$$

$$v_{2,2}(t)=a_2\int_0^t v_{2,1}(t)dt-2a_{22}\int_0^t v_{2,0}(t)v_{2,1}(t)dt$$

$$=[a_2(a_2-a_{22}c_2)c_2-2a_{22}c_2(a_2-a_{22}c_2)c_2]\frac{t^2}{2}$$

$$\therefore v_{2,2}(t)=[(a_2-a_{22}c_2)(a_2-2a_{22}c_2)c_2]\frac{t^2}{2}$$

$$v_{1,3}(t)=a_1\int_0^t v_{1,2}(t)dt-2a_{11}c_1\int_0^t v_{1,2}(t)dt-a_{11}\int_0^t v_{1,1}^2(t)dt+a_{12}c_1\int_0^t v_{2,2}(t)dt$$

$$+a_{12}c_2\int_0^t v_{1,2}(t)dt+a_{12}\int_0^t v_{1,1}(t)v_{2,1}(t)dt$$

$$=(a_1-2a_{11}c_1+a_{12}c_2)\{(a_1-2a_{11}c_1+a_{12}c_2)(a_1-a_{11}c_1+a_{12}c_2-\frac{a_{11}H_1}{c_1})c_1$$

$$+a_{12}c_1c_2(a_2-a_{22}c_2)\}\frac{t^3}{6}-a_{11}(a_1-a_{11}c_1+a_{12}c_2-\frac{a_{11}H_1}{c_1})(a_1-a_{11}c_1+a_{12}c_2-\frac{a_{11}H_1}{c_1})c_1^2\frac{t^3}{3}$$

$$+a_{12}c_1\{(a_2-a_{22}c_2)(a_2-2a_{22}c_2)c_2\}\frac{t^3}{6}+a_{12}c_2(a_2-a_{22}c_2)(a_1-a_{11}c_1+a_{12}c_2-\frac{a_{11}H_1}{c_1})c_1\frac{t^3}{3}$$

$$\therefore v_{1,3}(t)=[(a_1-2a_{11}c_1+a_{12}c_2)\{(a_1-2a_{11}c_1+a_{12}c_2)(a_1-a_{11}c_1+a_{12}c_2-\frac{a_{11}H_1}{c_1})c_1$$

$$+a_{12}c_1c_2(a_2-a_{22}c_2)\}+\left(a_1-a_{11}c_1+a_{12}c_2-\frac{a_{11}H_1}{c_1}\right)c_1\{2a_{12}c_2(a_2-a_{22}c_2)$$

$$-2a_{11}\left(a_1-a_{11}c_1+a_{12}c_2-\frac{a_{11}H_1}{c_1}\right)c_1\}+a_{12}c_1c_2\{(a_2-a_{22}c_2)(a_2-2a_{22}c_2)\}]\frac{t^3}{6}$$

$$v_{2,3}(t)=a_2\int_0^t v_{2,2}(t)dt-2a_{22}\int_0^t v_{2,0}(t)v_{2,2}(t)dt-a_{22}\int_0^t v_{2,1}^2(t)dt$$

$$=(a_2-2a_{22}c_2)\{(a_2-a_{22}c_2)(a_2-2a_{22}c_2)c_2\}\frac{t^3}{6}-a_{22}(a_2-a_{22}c_2)^2c_2^2\frac{t^3}{3}$$

$$\therefore v_{2,3}(t)=[(a_2-a_{22}c_2)c_2\{(a_2-2a_{22}c_2)(a_2-2a_{22}c_2)-2a_{22}(a_2-a_{22}c_2)c_2\}\frac{t^3}{6}$$

$$v_{1,4}(t)=(a_1-2a_{11}c_1+a_{12}c_2)\int_0^t v_{1,3}(t)dt-2a_{11}\int_0^t v_{1,1}(t)v_{1,2}(t)dt+a_{12}\int_0^t v_{1,1}(t)v_{2,2}(t)dt$$

$$+a_{12}\int_0^t v_{1,2}(t)v_{2,1}(t)dt+a_{12}c_1\int_0^t v_{2,3}(t)dt$$

$$=[(a_1-2a_{11}c_1+a_{12}c_2)\{(a_1-2a_{11}c_1+a_{12}c_2)\{(a_1-2a_{11}c_1+a_{12}c_2)(a_1-a_{11}c_1+a_{12}c_2$$

$$-\frac{a_{11}H_1}{c_1})c_1+a_{12}c_1c_2(a_2-a_{22}c_2)\}+(a_1-a_{11}c_1+a_{12}c_2-\frac{a_{11}H_1}{c_1})c_1\{2a_{12}c_2(a_2-a_{22}c_2)$$

$-2a_{11}(a_1 - a_{11}c_1 + a_{12}c_2 - \frac{a_{11}H_1}{c_1})c_1\} + a_{12}c_1c_2\{(a_2 - a_{22}c_2)(a_2 - 2a_{22}c_2)\}]\frac{t^4}{24} - 2a_{11}(a_1 - a_{11}c_1 + a_{12}c_2 - \frac{a_{11}H_1}{c_1})c_1\{(a_1 - 2a_{11}c_1 + a_{12}c_2)(a_1 - a_{11}c_1 + a_{12}c_2 - \frac{a_{11}H_1}{c_1})c_1 + a_{12}c_1(a_2 - a_{22}c_2)c_2\}\frac{t^4}{8} + a_{12}c_1\{(a_2 - 2a_{22}c_2)[(a_2 - a_{22}c_2)c_2(a_2 - 2a_{22}c_2)] - 2(a_2 - a_{22}c_2)c_2a_{22}(a_2 -$
a22c2c2}t424+a12c1a1− a11c1+a12c2− a11H1c1[a2− a22c2c2a2− 2a22c2]t48+ a12a2− a22c 2c2[(a1− 2a11c1+a12c2) (a1− a11c1+a12c2− a11H1c1)c1+ a12c1c2(a2− 2a22c2)] t48

$$\therefore v_{1,4}(t) = \left\{[(a_1 - 2a_{11}c_1 + a_{12}c_2)(a_1 - a_{11}c_1 + a_{12}c_2 - \frac{a_{11}H_1}{c_1})c_1 + a_{12}c_1c_2(a_2 - a_{22}c_2)\right)$$

$$[(a_1 - 2a_{11}c_1 + a_{12}c_2)^2 - c_1 6a_{11}\,(a_1 - a_{11}c_1 + a_{12}c_2 - \frac{a_{11}H_1}{c_1})]$$

$$+(a_1 - 2a_{11}c_1 + a_{12}c_2)\{2\left(a_1 - a_{11}c_1 + a_{12}c_1 - \frac{a_{11}H_1}{c_1}\right)c_1[a_{12}c_2(a_2 - a_{22}c_2)$$

$-a_{11}\left(a_1 - a_{11}c_1 + a_{12}c_2 - \frac{a_{11}H_1}{c_1}\right)c_1] + a_{12}c_1c_2[(a_2 - 2a_{22}c_2)(a_2 - a_{22}c_2)\ \ ]\ \ \} + [(a_2 -$
a22c2c2a2− 2a22c2][a12c1a2− 2a22c2+3a12c1a1− a11c1+a12c2− a11H1c1]+a2− a22c2c2{[
$(a_1 - a_{11}c_1 + a_{12}c_2 - \frac{a_{11}H_1}{c_1})c_1[3a_{12}(a_1 - 2a_{11}c_1 + a_{12}c_2)] + (a_2 - a_{22}c_2)c_2[3a_{12}^2c_1 - 2a_{22}a_{12}c_1]\}\frac{t^4}{24}$

$$v_{2,4}(t) = a_2\int_0^t v_{2,3}(t)dt - 2a_{22}c_2\int_0^t v_{2,3}(t)dt - 2a_{22}\int_0^t v_{2,1}(t)v_{2,2}(t)dt$$

$$= (a_2 - 2a_{22}c_2)\,(a_2 - a_{22}c_2)c_2\{(a_2 - 2a_{22}c_2)^2\text{-}2a_{22}c_2(a_2 - a_{22}c_2)\}\,\frac{t^4}{24}$$

$$-2a_{22}(a_2 - a_{22}c_2)c_2\{(a_2 - a_{22}c_2)c_2(a_2 - 2a_{22}c_2)\}\frac{t^4}{8}$$

$$\therefore v_{2,4}(t) = (a_2 - a_{22}c_2)c_2(a_2 - 2a_{22}c_2)\{(a_2 - 2a_{22}c_2)^2\text{-}8a_{22}c_2(a_2 - a_{22}c_2)\}\,\frac{t^4}{24}$$

Up to the terms which contain maximum the power of four, we obtain

$$N_1(t) = \lim_{p\to 1} v_1(t) = \sum_{x=0}^{4} v_{1,x}(t) = v_{1,0}(t) + v_{1,1}(t) + v_{1,2}(t) + v_{1,3}(t) + v_{1,4}(t)$$

$$N_1(t) = \lim_{p\to 1} v_2(t) = \sum_{x=0}^{4} v_{2,x}(t) = v_{2,0}(t) + v_{2,1}(t) + v_{2,2}(t) + v_{2,3}(t) + v_{2,4}(t)$$

The solutions by Homotopy Perturbation Method are derived as

$$N_1(t) = c_1 + \left[\left(a_1 - a_{11}c_1 + a_{12}c_2 - \frac{a_{11}H_1}{c_1}\right)c_1\right]t$$

$$+[(a_1 - 2a_{11}c_1 + a_{12}c_2)\left(a_1 - a_{11}c_1 + a_{12}c_2 - \frac{a_{11}H_1}{c_1}\right)c_1 + a_{12}c_1(a_2 - a_{22}c_2)c_2]\frac{t^2}{2} + \{(a_1$$

$$-2a_{11}c_1 + a_{12}c_2)[(a_1 - 2a_{11}c_1 + a_{12}c_2)\left(a_1 - a_{11}c_1 + a_{12}c_2 - \frac{a_{11}H_1}{c_1}\right)c_1$$

$$+a_{12}c_1c_2(a_2 - a_{22}c_2)] + \left(a_1 - a_{11}c_1 + a_{12}c_2 - \frac{a_{11}H_1}{c_1}\right)$$

$$c_1[2a_{12}c_2(a_2 - a_{22}c_2) - 2a_{11}\left(a_1 - a_{11}c_1 + a_{12}c_2 - \frac{a_{11}H_1}{c_1}\right)c_1] + a_{12}c_1c_2$$

$$[(a_2 - a_{22}c_2)(a_2 - 2a_{22}c_2)]\frac{t^3}{6} + \{[(a_1 - 2a_{11}c_1 + a_{12}c_2)\left(a_1 - a_{11}c_1 + a_{12}c_2 - \frac{a_{11}H_1}{c_1}\right)c_1$$

$$+a_{12}c_1c_2(a_2 - a_{22}c_2)]\left[(a_1 - 2a_{11}c_1 + a_{12}c_2)^2 - 6a_{11}c_1\left(a_1 - a_{11}c_1 + a_{12}c_2 - \frac{a_{11}H_1}{c_1}\right)\right]$$

$$+(a_1 - 2a_{11}c_1 + a_{12}c_2)\{2(a_1 - a_{11}c_1 + a_{12}c_2 - \frac{a_{11}H_1}{c_1})c_1[a_{12}c_2(a\_2 - a\_22\ c\_2\ )$$

$$-a_{11}\left(a_1 - a_{11}c_1 + a_{12}c_2 - \frac{a_{11}H_1}{c_1}\right)c_1] + a_{12}c_1c_2[(a_2 - a_{22}c_2)(a_2 - 2a_{22}c_2)]\}$$

$$+[(a_2 - a_{22}c_2)c_2(a_2 - 2a_{22}c_2)]\left[a_{12}c_1(a_2 - 2a_{22}c_2) + 3a_{12}c_1\left(a_1 - a_{11}c_1 + a_{12}c_2 - \frac{a_{11}H_1}{c_1}\right)\right]$$

$$+(a_2 - a_{22}c_2)c_2\{\left(a_1 - a_{11}c_1 + a_{12}c_2 - \frac{a_{11}H_1}{c_1}\right)c_1[3(a_1 - 2a_{11}c_1 + a_{12}c_2)a_{12}]$$

$$+(a_2 - a_{22}c_2)c_2(3a_{12}^2c_1 - 2a_{22}a_{12}c_1))\}\ \frac{t^4}{24}$$

$\therefore\ N_2(t) = c_2 + [(a_2 - a_{22}c_2)c_2]t + [(a_2 - a_{22}c_2)(a_2 - 2a_{22}c_2)c_2]\frac{t^2}{2} + [(a_2 - a_{22}c_2)c_2\{(a_2 -$
2a22c2a2− 2a22c2−2a22c2a2− a22c2}]t36+[a2− a22c2(a2− 2a22c2)c2a2− 2a22c2a2− 2a22c 2−8a22c2a2− a22c2]t424

5. Numerical Illustrations

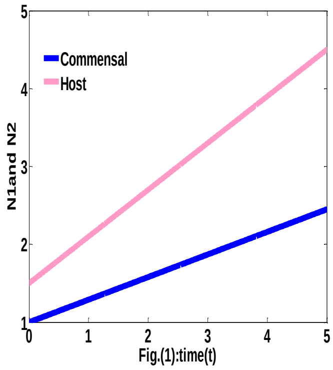
The nature of the ecological commensalism is to be established with a set of numerical solutions which can be illustrated in a course of specified time interval.

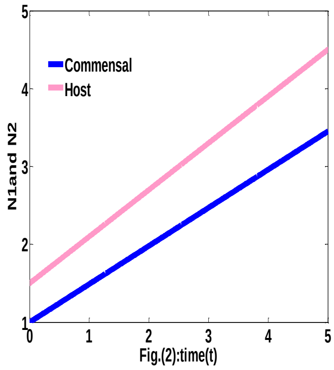
The fixed parameters are assumed as **a_{11}=0.8, a_{12}=0.9, a_2=1.3,a_{22}= 0.6, H_1=0.7, c $_1$=1.0 and c_2=1.5**

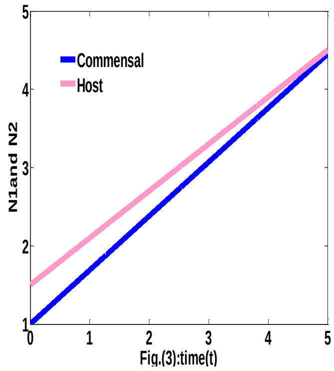
The varying variable is **a_1, i.e a_1= from 0.3 to 3.7 with difference 0.2**
and then t* is derived(1.73,1.03,0.74,0.56,0.48,0.41,0.38,0.33,0.29,0.25,0.21,0.20,0.18,0.16,0.14)

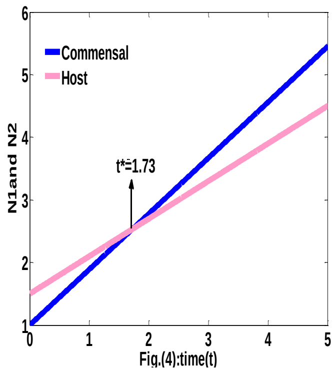
The obtained solutions are illustrated from Fig. (1) to Fig.(18).

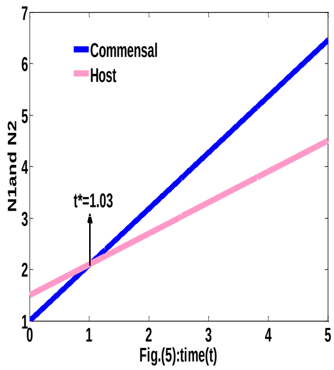
***Case(1):*In the case where natural growth rate of Ammonals Species is less than the growth rate of enemy species(a_1<a_2)-From Fig(1) to Fig(5)**



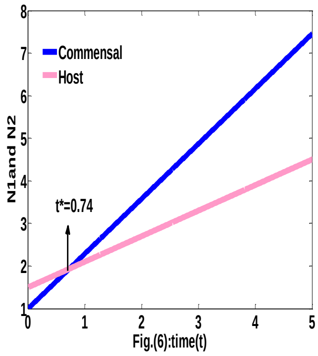


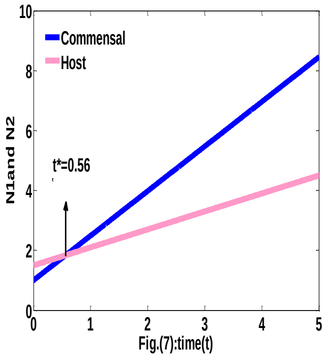


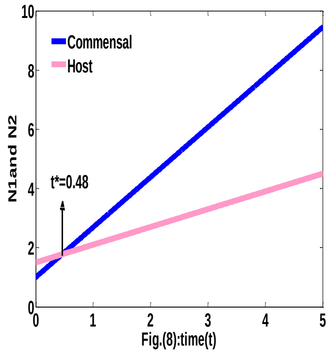


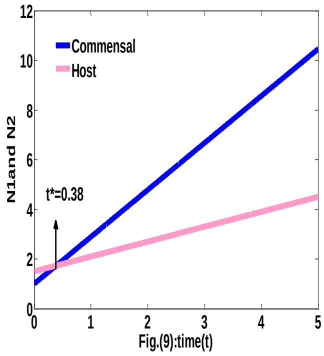


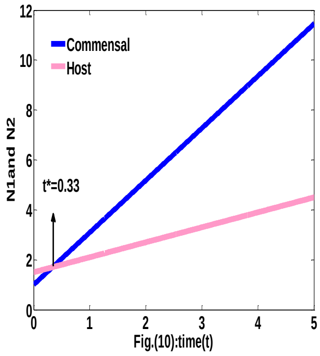
Case(2): **In the case where natural growth rate of Ammonals Species is greater than or equal to the growth rate of Enemy species($a_1 \geq a_2$) -From Fig(6) to Fig(18)**

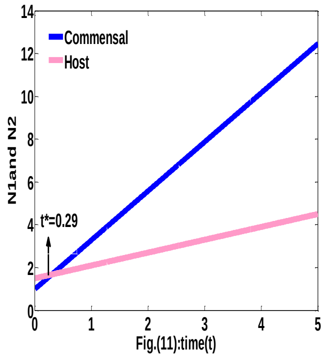


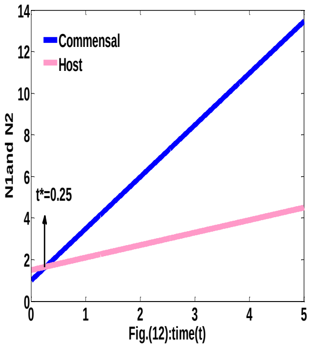


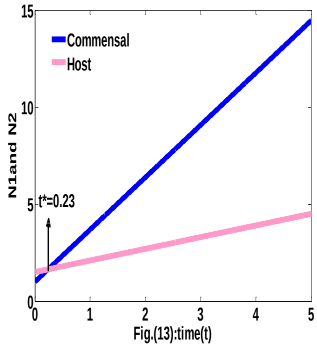


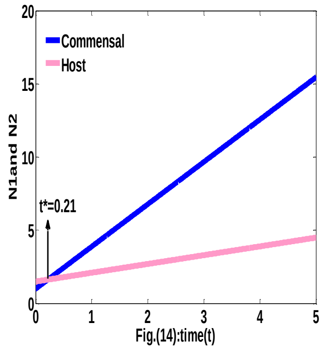


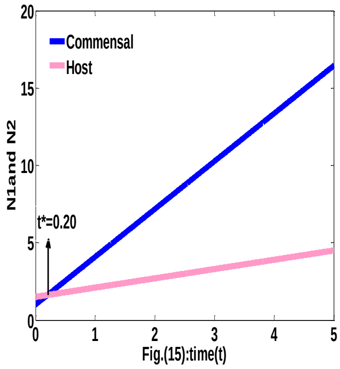


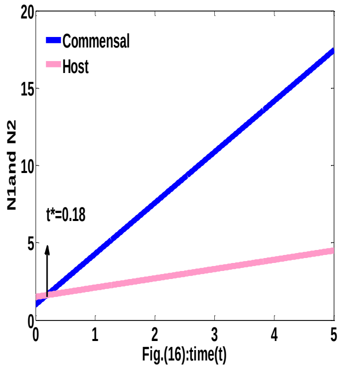


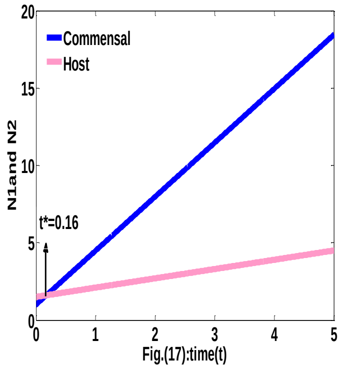


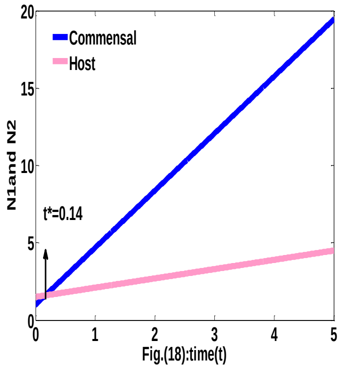












6. CONCLUSIONS

Case(1):From Fig(1) to Fig(5)

The following fruitful results are obtained

(i).At initial stage, Commensal species dominates over Host species and there is no observable direct interaction between the two species. The Host species has no considerable growth rate. Afterwards, Commensal strengthens in it's growth rate by the positive influence of Host species.

(ii).Commensal species has an exponential growth rate than the growth rate of Host species. In the period of time, it is also observed that the Commensal species has a steady growth rate after dominance reversal time (t*).

Case(2):From Fig(6) to Fig(18)

The obtained conclusions are given as below:

(i).Host species outnumbers Commensal species up to time distinct(t*).After the dominance reversal time, the Commensal species flourishes throughout the interval with a constant growth rate.

(ii).In the beginning phase, Host species has a minuscule amount of growth rate than Commensal Species. It reins over the enemy species up to t*, then after Commensal species dominates Host species in the rest of the interval.

Over All Conclusions: A mathematical model of harvested Commensal Species with limited resources is constructed by a couple of first order nonlinear differential equations. A series solution in the special case of ecological Commensalism is obtained by Homotopy Perturbation Method. Some numerical solutions are utilized for analyzing various interactions between Commensal and Host species.

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