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Apply Nonlinear Approaches to Linear Problems

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Abstract: *The article demonstrates importance and necessity of applying non-linear approaches to solving linear problems. It first presents several successful applications of non-linear methods that are applied in computation of determinants and inverses of matrices, calculations of discrete integrals and parallel pipeline computations in industrial developments, and then from viewpoint of scientific philosophy points out that non-linearization of linear problems and linearization of non-linear problems are complementary each other and might form an entia of methodology in scientific researches. The complementary logic relationship is shown with a diagram. Key issues in realizing the non-linearization of linear problems are investigated. In the end, the article proposes a study on the issue of non-linearization of linear problems for it revealing polymorphism of linear problems and providing a new channel in technical developments.*

Keywords: *Linear problems, Non-linearization, Methodology, Inverse thinking*

1. INTRODUCTION

Linearization of nonlinear problems has been a habituated mode of thinking in every person who works on science research and technology development in the past years. Human being have created a various linear tools such as differential, wedge differential, manifold and Lie-algebra and have solved kinds of problems in advances of human life [1]-[3].

The idea of applying non-linear approaches on linear problems is an academic thought reversing to the idea that using linear approaches to solve non-linear problems. From Ronald A Devore's summarization [4], Birman and Solomyak were pioneers of such thoughts owe to their raising non-linear approaches in 1967. Afterwards, people have done their researches in many related fields and gradually form systematic method [5]. According to Albert Cohen's report in 2007 Zhuhai Conference, non-linear methods are widely applied and achieved in aspects of analysis of isolated particles, wavelet transformations, Fourier transformation and image process [6].

Though the non-linearization of linear problems may be a branch of the previously stated non-linearization, which I call classical non-linearization, its development and advance are a little slower and more difficult than the classical non-linearization due to the resistances and queries from the thoughts of linearization that is supported with rich theories and methods that exhibit successes in solving kinds of problems. Nevertheless, it has showed its advantages in computation of matrix reverses, parallel pipeline computation, computation of discrete integrals, discrete Fourier transformation. Based on these facts, I propose the topic in this article to call more studies on this issue.

2. ANOTHER PATH TO ROMAN

This section shows via examples that, non-linearization of linear problems may obtain unexpected effects, sometimes even better than the classical approaches.

2.1 Computation of Determinants

It is known that determinant is a basic object in linear algebra but its computation is never a simple linear process. Non-linear approaches have been major means. There are many literatures to support this point of view. In 1975, Miklosko J A raised a recursion method to compute the determinant of a pentadiagonal matrix, as reference [7] shows. In 1999, Abbott J, Bronstein M and Mulders T uses Chinese remaindering and Hensel lifting techniques to develop a fast algorithm that is introduced in

[8]. In 2001, Rote G put forward algebraic & combinatorial approaches to develop division-free Algorithms as seen in [9]. In 2012, Ogita T proposed a robust non-linear algorithm that involves Monte Carlo's approach [10]. In 2014, Jia L and Yao GT proposed a way for computation determinant of circulant matrices by solving a polynomial algebraic equation [11]. Elouafi M , Ahmed DAH put forward a new recurrence formula for the Determinant of banded Matrices[12].

2.2 Computing Inverses of Matrices

Calculation of matrices is undoubtedly a linear issue and computing an inverse of a matrix is also a classical linear problem that directly determines efficiency, stability of solution to a linear systems. However, everyone who has an experience in finding a fast computation of reverse of a matrix knows heartfeltly that it is never a simple problem. To reach a faster goal, non-linearization is applied.

In 2006, Bhavanam Rami Reddy and his colleagues put forward a recursive approach to calculate inverse of a bordered matrix [13]. In 2007, Cao Jianshu raised a recursive method to compute reverse of a general sparse matrix [14]. In 2009 to 2011, I found a high-efficient recursive approach to calculate reverse of a matrix [15]-[17].

2.3 Construction of Parallel Pipeline CNC interpolator

Interpolation means to calculation objective points one by one and it is a fundamental function of a computer numerical control (CNC) system. Classical approach is to arrange the interpolated points by a linear mapping that the first computed point is at the first position and the second to a second and so on, and then compute and manufacture the points sequentially one by one, as illustrated in figure 1.

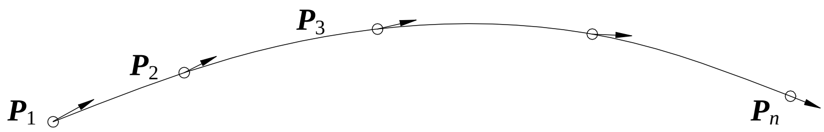

Fig.1. *A linear arrayed points*

Obviously the classical treatment cannot be applied by a parallel system because computation of a midpoint depends on its prior one although these points are relatively independent each other. Since 2009, I began to find a new way to compute the points in parallel mode and finally developed a method of high speed and high accuracy based on binary tree structure. Literatures [18] to [23] are of my related work.

Binary tree structure is of course not a linear one. However it can be used to solve linear problems.

2.4 Non-linearization of Discrete Integrals

Calculation of integrals is a typical linear one. It is also known that integrals cannot all be analytic. Recently study shows that, by a Darbox transformation, a discrete integral can be changed into an easier one. In the process, non-linearization plays an important role. An infinite dimensional integral can be transformed to be two finite integrals, as seen in literatures [24] and [25].

3. VALUABLE FOR AN ATTEMPT

Study of non-linearization of linear problems is valuable in science researches. In addition that it is a complement of methodology in science research, it also reveals polymorphism of linear problems and provides a new channel in technical developments.

3.1 Complement of Methodology in Science Researches

Scientific philosophy tells us that any essence will have multiple manifestations. Therefore science never excludes a new approach to solve a problem in case that there already existed one or more mature ways to the problem. This is what science means. Ever since the age of middle school, we were taught that both an original proposition and its counter proposition are all fit for solution of a problem. Theories of inverse problems together their methodologies [26][27] show that, it might not obtain a best result for a complicated problem if it is restricted in an exploration of single direction or a fixed thinking. From this point of view, non-linearization of linear problems and classical linearization of the problems might form an entia that are complementary each other. Figure 2 can illustrates this complementary logic relationship.

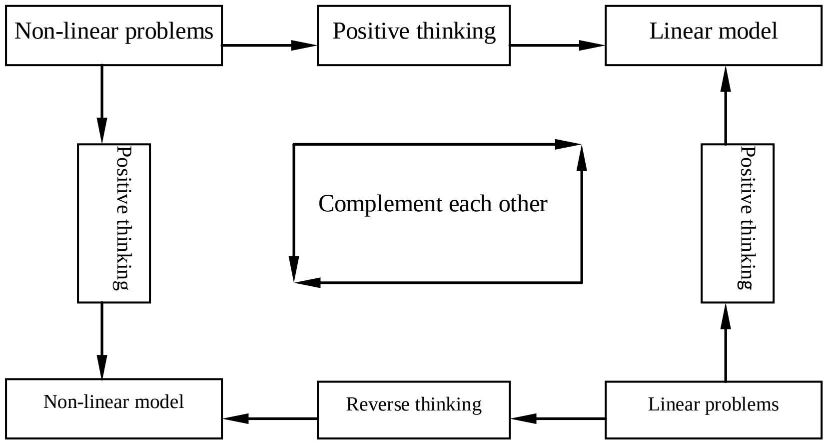

Fig. 2. *Complement between Non-linearization and linearization*

3.2 Reveal of Polymorphism of Linear problems

Process of Scientific research is a process to reveal essence of researched objects. Historically, a linear problem has been regarded to a simplest problem. There is little evaluation to a problem and its solution approach. However, results of non-linearization of linear problem show that, even a linear problem has its polymorphism: it may map many non-linear issues, as shown in figure 3. Hence its essence is revealed only when researches are done from different angles, different levels and different systems.

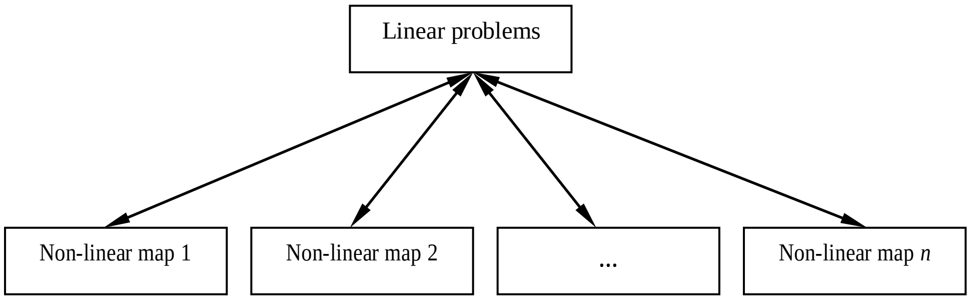

Fig. 3. *Polymorphism of linear problems*

3.3 New Channel in Technical Developments

The ultimate goal of scientific researches is to guide and serve technical developments. Classical linear methods have already produced numerous advanced ways for technical developments. However, as they are known, almost every one has its insufficient in this or that way. Therefore, non-linearization of linear problems can enable us to find a way for solving classical problems from another point of view. Take an example of data storage in a system on chip (SoC). Due to limitation of memory, many classical algorithms are restricted or forbidden to use in a SoC. Classical linear approaches such as array, stack and queue cannot provide a way to access mass data stored in form of external data. Then I develop a binary storage and accessing approach to solve the problem, as literature [22] shows. It is of course a new channel.

4. WHAT NEEDS TO DO

Non-linearization of linear problems needs a lot of research work to realize. In my opinion, the most important ones are the following three

(i). Objectives. What kind of linear problems need to be non-linearized?

(ii). Valid approaches. Which approach is suit for non-linearizing a linear problem?

(iii). Applications. Where and how can the researched resulted be applied?

4.1 Research Objectives

In comparison to non-linear problems, descriptions of linear problems are relatively simple and their classification is also simple. But it does not mean that it is simple to non-linearize a linear problem. In fact, we have not got a general rule to what a linear problem can be non-linearized, neither we have a rule to judge if a non-linearization is superior to its original linear solution. Therefore, we need first classify linear problems to see if a kind can be non-linearized, as illustrated in figure 4.

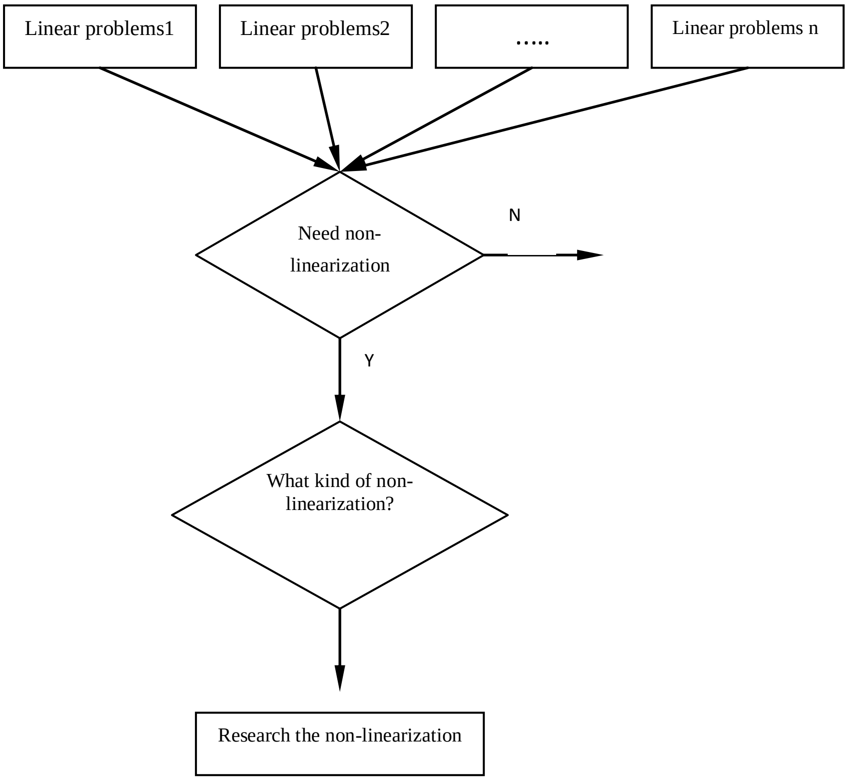

Fig. 4. *Classification of non-linearization of linear problems*

4.2 Valid Approaches

Once a research objective is determined, approach to non-linearize it becomes a key research issue. Due to the fact that many non-linear problems may correspond to one linear model, as shown in figure 5, while a linear problem may correspond to multiple non-linearizing approaches, as shown in figure 6, approach to find a non-linearization of a linear model is actually an optimizational process. And this enhances difficulty in non-linearization of a linear model.

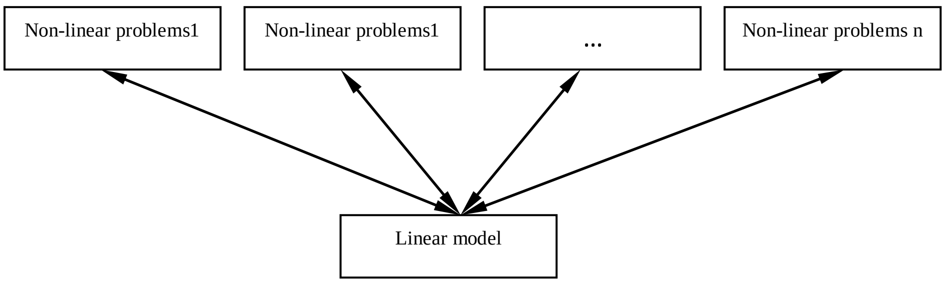

Fig. 5. *One to multiple mapping of a linear model to its original problems*

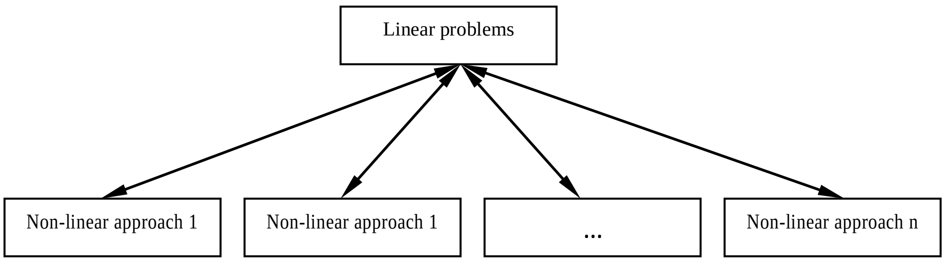

Fig. 6. *One to multiple mapping of a linear problem to its non-linear approaches*

4.3 Application Occasions

Core of application of a scientific research is to apply the science theory to concrete engineering. Non-linearization of a linear problem also faces such demands. Non-linearization may change classical process to final research results. For example, a binary storage structure must have its own physical structure as well as logic access mode. Hence there exists difference between a linear mode and non-linear mode in engineering design and implementation. This requires scientific researchers to combine their researches with engineering developments, which is a long historical issue both in science researches and in engineering developments.

5. CONCLUSION

Non-linearization of linear problems is essentially a complementary approach to study linear problems. The non-linearization of linear problems and linearization of non-linear problems are two sides of a coin and should form an entia of methodology in scientific researches. Hence I propose a systematic study on the non-linearization of linear problems because it reveals polymorphism of linear problems and provides a new channel in technical developments. It is worthy of an attempt in viewpoint of scientific philosophy.

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WANG Xingbo, was born in Hubei, China. He got his Master and Doctor's degree at National University of Defense Technology of China and had been a staff in charge of researching and developing CAD/CAM/NC technologies in the university. Since 2010, he has been a professor in Foshan University, still in charge of researching and developing CAD/CAM/NC technologies. Wang has published 8 books, over 80 papers and obtained more than 20 patents in mechanical engineering.