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Non-Darcy Convective Heat and Mass Transfer Flow in a Circular Annulus with Soret, Dufour Effects and Constant Heat Flux

C. Sulochana
Professor, Department of Mathematics
Gulbarga University
Gulbarga-06, Karnataka, India
sulo_laxman@yahoo.com

Tayappa H.
Assistant Professor of Mathematics
SSA Govt. First Grade College
Bellary, Karnataka, India
tayappa_h@rediffmail.com

Abstract: *We investigate the combined effect of Soret and Dufour effects on free and forced convection flow through a porous medium in a co-axial cylindrical duct where the boundaries are maintained at constant temperature and concentration. The Brinkman-Forchhimer extended Darcy equations which takes into account the boundary and inertia effects are used in the governing linear momentum equations. The effect of density variation is confined to the buoyancy term under Boussinesq approximation. The momentum, energy and diffusion equations are coupled equations. In order to obtain a better insight into this complex problem, we make use of Galerkin finite element analysis with quadratic polynomial approximations. The behaviour of velocity, temperature and concentration is analysed at different axial positions. The rates of heat and mass transfer have also been obtained for variations in the governing parameters.*

Keywords: *Soret effect, Dufour effect, Chemical reaction, Porous medium, Circular annulus, Finite element technique*

1. INTRODUCTION

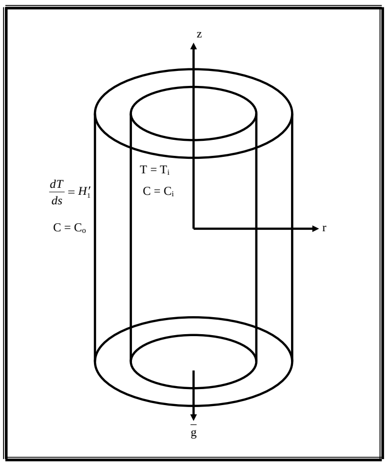
Transport phenomena involving the combined influence of thermal and concentration buoyancy are often encountered in many engineering systems and natural environments. There are many applications of such transport processes in the industry, notably in chemical distilleries, heat exchangers, solar energy collectors and thermal protection systems. In all such classes of flows, the driving force is provided by a combination of thermal and chemical diffusion effects. In atmosphere flows, thermal convection of the earth by sunlight is affected by differences in water vapour concentration. This buoyancy driven convection due to coupled heat and mass transfer in porous media has also many important applications in energy related engineering. These include moisture migration, fibrous insulation, spreading of chemical pollution in saturated soils, extraction of geothermal energy and underground disposal of natural waste.

The increasing cost of energy has led technologists to examine measures which could considerably reduce the usage of the natural source energy. Thermal insulations will continue to find increased use as engineers seek to reduce cost. Heat transfer in porous thermal insulation within vertical cylindrical annuli provides us insight into the mechanism of energy transport and enables engineers to use insulation more efficiently. In particular, design engineers require relationships between heat transfer, geometry and boundary conditions which can be utilized in cost-benefit analysis to determine the amount of insulation that will yield the maximum investment. Apart from this, the study of flow and heat transfer in the annular region between the concentric cylinders has applications in nuclear waste disposal research. It is known that canisters filled with radioactive rays be buried in earth so as to isolate them from human population and is of interest to determine the surface temperature of these canisters. This surface temperature strongly depends on the buoyancy driven flows sustained by the heated surface and the possible moment of groundwater past it. This phenomenon is ideal to the study of convection flow in a porous medium contained in a cylindrical annulus [18, 17, 16].

Free convection in a vertical porous annulus has been extensively studied by Prasad [17], Prasad and Kulacki [16] and Prasad et al [18] both theoretically and experimentally. Caltagirone [3] has published a detailed theoretical study of free convection in a horizontal porous annulus including possible three dimensional and transient effects. Convection through annulus region under steady state conditions has also been discussed with two cylindrical surface kept at different temperatures [8]. This work has been extended in temperature dependent convection flow [5,6,8] as well as convection flows through horizontal porous channel whose inner surface is maintained at constant temperature while the other surface is maintained at circumferentially varying sinusoidal temperature[11,20,29].

Free convection flow and heat transfer in hydromagnetic case is important in nuclear and space technology [8,12,15,23,32,31]. In particular, such convection flow in a vertical annulus region in the presence of radial magnetic field has been studied by Sastry and Bhadram[21]. Nanda and Purushotham [9] have analysed the free convection of a thermal conducting viscous incompressible fluid induced by traveling thermal waves on the circumference of a long vertical circular cylindrical pipe. Whitehead [30], Neeraja[10] has made a study of the fluid flow and heat transfer in a viscous incompressible fluid confined in an annulus bounded by two rigid cylinders. The flow is generated by periodical traveling waves imposed on the outer cylinder and the inner cylinder is maintained at constant temperature.

Chen and Yuh[4] have investigated the heat and mass transfer characteristics of natural convection flow along a vertical cylinder under the combined buoyancy effects of thermal and species diffusion. Sivanjaneya Prasad [24] has investigated the free convection flow of an incompressible, viscous fluid through a porous medium in the annulus between the porous concentric cylinders under the influence of a radial magnetic field. Antonio[2] has investigated the laminar flow, heat transfer in a vertical cylindrical duct by taking into account both viscous dissipation and the effect of buoyancy, The limiting case of fully developed natural convection in porous annuli is solved analytically for steady and transient cases by E. Sharawi and Al-Nimir[22] and Al-Nimir[1]. Philip[14] has obtained solutions for the annular porous media valid for low modified Reynolds number. Ravi[19] has analysed the unsteady convective heat and mass transfer through a cylindrical annulus with constant heat sources. Sreevani[26] has studied the convective heat and mass transfer through a porous medium in a cylindrical annulus under radial magnetic field with Soret effect. Prasad [17] has analysed the convective heat and mass transfer in an annulus in the presence of heat generating source under radial magnetic field. Reddy[25] has discussed the Soret effect on mixed convective heat and mass transfer through a porous cylindrical annulus. For natural convection, the existence of large temperature differences between the surfaces is important. Keeping the applications in view, Sudheer Kumar et al [28] have studied the effect of radiation on natural convection over a vertical cylinder in a porous media. Padmavathi [13] has analysed the convective heat transfer in a cylindrical annulus by using finite element method. Recently Mahesha Narayana et al [7] have discussed viscous dissipation and thermal radiation effects on mixed convection from a vertical plate in a non-darcy porous medium.



SCHEMATIC DIAGRAM OF THE CONFIGURATION

In this paper, we discuss the effect of Soret and Dufour effects on free and forced convection flow through a porous medium in a co-axial cylindrical duct where the boundaries are maintained at constant temperature and concentration. The Brinkman-Forchhimer extended Darcy equations which take into account the boundary and inertia effects are used in the governing linear momentum equations. The effect of density variation is confined to the buoyancy term under Boussinesq approximation. The momentum, energy and diffusion equations are coupled equations. In order to obtain a better insight into this complex problem, we make use of Galerkin finite element analysis with quadratic polynomial approximations. The Galerkin finite element analysis has two important features. The first is that the approximation solution is written directly as a linear combination of approximation functions with unknown nodal values as coefficients. Secondly, the approximation polynomials are chosen exclusively from the lower order piecewise polynomials restricted to contiguous elements. The behaviour of velocity, temperature and concentration is analysed at different axial positions. The rates of heat and mass transfer have been obtained for variations in the governing parameters.

2. FORMULATION OF THE PROBLEM

We consider the free and forced convection flow in a vertical circular annulus through a porous medium whose walls are maintained at a constant temperature and concentration. The flow, temperature and concentration in the fluid are assumed to be fully developed. Both the fluid and porous region have constant physical properties and the flow is a mixed convection flow taking place under thermal and molecular buoyancies and uniform axial pressure gradient. The Boussinesq approximation is invoked so that the density variation is confined to the thermal and molecular buoyancy forces. The Brinkman-Forchhimer-Extended Darcy model which accounts for the inertia and boundary effects has been used for the momentum equation in the porous region. The momentum, energy and diffusion equations are coupled and non-linear. Also the flow is unidirectional along the axial direction of the cylindrical annulus. Making use of the above assumptions the governing equations are

$$\left.\begin{aligned}&-\frac{\partial p}{\partial z}+\frac{\mu}{\delta}\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}\right)-(\frac{\mu}{k})u-(\frac{\sigma\mu_e^2 H_o^2}{r^2})u+\frac{\rho\delta F}{\sqrt{k}}u^2+\rho g\beta(T-T_0)\\&\qquad\qquad+\rho g\beta^{\bullet}(C-C_0)=0\end{aligned}\right| \tag{1}$$

$$\rho c_p u\frac{\partial T}{\partial z}=\lambda\left(\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}\right)+Q(T_o-T)+k_{12}\left(\frac{\partial^2 C}{\partial r^2}+\frac{1}{r}\frac{\partial C}{\partial r}\right)+Q_1^{'}(C-C_o) \tag{2}$$

$$u\frac{\partial C}{\partial z}=D_1\left(\frac{\partial^2 C}{\partial r^2}+\frac{1}{r}\frac{\partial C}{\partial r}\right)-kC+k_{11}\left(\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}\right) \tag{3}$$

where u is the axial velocity in the porous region, T, C are the temperature and concentration of the fluid, k is the permeability of porous medium, F is a function that depends on Reynolds number, the microstructure of the porous medium and D_1 is the molecular diffusivity, β is the coefficient of the thermal expansion, β* is the coefficient of volume expansion, Cp is the specific heat, ρ is density and g is gravity.

The relevant boundary conditions are

$$\begin{aligned}&u=0, \qquad T=T_i\ , C=Ci \quad \text{at} \quad r=a\\&u=0, \qquad \frac{\partial T}{\partial r}=H_1^{'}, C=Co \quad \text{at} \quad r=a+s\end{aligned} \tag{4}$$

We now define the following non-dimensional variables

$$z^*=\frac{z}{a} \quad , \quad r^*=\frac{r}{a} \quad , \quad u^*=\frac{a}{\gamma}u$$

$$p^* = \frac{pa\delta}{\rho\gamma^2} \quad , \quad \theta^* = \frac{T - T_i}{\Delta T} \quad , \quad s^* = \frac{s}{a}$$

$$C^* = \frac{C - C_i}{C_o - C_i}$$

Introducing these non-dimensional variables, the governing equations in the non-dimensional form are (on removing the stars)

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} = \pi + \delta(D^{-1} + \frac{M^2}{r^2})u + \delta^2 \Lambda u^2 - \delta G(\theta + N\,C) \tag{5}$$

$$\frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r}\frac{\partial \theta}{\partial r} - \alpha\theta = P_r N_1 u + Du(\frac{\partial^2 C}{\partial r^2} + \frac{1}{r}\frac{\partial C}{\partial r}) + Q_1 C \tag{6}$$

$$\frac{\partial^2 C}{\partial r^2} + \frac{1}{r}\frac{\partial C}{\partial r} - \gamma\, C = Sc\, N_2 u + ScSr(\frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r}\frac{\partial \theta}{\partial r}) \tag{7}$$

where

$\Lambda = FD^{-1}$ (Inertia parameter or Forchhimer number)

$G = \frac{g\beta(T_1 - T_0)a^3}{\gamma^2}$ (Grashoff number)

$D^{-1} = \frac{a^2}{k}$ (Inverse Darcy parameter), $N_1 = \frac{Aa}{T_1 - T_0}$ (Non-dimensional temperature gradient)

$N_2 = \frac{Ba}{C_1 - C_0}$ (Non-dimensional concentration gradient), $P_r = \frac{\rho c_p \gamma}{\lambda}$ (Prandtl number)

$Sc = \frac{\nu}{D_1}$ (Schmidt number), $S_o = \frac{k_{11}\Delta C}{\nu \Delta T}$ (Soret parameter)

$Du = \frac{k_{12}\Delta T}{\lambda \Delta C}$ (Dufour parameter), $\gamma = \frac{ka^2}{D_1}$ (Chemical reaction parameter)

$\alpha = \frac{Qa^2}{\lambda Cp}$ (Source parameter), $Q_1 = \frac{Q_1' \Delta C a^2}{\lambda \Delta T}$ (Radiation absorption parameter), $\pi = \frac{\partial P}{\partial Z}$

The corresponding non-dimensional conditions are

$$u = 0 \quad , \quad \theta = 0, \qquad C=0 \quad \text{at} \quad r=1 \tag{7}$$

$$u = 0 \quad , \quad \frac{\partial \theta}{\partial r} = H_1, \quad C=1 \quad \text{at} \quad r=1+s \tag{8}$$

For N=0 the equations (5) – (7) reduce to that of Padmavathi (12)

For $\alpha = 0$ they are in good agreement with Sudha (26)

3. FINITE ELEMENT ANALYSIS

The finite element analysis with quadratic polynomial approximation functions is carried out along the radial distance across the circular duct. The behavior of the velocity, temperature and concentration profiles has been discussed computationally for different variations in governing parameters. The Gelarkin method has been adopted in the variational formulation in each element to obtain the global coupled matrices for the velocity, temperature and concentration in course of the finite element analysis.

Choose an arbitrary element e_k and let u^k, θ^k and C^k be the values of u, θ and C in the element e_k

We define the error residuals as

$$E_u^k = \frac{d}{dr}\left(r\frac{du^k}{dr}\right) + \delta G(\theta^k + NC^k) - \delta(D^{-1} + \frac{M^2}{r^2})ru^k - \delta^2\Lambda r(u^k)^2 \tag{9}$$

$$E^k{}_\theta = \frac{d}{dr}\left(r\frac{d\theta^k}{dr}\right) - rP_rN_1u^k - \alpha\theta + Du\frac{d}{dr}\left(r\frac{dC^k}{dr}\right) + Q_1C \tag{10}$$

$$E^k{}_c = \frac{d}{dr}\left(r\frac{dC^k}{dr}\right) - rScN_2u^k + ScSo\frac{d}{dr}\left(r\frac{d\theta^k}{dr}\right) \tag{11}$$

where u^k, θ^k & C^k are values of u, θ & C in the arbitrary element e_k. These are expressed as linear combinations in terms of respective local nodal values.

$$u^k = u_1^k\psi_1^k + u_3^k\psi_1^k + u_3^k\psi_3^k$$
$$\theta^k = \theta_1^k\psi_1^k + \theta_2^k\psi_2^k + \theta_3^k\psi_3^k$$
$$C^k = C_1^k\psi_1^k + C_2^k\psi_2^k + C_3^k\psi_3^k$$

where ψ_1^k, ψ_2^k --------- etc are Lagrange's quadratic polynomials.

Following the Gelarkin weighted residual method and integrating by parts equations (9) -(11) we obtain

$$\begin{aligned}&\int_{rA_1}^{rB_1} r\frac{du^k}{dr}\frac{d\psi_j^k}{dr}dr - \delta G\int_{rA_1}^{rB_1} r(\theta^k + NC^k)\psi_j^k dr + \delta(D^{-1})\int_{rA_1}^{rB_1} ru^k\psi_j^k dr + \\ &\quad + \delta(M^2)\int_{rA_1}^{rB_1}((u^k\psi_j^k)/r)dr + \delta^2\Lambda\int_{rA_1}^{rB_1} r(u^k)^2\psi_j^k dr \\ &= Q_{2\ j}^k + Q_{1\ j}^k - P\int_{rA_1}^{rB_1} r\psi_j^k dr\end{aligned} \tag{12}$$

$$-Q_{1\ j}^k = \left[\left(\frac{du^k}{dr}\right)(r\psi_j^k)\right]r_{A_1} \quad , \quad -Q_{2\ j}^k = \left[\left(\frac{du^k}{dr}\right)(r\psi_j^k)\right]r_{B_1}$$

$$\begin{aligned}&\int_{rA_1}^{rB_1} r\frac{d\theta^k}{dr}\frac{d\psi_j{}^k}{dr}dr - \alpha\int_{rA_1}^{rB_1} r\theta^k\psi_i^k dr = -Du\int_{rA_1}^{rB_1} r\frac{dc^k}{dr}\frac{d\psi_j{}^k}{dr}dr + \\ &\quad - Q_1\int_{rA_1}^{rB_1} rC^k\psi_i^k dr + N_1P_r\int_{rA_1}^{rB_1} ru^k\psi_j^k dr + R_{2\ j}^k + R_{1\ j}^k\end{aligned} \tag{13}$$

$$-R_{1\ j}^k = \left[\left(\frac{d\theta^k}{dr}\right)(r\psi_j^k)\right]r_{A_1} \quad , \quad R_{2\ j}^k = \left[\left(\frac{d\theta^k}{dr}\right)(r\psi_j^k)\right]r_{B_1}$$

$$\begin{aligned}&\int_{rA_1}^{rB_1} r\frac{dC^k}{dr}\frac{d\psi_j{}^k}{dr}dr - \gamma\int_{rA_1}^{rB_1} rC^k\psi_j^k d = -ScSo\int_{rA_1}^{rB_1} r\frac{d\theta^k}{dr}\frac{d\psi_j{}^k}{dr}dr + \\ &\quad + N_2Sc\int_{rA_1}^{rB_1} ru^k\psi_j^k dr + S_{2\ j}^k + S_{1\ j}^k\end{aligned} \tag{14}$$

$$-S_{1\ j}^k = \left[\left(\frac{dC^k}{dr}\right)(r\psi_j^k)\right]r_{A_1} \quad , \quad S_{2\ j}^k = \left[\left(\frac{dC^k}{dr}\right)(r\psi_j^k)\right]r_{B_1}$$

Expressing u^k, θ^k, C^k in terms of local nodal values in (12) - (14) we obtain

$$\left.\begin{array}{l}\sum_{i=1}^{3} u^k \int_{rA_1}^{rB_1} r \frac{d\psi_i^k}{dr}\frac{d\psi_j^k}{dr}dr - \delta G\sum_{i=1}^{3}(\theta_i^k + NC_i^k)\int_{rA_1}^{rB_1} r\psi_i^k\psi_j^k dr + \delta D^{-1}\sum_{i=1}^{3}\int_{rA_1}^{rB_1} r\psi_i^k\psi_j^k dr \\ \quad + \delta M^2\sum_{i=1}^{3}\int_{rA_1}^{rB_1}((\psi_i^k\psi_j^k)/r)dr + \delta^2\Lambda\sum_{i=1}^{3}u_i^k\int_{rA_1}^{rB_1} rU_i^k\psi_i^k\psi_j^k dr = Q_{2\,j}^{j} + Q_{1\,j}^{k} - P\int_{rA_1}^{rB_1} r\psi_i^k\psi_j^k dr\end{array}\right| \tag{15}$$

$$\left.\begin{array}{l}\sum_{i=1}^{3}\theta_i^k\int_{rA_1}^{rB_1} r\frac{d\psi_i^k}{dr}\frac{d\psi_j^k}{dr}dr - N_1P_r\sum_{i=1}^{3}u_i^k\int_{rA_1}^{rB_1} r\psi_i^k\psi_j^k dr + \alpha\int_{rA_1}^{rB_1}\theta dr + \\ \quad + Q_1\int_{rA_1}^{rB_1} rC^k\psi_i^k dr + Du\sum_{i=1}^{3}C_i^k\int_{rA_1}^{rB_1} r\frac{d\psi_i^k}{dr}\frac{d\psi_j^k}{dr}dr = R_{2\,j}^k + R_{1\,j}^k\end{array}\right| \tag{16}$$

$$\left.\begin{array}{l}\sum_{i=1}^{3}C_i^k\int_{rA_1}^{rB_1} r\frac{d\psi_i^k}{dr}\frac{d\psi_j^k}{dr}dr - \gamma\sum_{i=1}^{3}C_i^k\int_{rA_1}^{rB_1} r\psi_i^k\psi_j^k d - N_2Sc\sum_{i=1}^{3}u_i^k\int_{rA_1}^{rB_1} r\psi_i^k\psi_j^k dr + \\ \quad + ScSo\sum_{i=1}^{3}\theta_i^k\int_{rA_1}^{rB_1} r\frac{d\psi_i^k}{dr}\frac{d\psi_j^k}{dr}dr = S_{2\,j}^k + S_{1\,j}^k\end{array}\right| \tag{17}$$

Choosing different ψ_j^k's corresponding to each element e_k in the equation (9) yields a local stiffness matrix of order 3×3 in the form

$$\begin{array}{l}(f_{i\,j}^k)(u_i^k) - \delta G(g_{i\,j}^k)(\theta_i^k + NC_i^k) + \delta D^{-1}(m_{i\,j}^k)(u_i^k) + \\ \quad + \delta^2\Lambda(n_{i\,j}^k)(u_i^k) = (Q_{2\,j}^k) + (Q_{1\,j}^k) + (v_j^k)\end{array} \tag{18}$$

Likewise the equation (16) & (17) give rise to stiffness matrices

$$(e_{i\,j}^k)(\theta_i^k) - N_1P_r(t_{i\,j}^k)(u_i^k) + Du(C_i^k) = R_{2\,j}^k + R_{1\,j}^k \tag{19}$$

$$(l_{i\,j}^k)(C_i^k) - N_2Sc(t_{i\,j}^k)(u_i^k) + ScSo((\theta_i^k)) = S_{2\,j}^k + S_{1\,j}^k \tag{20}$$

where

$(f_{i\,j}^k)$, $(g_{i\,j}^k)$, $(m_{i\,j}^k)$, $(n_{i\,j}^k)$, $(e_{i\,j}^k)$, $(l_{i\,j}^k)$ and $(t_{i\,j}^k)$ are 3×3 matrices and

$$v_j^k = -P_1\int_{rA_1}^{rB_1} r\psi_i^k\psi_j^k dr$$

and $(Q_{2\,j}^k)$, $(Q_{1\,j}^k)$, $(R_{2\,j}^k)$ & $(R_{1\,j}^k)$, $(S_{2\,j}^k)$ & $(S_{1\,j}^k)$ are 3×1 column matrices. Such stiffness matrices (18) - (20) in terms of local nodes in each element are assembled using interelement continuity and equilibrium conditions to obtain the coupled global matrices in terms of the global nodal values of u, θ & C in the region.

In case we choose n quadratic elements, then the global matrices are of order 2n+1. The ultimate coupled global matrices are solved to determine the unknown global nodal values of the velocity, temperature and concentration in fluid region. In solving these global matrices an iteration procedure has been adopted to include the boundary and effects in the porous medium.

In fact, the non-linear term arises in the modified Brinkman linear momentum equation (12) of the porous medium. The iteration procedure in taking the global matrices is as follows. We split the square term into a product term and keeping one of them say U_i's under integration, the other is expanded in terms of local nodal values as in(15), resulting in the corresponding coefficient matrix $(n_{i\ j}^{k}{}'s)$ in (18), whose coefficients involve the unknown U_i's . To evaluate (18), to begin with, choose the initial global nodal values of U_i's as zeros in the zeroth approximation. We evaluate u_i's , θ_i's and C_i's in the usual procedure mentioned earlier. Later choosing these values of u_i's as first order approximation calculate θ_i's, C_i's. In the second iteration, we substitute for U_i's the first order approximation of and u_i's and the first approximation of θ_i's and C_i's and obtain second order approximation. This procedure is repeated till the consecutive values of u_i's , θ_i's and C_i's differ by a pre-assigned percentage.

For computational purpose we choose five elements in flow region

The shape functions in the region are

$$\psi_1^1 = \frac{50(-1+r-\frac{s}{5})(-1+r-\frac{s}{10})}{s^2} \qquad \psi_2^1 = \frac{100(-1+r)(-1+r-\frac{s}{5})}{s^2}$$

$$\psi_3^1 = \frac{50(-1+r)(-1+r-\frac{s}{10})}{s^2} \qquad \psi_1^2 = \frac{50(-1+r-\frac{2s}{5})(-1+r-\frac{3s}{10})}{s^2}$$

$$\psi_2^2 = -\frac{100(-1+r-\frac{2s}{5})(-1+r-\frac{s}{5})}{s^2} \qquad \psi_3^2 = \frac{50(-1+r-\frac{3s}{5})(-1+r-\frac{s}{5})}{s^2}$$

$$\psi_1^3 = \frac{50(-1+r-\frac{3s}{5})(-1+r-\frac{s}{2})}{s^2} \qquad \psi_2^3 = -\frac{100(-1+r-\frac{3s}{5})(-1+r-\frac{2s}{5})}{s^2}$$

$$\psi_3^3 = \frac{50(-1+r-\frac{s}{2})(-1+r-\frac{2s}{5})}{s^2} \qquad \psi_1^4 = \frac{50(-1+r-\frac{4s}{5})(-1+r-\frac{7s}{10})}{s^2}$$

$$\psi_2^4 = -\frac{100(-1+r-\frac{4s}{5})(-1+r-\frac{3s}{5})}{s^2} \qquad \psi_3^4 = \frac{50(-1+r-\frac{7s}{10})(-1+r-\frac{3s}{5})}{s^2}$$

$$\psi_1^5 = \frac{50(-1+r-s)(-1+r-\frac{9s}{10})}{s^2} \qquad \psi_2^5 = -\frac{100(-1+r-s)(-1+r-\frac{4s}{5})}{s^2}$$

$$\psi_3^5 = \frac{50(-1+r-\frac{9s}{10})(-1+r-\frac{4s}{5})}{s^2}$$

The global matrix for θ is

$$A_3 X_3 = B_3 \tag{21}$$

The global matrix for C is

$$A_4 X_4 = B_4 \tag{22}$$

The global matrix for u is

$$A_5 X_5 = B_5 \tag{23}$$

where

$$A_3 = \begin{bmatrix} e_{11} & e_{12} & e_{13} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ e_{21} & e_{22} & e_{23} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ e_{31} & e_{32} & e_{33} & e_{34} & e_{35} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & e_{43} & e_{44} & e_{45} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & e_{53} & e_{54} & e_{55} & e_{56} & e_{57} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & e_{65} & e_{66} & e_{67} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & e_{75} & e_{76} & e_{77} & e_{78} & e_{79} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & e_{87} & e_{88} & e_{89} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & e_{97} & e_{98} & e_{99} & e_{910} & e_{911} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & e_{109} & e_{1010} & e_{1011} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & e_{119} & e_{1110} & e_{1111} \end{bmatrix} \quad A_4 = \begin{bmatrix} k_{11} & k_{12} & k_{13} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ k_{21} & k_{22} & k_{23} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ k_{31} & k_{32} & k_{33} & k_{34} & k_{35} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & k_{43} & k_{44} & k_{45} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & k_{53} & k_{54} & k_{55} & k_{56} & k_{57} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & k_{65} & k_{66} & k_{67} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & k_{75} & k_{76} & k_{77} & k_{78} & k_{79} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & k_{87} & k_{88} & k_{89} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & k_{97} & k_{98} & k_{99} & k_{910} & k_{911} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & k_{109} & k_{1010} & k_{1011} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & k_{119} & k_{1110} & k_{1111} \end{bmatrix}$$

$$A_5 = \begin{bmatrix} h_{11} & h_{12} & h_{13} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ h_{21} & h_{22} & h_{23} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ h_{31} & h_{32} & h_{33} & h_{34} & h_{35} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_{43} & h_{44} & h_{45} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_{53} & h_{54} & h_{55} & h_{56} & h_{57} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & h_{65} & h_{66} & h_{67} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & h_{75} & h_{76} & h_{77} & h_{78} & h_{79} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & h_{87} & h_{88} & h_{89} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & h_{97} & h_{98} & h_{99} & h_{910} & h_{911} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & h_{109} & h_{1010} & h_{1011} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & h_{119} & h_{1110} & h_{1111} \end{bmatrix} \qquad h_{ij} = f_{ij} + \delta D^{-1} m_{ij} + \delta^2 \Lambda n_{ij}$$

$$X_3 = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \\ \theta_5 \\ \theta_6 \\ \theta_7 \\ \theta_8 \\ \theta_9 \\ \theta_{10} \\ \theta_{11} \end{bmatrix} \quad X_4 = \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \\ C_7 \\ C_8 \\ C_9 \\ C_{10} \\ C_{11} \end{bmatrix} \quad X_5 = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \\ u_7 \\ u_8 \\ u_9 \\ u_{10} \\ u_{11} \end{bmatrix} \quad B_3 = \begin{bmatrix} F_u^1 \\ F_u^2 \\ F_u^3 \\ F_u^4 \\ F_u^5 \\ F_u^6 \\ F_u^7 \\ F_u^8 \\ F_u^9 \\ F_u^{10} \\ F_u^{11} \end{bmatrix} \quad B_4 = \begin{bmatrix} F_c^1 \\ F_c^2 \\ F_c^3 \\ F_c^4 \\ F_c^5 \\ F_c^6 \\ F_c^7 \\ F_c^8 \\ F_c^9 \\ F_c^{10} \\ F_c^{11} \end{bmatrix} \quad B_5 = \begin{bmatrix} F_\theta^1 \\ F_\theta^2 \\ F_\theta^3 \\ F_\theta^4 \\ F_\theta^5 \\ F_\theta^6 \\ F_\theta^7 \\ F_\theta^8 \\ F_\theta^9 \\ F_\theta^{10} \\ F_\theta^{11} \end{bmatrix}$$

(The details of e_{11} etc, h_{11} etc, F_u^1 etc, F_θ^1 etc , F_c^1 are constants).

The equilibrium conditions are

$$\begin{aligned} &R_3^1 + R_1^2 = 0, \quad R_3^2 + R_1^3 = 0, \quad R_3^3 + R_1^4 = 0, \quad R_3^4 + R_1^5 = 0, \\ &Q_3^1 + Q_1^2 = 0, \quad Q_3^2 + Q_1^3 = 0, \quad Q_3^3 + Q_1^4 = 0, \quad Q_3^4 + Q_1^5 = 0, \\ &S_3^1 + S_1^2 = 0, \quad S_3^2 + S_1^3 = 0, \quad S_3^3 + S_1^4 = 0, \quad S_3^4 + S_1^5 = 0 \end{aligned} \tag{24}$$

4. Solution of the Problem

Solving these coupled global matrices for temperature, concentration and velocity (3.13)-(3.15) respectively and using the iteration procedure we determine the unknown global nodes through which the temperature, concentration and velocity at different radial intervals at any arbitrary axial cross sections are obtained. The respective expressions are given by

$$\begin{aligned}
\theta(r) &= \psi_1^1\,\theta_{11} + \psi_{21}^1\,\theta_{12} + \psi_3^1\,\theta_{13} && 1 \le r \le 1 + S*0.2 \\
&= \psi_1^2\,\theta_{13} + \psi_2^2\,\theta_{14} + \psi_3^2\,\theta_{15} && 1 + S*0.2 \le r \le 1 + S*0.4 \\
&= \psi_1^3\,\theta_{15} + \psi_2^3\,\theta_{16} + \psi_3^3\,\theta_{17} && 1 + S*0.4 \le r \le 1 + S*0.6 \\
&= \psi_1^4\,\theta_{17} + \psi_2^4\,\theta_{18} + \psi_3^4\,\theta_{19} && 1 + S*0.6 \le r \le 1 + S*0.8 \\
&= \psi_1^{15}\,\theta_{19} + \psi_2^5\,\theta_{20} + \psi_3^5\,\theta_{21} && 1 + S*0.8 \le r \le 1 + S
\end{aligned}$$

$$\begin{aligned}
C(r) &= \psi_1^1\,C_{11} + \psi_{21}^1\,C_{12} + \psi_3^1\,C_{13} && 1 \le r \le 1 + S*0.2 \\
&= \psi_1^2\,C_{13} + \psi_2^2\,C_{14} + \psi_3^2\,C_{15} && 1 + S*0.2 \le r \le 1 + S*0.4 \\
&= \psi_1^3\,C_{15} + \psi_2^3\,C_{16} + \psi_3^3\,C_{17} && 1 + S*0.4 \le r \le 1 + S*0.6 \\
&= \psi_1^4\,C_{17} + \psi_2^4\,C_{18} + \psi_3^4\,C_{19} && 1 + S*0.6 \le r \le 1 + S*0.8 \\
&= \psi_1^{15}\,C_{19} + \psi_2^5\,C_{20} + \psi_3^5\,C_{21} && 1 + S*0.8 \le r \le 1 + S
\end{aligned}$$

$$\begin{aligned}
w(r) &= \psi_1^1\,u_{11} + \psi_{21}^1\,u_{12} + \psi_3^1\,u_{13} && 1 \le r \le 1 + S*0.2 \\
&= \psi_1^2\,u_{13} + \psi_2^2\,u_{14} + \psi_3^2\,u_{15} && 1 + S*0.2 \le r \le 1 + S*0.4 \\
&= \psi_1^3\,c_{15} + \psi_2^3\,c_{16} + \psi_3^3\,c_{17} && 1 + S*0.4 \le r \le 1 + S*0.6 \\
&= \psi_1^4\,u_{17} + \psi_2^4\,u_{18} + \psi_3^4\,u_{19} && 1 + S*0.6 \le r \le 1 + S*0.8 \\
&= \psi_1^{15}\,u_{19} + \psi_2^5\,u_{20} + \psi_3^5\,u_{21} && 1 + S*0.8 \le r \le 1 + S
\end{aligned}$$

5. Nusselt Number and Sherwood Number

The rate of heat transfer (Nusselt number) is evaluated using the formula

$$Nu = -\left(\frac{d\theta}{dr}\right)_{r=1,1+s}$$

The rate of mass transfer (Sherwood number) is evaluated using the formula

$$Sh = -\left(\frac{dC}{dr}\right)_{r=1,1+s}$$

6. Discussion of the Numerical Results

In this analysis we discuss the combined influence of Soret and Dufour effects on convective heat and mass transfer flow of a viscous electrically conducting fluid through a porous medium confined in an annular region between the cylinders r = a and r = b in the presence of heat generating sources. The governing equations of flow, heat and mass transfer are solved by employing Galerkin finite element analysis. Also we consider the chemical reaction effect on flow phenomenon.

The axial velocity (w) is shown in figures 1-4 for different values of S_0, Du, Q_1, and γ. The actual axial velocity is in vertically upward direction and hence w<0 represents a reversal flow. Fig (1) represents the variation w with Soret parameter S_0. It is found that |w| depreciates with increase in $|S_0|$. The variation w with Dufour parameter is shown in fig (2). It is found that |w| enhances with increase in Du≤0.1 and depreciates with higher Du≥0.1 and again depreciates with higher Du≥0.75 (fig. 2). Fig (3) represents the variation of w with chemical reaction parameter γ. It is noticed that |w| enhances in the degenerating chemical reaction case and depreciates in the generating chemical reaction case. An increase in the radiation absorption parameter ($Q_1 \le 1.5$) results in depreciation in |w| and enhances with higher $Q_1 \ge 2.5$ (fig.4).

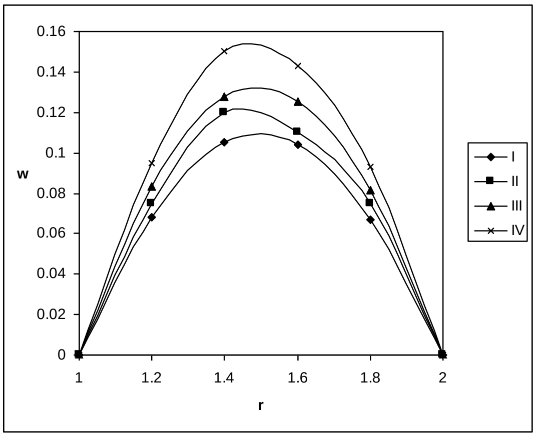


Fig. 1 : The variation of w with S_0

	I	II	III	IV
S_0	0.5	1	-0.5	-1

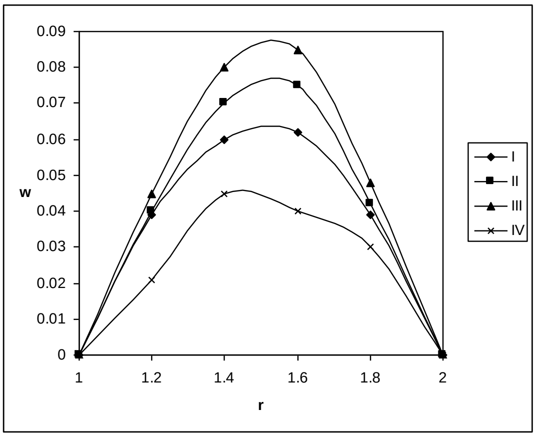


Fig. 2 : The variation of w with Du

	I	II	III	IV
Du	0.01	0.03	0.1	0.3

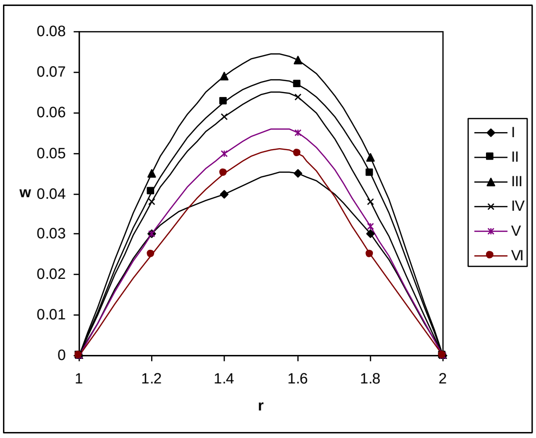


Fig. 3 : The variation of w with γ

	I	II	III	IV	V	VI
γ	0.5	1.5	2.5	-0.5	-1.5	-2.5

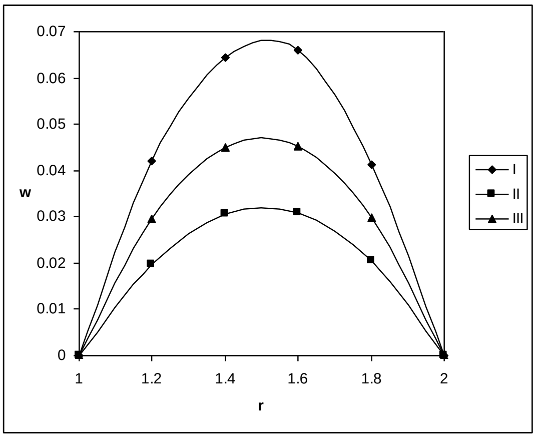


Fig. 4 : The variation of w with Q_1

	I	II	III
Q_1	0.5	1.5	2.5

The non-dimensional temperature (θ) is shown in figs. (5-8) for different parametric values. The variation θ with Soret parameter S_0 is shown in (Fig. 5). It is observed that the actual temperature reduces with $S_0 \leq 0.075$ and enhances with higher $S_0 \geq 0.1$ while an increase in $|S_0|$ we notice an enhancement in (Fig. 5). It is noticed that the actual temperature experiences an enhancement with increase in Du (Fig.6).

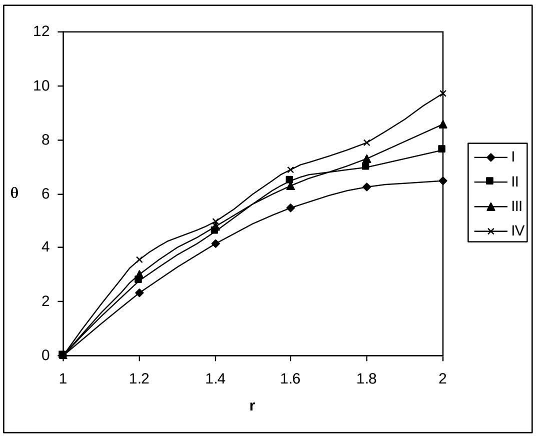


Fig. 5 : The variation of θ with S_0

	I	II	III	IV
S_0	0.5	1	-0.5	-1

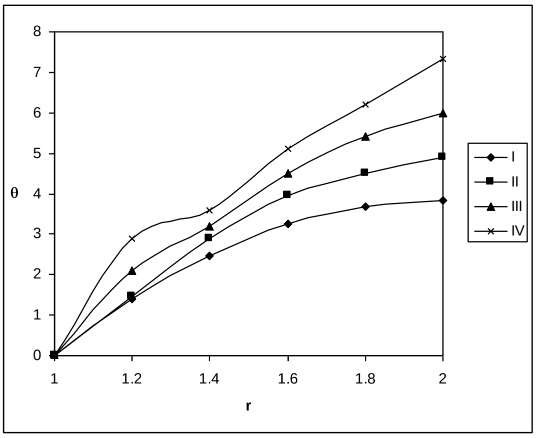


Fig. 6 : The variation of θ with Du

	I	II	III	IV
Du	0.01	0.03	0.1	0.3

The variation θ with chemical reaction parameter γ is shown in (Fig. 7). It is found that the actual temperature reduces with γ≤1.5 and enhances with γ≥2.5, while it reduces in the generating chemical reaction case. The actual temperature reduces with increase in (Q_1≤1.5) and enhances with Q_1≥2.5 in entire flow region (fig.8)

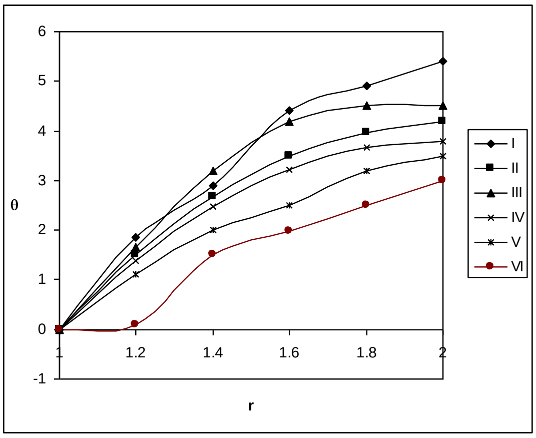


Fig. 7 : The variation of θ with γ

	I	II	III	IV	V	VI
γ	0.5	1.5	2.5	-0.5	-1.5	-2.5

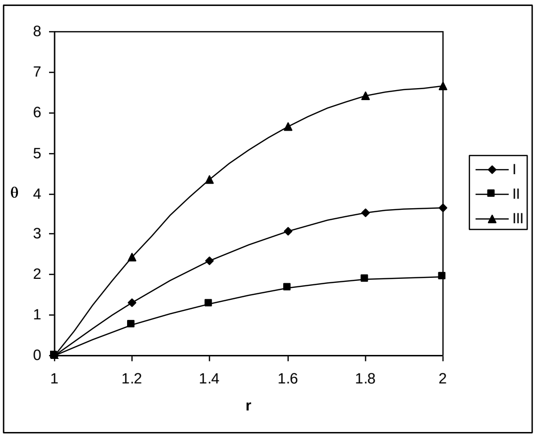


Fig. 8 : The variation of θ with Q_1

	I	II	III
Q_1	0.5	1.5	2.5

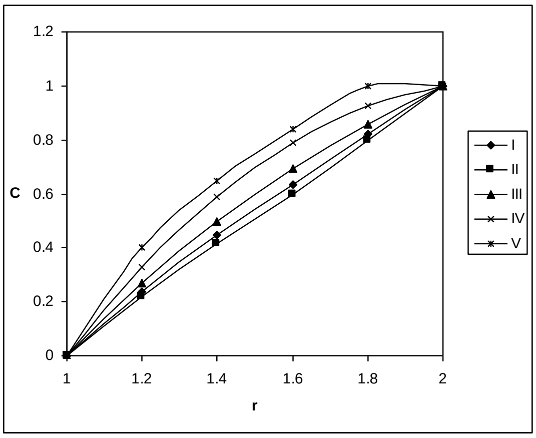


Fig. 9 : The variation of C with S_0

	I	II	III	IV	V
S_0	0.05	0.075	0.1	-0.5	-0.75

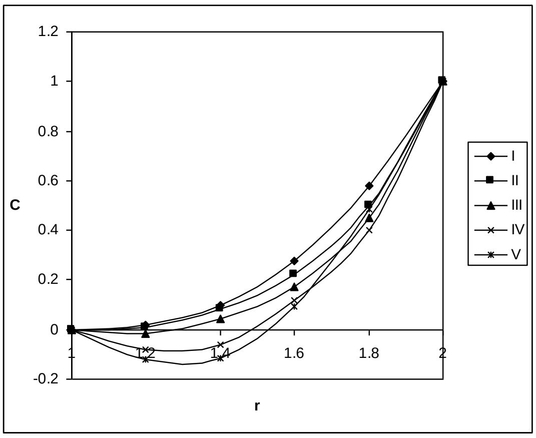


Fig. 10 : The variation of C with Du

	I	II	III	IV	V
Du	0.01	0.03	0.1	0.3	0.5

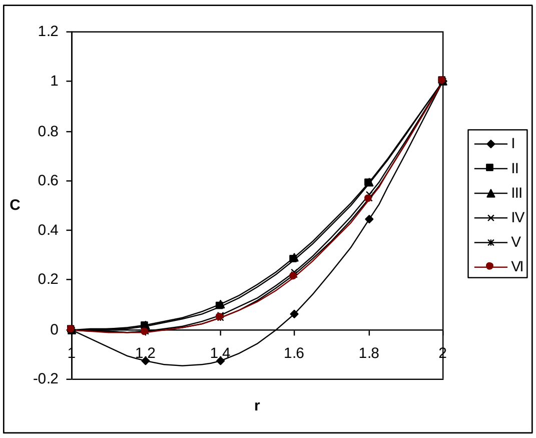


Fig. 11 : The variation of C with γ

	I	II	III	IV	V	VI
γ	0.5	1.5	2.5	-0.5	-1.5	-2.5

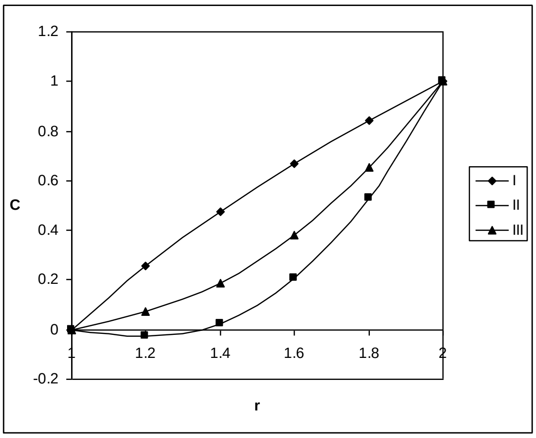


Fig. 12 : The variation of C with Q_1

	I	II	III
Q_1	0.5	1.5	2.5

The non-dimensional concentration (C) is shown in Figs (9-12) for a different parametric values. We follow the convention that the non-dimensional concentration positive (or) negative according as actual concentration $>< C_i$. Fig. 9 represents the variation of C with Soret parameter S_0, it is observed that the actual concentration reduces with $S_0>0$ and enhances with $|S_0|$ in the entire flow region. From (Fig. 10) we conclude that the actual concentration experiences an enhancement with increase in Dufour parameter Du. The actual concentration depreciates with increase $\gamma\leq1.5$ and enhances with $\gamma\geq2.5$ while it enhances in the generating chemical reaction case (Fig. 11). From fig.12, it can be seen that the actual concentration experiences a depreciation with $Q_1\leq1.5$ and enhances with higher value of $Q_1\geq2.5$.

The rate of heat transfer (Nusselt number) at the inner cylinder r = 1 is shown in tables 1-2 for different parametric values. An increase in $S_0\leq0.5$ results in a depreciation in |Nu| and for higher $S_0\geq1$, we notice an enhancement in |Nu|. The variation of Nu with Dufour parameter Du reveals that |Nu| reduces with increase Du≤0.05 and enhances with higher Du≥0.1. With respect to the chemical reaction parameter γ we find that the rate of heat transfer enhances in the degenerating chemical reaction case and reduces in the generating chemical reaction case at the inner cylinder (table. 1 & 2). The rate of heat transfer experiences an enhancement with increase in the radiation absorption parameter Q_1 at r=1.

Table1. Nusselt Number (Nu) at r = 1

G	I	II	III	IV	V	VI	VII	VIII	IX
10^3	19.8819	498.278	255.971	692.072	7.7902	8.10576	9.2184	51.4384	56.646
$3x10^3$	21.8676	365.041	354.411	1013.82	7.84019	8.15151	9.21800	31.3305	34.018
-10^3	18.2388	672.713	192.773	488.045	7.74167	8.06183	9.22456	89.3098	91.375
$-3x10^3$	16.857	896.39	150.835	356.153	7.69464	8.01981	9.23684	98.119	107.43
S_0	0.5	1	-0.5	-1	0.5	0.5	0.5	0.5	0.5
Q_1	1	1	1	1	1	1	1	1.5	2.5

Table 2. Nusselt Number (Nu) at r = 1

G	I	II	III	IV	V	VI	VII	VIII	IX
10^3	10.3299	16.0733	37.8959	151.765	9.34957	8.79618	8.53041	7.7902	9.2185
$3x10^3$	9.9441	15.5384	35.9163	143.349	9.32431	8.8135	8.5644	7.8402	9.2181
-10^3	12.0297	16.6548	40.0109	160.67	9.38249	8.78259	8.49851	7.7417	9.2246
$-3x10^3$	16.5391	17.2863	42.276	170.09	9.42371	8.77306	8.46888	7.6946	9.2368
γ	0.5	1.5	2.5	-0.5	-1.5	-2.5	0.5	0.5	0.5
Du	0.01	0.01	0.01	0.01	0.01	0.01	0.05	0.1	0.3

The rate of mass transfer (Sherwood number) at r = 1&2 shows in tables 3-6 for different parametric values. An increase in $|S_0|$ enhances |Sh| at r =1&2. The variation of Sh with Du shows that |Sh| at r = 1, reduces with increase in Du≤0.05 and enhances with higher Du≥0.01. At r = 2, |Sh| enhances with increase in Du. We find that |Sh| reduces with increase in the chemical reaction parameter $\gamma\leq1.5$ and enhances with higher $\gamma\geq2.5$ at r = 1&2. In the generating chemical reaction case, |Sh| at r = 1 enhances with $|\gamma| \leq 1.5$ and reduces with $|\gamma|\leq2.5$. At r = 2, |Sh| enhances in the generating chemical reaction case. An increasing Q_1 leads to an increase in the rate of mass transfer at both the cylinders

Table 3. Sherwood number(Sh) at r = 1

G	I	II	III	IV	V	VI	VII	VIII	IX
10^3	0.78362	81.3324	37.4524	105.422	-0.05284	-0.16477	-0.53913	4.41655	60.7864
$3x10^3$	0.7374	62.7674	46.1832	138.24	-0.060321	-0.17292	-0.553349	1.92211	39.9443
-10^3	0.82174	103.922	31.3816	82.6009	-0.045336	-0.15652	-0.52429	8.48369	90.8583
$-3x10^3$	0.85368	130.798	27.08	66.5633	-0.03777	-0.148147	-0.50875	14.9576	133.561
S_0	0.5	1	-0.5	-1	0.5	0.5	0.5	0.5	0.5
Q_1	1	1	1	1	1	1	1	1.5	2.5

Table 4. Sherwood number(Sh) at r = 1

G	I	II	III	IV	V	VI	VIII	IX	X
10^3	-0.8585	0.04125	3.68071	23.541	-0.887841	-0.890199	-0.0528	-0.1648	-0.5392
$3x10^3$	-1.04754	-0.0457	3.39697	22.3118	-0.90153	-0.92774	-0.0603	-0.1729	-0.5534
-10^3	-0.5249	0.132804	3.9903	24.8263	-0.8734	-0.88235	-0.0453	-0.1565	-0.5243
$-3x10^3$	0.083115	0.229838	4.31649	26.1699	-0.85816	-0.87417	-0.0378	-0.1782	-0.5087
γ	0.5	1.5	2.5	-0.5	-1.5	-2.5	0.5	0.5	0.5
Du	0.01	0.01	0.01	0.01	0.01	0.01	0.05	0.01	0.03

Table 5. Sherwood number(Sh) at r = 2

G	I	II	III	IV	V	VI	VII	VIII	IX
10^3	1.48682	-104.153	-46.1661	-134.92	2.43222	2.54885	2.89421	-3.7443	-75.819
$3x10^3$	1.55603	-79.449	-59.3995	-182.715	2.44332	2.56094	2.91475	-4.44808	-48.2425
-10^3	1.42928	-134.604	-37.1509	-102.439	2.42107	2.53662	2.8728	-9.2325	-116.465
$-3x10^3$	1.38063	-171.342	-30.8628	-80.0621	2.40985	2.52423	2.85045	-18.1787	-175.504
S_0	0.5	1	-0.5	-1	0.5	0.5	0.5	0.5	0.5
Q_1	1	1	1	1	1	1	1	1.5	2.5

Table 6. Sherwood number(Sh) at r = 2

G	I	II	III	IV	V	VI	VII	VIII	IX
10^3	3.11955	1.7254	-2.8619	-25.6851	3.39202	3.63284	3.8072	2.4322	2.8942
$3x10^3$	3.38877	1.83844	-2.49808	-24.234	3.41317	3.64603	3.81719	2.4433	2.9147
-10^3	2.65277	1.60604	-3.24624	-27.2051	3.36981	3.61922	3.79706	2.4211	2.8728
$-3x10^3$	1.79612	1.479	-3.65189	-28.7968	3.34645	3.60512	3.78672	2.4098	2.8505
γ	0.5	1.5	2.5	-0.5	-1.5	-2.5	0.5	0.5	0.5
Du	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.05	0.3

7. CONCLUSION

In this analysis the following conclusions are drawn: (1) The axial velocity depreciates with increase in Soret parameter. (2) The velocity enhances in the degenerating chemical reaction case and depreciates in the generating chemical reaction case. (3) The actual temperature experiences an enhancement with increase in Dufour parameter. (4) The actual concentration reduces with positive values of Soret parameter and enhances with absolute values of Soret parameter. (5) The actual concentration experiences an enhancement with increase in Dufour parameter.

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