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Elementary Characteristics of P-Lattice Measurable Sets Analogs to Boolean Structures

D. V. S. R. Anil Kumar

T. K. R. College of Engineering & Technology,
Meer pet, Hyderabad, A.P. India
dvsranilkumar@gmail.com

Gopal Kulkarni

Vignan Bharati Institute of Technology,
Hyderabad, A.P. India
kulkigplrao@yahoo.co.in

T. Eswar Lal

Department of Mathematics, K.L. University, Vaddeswaram,
Guntur, A.P., India
eswarlal@kluniversity.in

Abstract: *This paper describes that the concepts of P- lattice measure space, countable union (intersection) of P-lattice measurable sets, and establish that the measurability's of these p-lattice measurable sets which analogs to Boolean structures.*

Keywords: *Lattice, sample space, Measure, Measurable.*

1. Introduction

By [2] Let S be any (non-empty) set that represents a sample space. Here we allow the set S to be arbitrary (either finite, countably infinite, or uncountably infinite). Let F denote a collection of subsets of S. We say a collection F is a sigma algebra (also called σ -algebra) if it satisfies the following three properties

1. The full sample space S is in F, and the empty set is ϕ in F.
2. If a set E is in F, then its complement E^c is also in F.
3. If $\{E_n\}_{n=1}^{M}$ is a finite sequence of subsets, each of which is in F, then the union $\bigcup_{n=1}^{M} E_n$ is also in F. Likewise, if $\{E_n\}_{n=1}^{\infty}$ is a countable sequence of subsets, each of which is in F, then the countable union $\bigcup_{n=1}^{\infty} E_n$ is also in F.

The set F consisting of all subsets of the sample space S is (trivially) a sigma algebra. However, it is possible to have sigma algebras that do not contain all subsets of S. For example, given any non-empty sample space S, the 2-element collection of subsets consisting of only ϕ and S is trivially a sigma algebra. For another example, if we have a finite sample space S = {1, 2, 3, 4}, it is easy to show that the following collection of sets F satisfies the defining properties of a sigma algebra: □ = {ϕ,{1,2},{3,4},{1,2,3,4}}. This particular sigma algebra F does not include the set {1, 2, 3}. The example illustrates sigma algebra for a finite sample space S. Of course, for the context of probability theory, there is no reason to talk about sigma algebras at all unless we consider sample spaces S that are uncountably infinite. In the case when S is the unit interval [0, 1], the Borel sigma algebra F is the collection of all open intervals (a, b) such that $0 \le a < b \le 1$, together with all complements of these, all finite or countably infinite unions of these, all complements of these unions, and so on. It is not obvious that this procedure does not include all possible subsets of [0, 1], but it can be shown that it does not. Clearly all sigma algebras are algebras, but the reverse is not true.

In section2 by[1]lattice σ-algebra, lattice measurable space, we define probability of lattice measure space, σ- P –lattice measurable set, δ - P –lattice measurable set, and insection3 we proved that these measurable sets are P –lattice measurable.

2. Preliminaries

Definition2.1. Let S is the sample space and F is a lattice of any subsets of S. If a lattice F satisfies the following conditions, then it is called a lattice σ-Algebra

(1) $\phi \in F$ (2) $\forall E \in F, E^c \in F$ (3) If $E_n \in F$ for n = 1, 2, 3 ....., then $\bigvee_{n=1}^{\infty} E_n \in F$

We denote σ (F), as the lattice σ -algebra generated by F.

Example of lattice sigma algebra

Power sets of sample spaces are always lattice sigma algebras. Suppose that S = {1, 2, 3}.

A lattice sigma algebra F is the collection of sets S where
F = {ϕ, {1, 2, 3}, {1}, {2}, {3}, {1, 2}, {1, 3}, {2, 3}}.

The pair (S, F) is called a lattice measurable space. Now define a probability lattice measure P (E) for all subsets E of the sample space S, including the empty set ϕ.

Definition2.2. A probability P on a lattice measurable space (S, F) is a function P:F→ [0,1] such that (1) P(S) = 1, P(ϕ) = 0.(2) P(E) ≥ 0 for all events E $\subseteq$ S.

(3) If $\{E_n\}_{n=1}^{M}$ is a finite sequence of mutually exclusive events (so that E_n □ E_m = ϕ for all n≠m), then $P\left(\bigvee_{n=1}^{M} E_n\right) = \sum_{n=1}^{M} P(E_n)$

Likewise, if $\{E_n\}_{n=1}^{\infty}$ is a countably infinite sequence of mutually exclusive events, then

$$P\left(\bigvee_{n=1}^{\infty} E_n\right) = \sum_{n=1}^{\infty} P(E_n)$$

The triple(S,F, P) is called a probability lattice measure space. The subsets E of S which belong to F are called probability of lattice measurable sets or simply **P**- measurable. In a probability context these sets are called events and we use the interpretation P (E) = "the probability that the event E occurs".

Definition2.3. By a σ - P –lattice measurable we mean a countable union of P –lattice measurable sets.

Definition2.4. By a δ - P –lattice measurable we mean a countable intersection of P –lattice measurable sets.

Result2.1 [1] If E is measurable set if only if E^c is also measurable.

3. σ - P-Lattice Measurable and δ - P –Lattice Measurable

Theorem3.1. If $E_1, E_2, \ldots\ldots$ are pair wise disjoint P-lattice measurable sets and $E = \bigvee_{k=1}^{\infty} E_k$, then E is P –lattice measurable (or) Every σ - P- lattice measurable is P-lattice measurable and also $P(E) = \sum_{k=1}^{\infty} P(E_k)$.

Proof. Part1. It is given that $E_i \wedge E_j = \phi$ for $i \neq j$, We have $P(\vee_{k=1}^{\infty} E_k) \le \sum_{k=1}^{\infty} P(E_k)$ ....(1). Clearly $\vee_{k=1}^{\infty} E_k \ge \vee_{k=1}^{n} E_k$.

Which implies $P(\vee_{k=1}^{\infty} E_k) \ge P(\vee_{k=1}^{n} E_k)$ …………(2). We know that if $E_i \wedge E_j = \phi$.

Which leads to $P(E_1 \vee E_2) = P(E_1) + P(E_2)$ (from definition 2.2.).

Extending, by induction, the result for infinite number of pair wise disjoint P-lattice measurable sets, we get

$P(\vee_{k=1}^{n} E_k) = \sum_{k=1}^{n} P(E_k)$. Then we have $P(\vee_{k=1}^{\infty} E_k) \ge \sum_{k=1}^{n} P(E_k)$ (By (2)).

Letting $n \to \infty$, $P(\vee_{k=1}^{\infty} E_k) \ge \sum_{k=1}^{\infty} P(E_k)$ ………..(3). From (1) and (3),

we have $P(\vee_{k=1}^{\infty} E_k) = \sum_{k=1}^{\infty} P(E_k)$.

Part2. Let $E_1, E_2, ...$ are pair wise disjoint P-lattice measurable sets.

Clearly $E = \vee_{k=1}^{\infty} E_k = E_1 \vee (E_2 \wedge E_1^c) \vee .... \vee (E_k \wedge (\vee_{k=1}^{n-1} E_k^c) \vee .....$

Evidently $E_1, E_2 \wedge E_1^c, ...$ are disjoint P-lattice measurable sets and hence

By part1, $\vee_{k=1}^{\infty} E_k$ is P-lattice measurable.

Hence every σ - P- lattice measurable is P-lattice measurable.

Theorem 3.2. First Valuation Theorem:

Suppose that $\{E_k\}$ is monotonic increasing sequence of P-lattice measurable sets and $E = \vee_{k=1}^{\infty} E_k$.

Then $P(E) = \underset{n\to\infty}{Lt} P(E_n)$.

Proof. Write $E = E_1 \vee (E_2 \wedge E_1^c) \vee .... \vee (E_k \wedge (\vee_{k=1}^{n-1} E_k^c) \vee .....$

So we have $E = E_1 \vee (\vee_{k=1}^{\infty} (E_{k+1} \wedge E_k^c)$ (A disjoint union) by theorem 3.1.

$$\text{Now, } P(E) = P(E_1) + \sum_{k=1}^{\infty} P(E_{k+1} - E_k)$$

$$= P(E_1) + \sum_{k=1}^{\infty} (P(E_{k+1}) - P(E_k))$$

$$= P(E_1) + \underset{n\to\infty}{Lt} \sum_{k=1}^{n} (P(E_{k+1}) - P(E_k))$$

$$= P(E_1) + \underset{n\to\infty}{Lt} [P(E_2) - P(E_1) + ... + P(E_n) - P(E_{n-1})]$$

$$= P(E_1) + \underset{n\to\infty}{Lt} [-P(E_1) + P(E_n)]$$

$$= P(E_1) - P(E_1) + \underset{n\to\infty}{Lt} P(E_n) = \underset{n\to\infty}{Lt} P(E_n).$$

Theorem3.3. If $E_1, E_{2,}.......$ are P-lattice measurable sets, then $\overset{\infty}{\underset{k=1}{\wedge}} E_k$ is P-lattice measurable (or) every δ - P-lattice measurable is P-lattice measurable.

Proof. By theorem 3.1. E = $\overset{\infty}{\underset{k=1}{\vee}} E_k$ is P-lattice measurable.

Let G = $\overset{\infty}{\underset{k=1}{\wedge}} E_k$. Then $G^c = (\overset{\infty}{\underset{k=1}{\wedge}} E_k)^c = \overset{\infty}{\underset{k=1}{\vee}} E_k^c$. Given that each E_k is a P-lattice measurable.

Hence by Result 2.1., each E_k^c is a P-lattice measurable.

Which implies $\overset{\infty}{\underset{k=1}{\vee}} E_k^c$ is P-lattice measurable (Every σ - P -lattice measurable).

This leads to G^c is P-lattice measurable.

Hence G is P-lattice measurable (By Result 2.1.).

Theorem 3.4. Second Valuation Theorem:

Suppose that $\{E_k\}$ is a monotonic decreasing sequence of P-lattice measurable sets and

$E = \overset{\infty}{\underset{k=1}{\wedge}} E_k$.

Then $P(E) = \underset{n\to\infty}{Lt} P(E_n)$.

Proof. Let $E = \overset{\infty}{\underset{k=1}{\wedge}} E_k$.

Evidently $E_1 = E \vee (E_1 \wedge E_2^c) \vee (E_2 \wedge E_3^c) \vee ....$

$$\text{Then } P(E_1) = P(E) + \sum_{k=1}^{\infty}(P(E_k) - P(E_{k+1}))$$

$$= P(E) + \underset{n\to\infty}{Lt} \sum_{k=1}^{n}(P(E_k) - P(E_{k+1}))$$

$$= P(E) + \underset{n\to\infty}{Lt} [P(E_1) - P(E_2) + ...+ P(E_n) - P(E_{n+1})]$$

$$= P(E) + \underset{n\to\infty}{Lt} [P(E_1) - P(E_{n+1})]$$

$$= P(E) + P(E_1) - \underset{n\to\infty}{Lt} P(E_{n+1}).$$

Which implies $P(E) = \underset{n\to\infty}{Lt} \mathbf{P}(E_n)$.

4. Conclusion

This paper described that the concepts of probability lattice measure space, countable union (intersection) of p-lattice measurable sets, and established that the measurability's of these p-lattice measurable sets.

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