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Applications on Generalized Two-Dimensional Fractional Sine Transform In the Range $0\ to\ \infty$

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Abstract: *Various transforms are employed for signal processing to obtain useful information, which is not explicitly available when the signal is in the time domain. Most of the real time signals such as speech, biomedical signals, etc., are non-stationary signals. The Fourier transform (FT), used for most of the signal processing applications, determines the frequency components present in the signal but with zero time resolution. The fractional cosine and sine transform closely related to the fractional Fourier transform which is now actively used in optics and signal processing.*

In this paper applications on generalized two dimensional fractional Sine transform are discussed. Also this paper presents Generalization of two dimensional fractional sine transform in the distributional sense.

Keywords: *fractional Fourier transform, fractional Cosine transform, fractional Sine transform.*

1. INTRODUCTION

Nowadays, fractional transform play an important role in information processing, image reconstruction, pattern recognition, and acrostic signal processing [6], [7] and the obvious question is: why do we need fractional transformation if we successfully apply the ordinary ones? First, because they naturally arise under the consideration of different problems for example, in optics and quantum mechanics and secondly, because fractionalization gives us a new degree of freedom (The fractional order), which can be used for more complete characterization of an object (A signal in general) or as an additional encoding parameter. Fourier analysis is one of the most frequently used tools in signal processing and many other scientific disciplines.

Namias [8] introduced the concept of Fourier transform of fractional order, which depends on a continuous parameter α . The generalization of ordinary Fourier transform and its properties were discussed in Cariolaro et.al [3] Zayed [1] Dragoman [4] etc. Fractional Fourier transform is further generalization to the integral with respect to new measure $d\rho$ and a new generalized integral transform was obtained by Zayed [1]. Bhosale and Chaudhary [2] had extended fractional Fourier transform to the distribution of compact support. The fractional Fourier transform with $\alpha = 1$ corresponds to the classical Fourier transform and fractional Fourier transform with $\alpha = 0$ corresponds to the identity operator. In [5] other integral transform of Fourier class that is Cosine transform and Sine transform, are also generalized to the corresponding fractional integral transform and studied by different mathematicians.

Pei Soo-Chang redefined the fractional cosine and sine transform based on fractional Fourier transform in 2001 [5] The idea of fractionalization of CT and ST was proposed in [9].There the real and imaginary parts of fractional FT kernel were chosen as kernel for a fractional CT and Fractional ST respectively.

The organization of this paper is as follows: We first provide the definition of distributional two dimensional fractional sine transform in section 2. In section 3 we are discussed applications on generalized two dimensional fractional Sine transform in the range $0\ to\ \infty$

2. Distributional Two-Dimensional Fractional Sine Transform

The two dimensional distributional fractional Sine transform of $f(x,y) \in E^*(R^n)$ defined by

$$F_s^\alpha\{f(x,y)\} = F^\alpha(u,v) = \langle f(x,y), K_\alpha(x,y,u,v)\rangle \quad (2.1)$$

$$K_s^\alpha(x,y,u,v) = \sqrt{\frac{1-icot\alpha}{2\pi}}\, e^{\frac{i(x^2+y^2+u^2+v^2)cot\alpha}{2}} e^{i(\theta-\frac{\pi}{2})} \sin(cosec\alpha.ux) . \sin(cosec\alpha.vy). \quad (2.2)$$

where, rhs of equation (2.1) has a meaning as the application of $f \in E^*$ to $K_\alpha(x,y,u,v) \in E$.

3. Examples in the Range 0 to ∞

3.1

If $F_c^\alpha\{f(x,y)\}(u,v)$ denotes generalized two dimensional fractional Sine transform of f(x, y) then $F_s^\alpha\{1\}(u,v)\}$

$$= i\sqrt{\frac{1-icot\alpha}{2\pi cot^2\alpha}}\, e^{i\left(\left(\frac{cot\alpha}{2}-css2\alpha\right)(u^2+v^2)+\left(\theta-\frac{\pi}{2}\right)\right)} \sum_{k=0}^{\infty}\frac{\left(csc\alpha.u\sqrt{\frac{i}{2cot\alpha}}\right)^{2k+1}}{k!\,(2k+1)} \sum_{l=0}^{\infty}\frac{\left(csc\alpha.v\sqrt{\frac{i}{2cot\alpha}}\right)^{2l+1}}{l!\,(2l+1)}$$

Solution:

$F_s^\alpha\{1\}(u,v) =$

$$\int_0^\infty\int_0^\infty 1\sqrt{\frac{1-icot\alpha}{2\pi}}\, e^{\frac{i(x^2+y^2+u^2+v^2)cot\alpha}{2}} e^{i(\theta-\frac{\pi}{2})}\sin(cosec\alpha.ux).\sin(cosec\alpha.vy)\,dx\,dy$$

$F_s^\alpha\{1\}(u,v) =$

$$\sqrt{\frac{1-icot\alpha}{2\pi}}\, e^{\frac{i(u^2+v^2)cot\alpha}{2}} e^{i(\theta-\frac{\pi}{2})}\int_0^\infty\int_0^\infty 1\, e^{\frac{i(x^2+y^2)cot\alpha}{2}}\sin(cosec\alpha.ux).\sin(cosec\alpha.vy)\,dx\,dy$$

Let, A= $\sqrt{\frac{1-icot\alpha}{2\pi}}\, e^{i(\theta-\frac{\pi}{2})}$ B= $e^{\frac{i(u^2+v^2)cot\alpha}{2}}$

$$F_s^\alpha\{1\}(u,v) = AB\int_0^\infty e^{\frac{i(x^2)cot\alpha}{2}}\sin(cosec\alpha.ux)\,dx\int_0^\infty e^{\frac{i(y^2)cot\alpha}{2}}\sin(cosec\alpha.vy)\,dy$$

Let, a= $\frac{cot\alpha}{2}$, b= $cosec\alpha.u$, c= $cosec\alpha.v$

$$F_s^\alpha\{1\}(u,v) = AB\int_0^\infty e^{ix^2a}\sin(bx)\,dx\int_0^\infty e^{iy^2a}\sin(cy)\,dy$$

$$F_s^\alpha\{1\}(u,v) = AB\left[\left(\frac{-1}{4\sqrt{a}}(-1)^{\frac14}\sqrt{\pi}e^{\frac{-ib^2}{4a}}\right)\left(erfi\left(\frac{\sqrt[4]{-1}}{2\sqrt{a}}(2ax+b)\right)-\left(erfi\left(\frac{\sqrt[4]{-1}}{2\sqrt{a}}(2ax-b)\right)\right)\right]_0^\infty\left[\left(\frac{-1}{4\sqrt{a}}(-1)^{\frac14}\sqrt{\pi}e^{\frac{-ic^2}{4a}}\right)\left(erfi\left(\frac{\sqrt[4]{-1}}{2\sqrt{a}}(2ay+c)\right)-\left(erfi\left(\frac{\sqrt[4]{-1}}{2\sqrt{a}}(2ay-c)\right)\right)\right]_0^\infty$$

$F_s^\alpha\{1\}(u,v) =$

$$\left(\frac{i}{16a}\pi e^{\frac{-i(b^2+c^2)}{4a}}\right)AB\left[erfi(\sqrt{i}\infty)-erfi(\sqrt{i}\infty)-erfi\left(\frac{\sqrt{i}b}{2\sqrt{a}}\right)+erfi\left(\frac{-\sqrt{i}b}{2\sqrt{a}}\right)\right]\left[erfi(\sqrt{i}\infty)-erfi(\sqrt{i}\infty)-erfi\left(\frac{\sqrt{i}c}{2\sqrt{a}}\right)+erfi\left(\frac{-\sqrt{i}c}{2\sqrt{a}}\right)\right]$$

Here $erfi(\sqrt{i}\infty) = \sqrt{i}$

$$F_s^\alpha\{1\}(u,v) = \left(\frac{ABi\pi}{4a}e^{\frac{-i(b^2+c^2)}{4a}}\right)\left[erfi\left(\frac{\sqrt{i}b}{2\sqrt{a}}\right)\right]\left[erfi\left(\frac{\sqrt{i}c}{2\sqrt{a}}\right)\right]$$

$$F_s^\alpha\{1\}(u,v) = \left(\frac{ABi\pi}{4a}e^{\frac{-i(b^2+c^2)}{4a}}\right)\left[erfi\left(\frac{b}{2}\sqrt{\frac{i}{a}}\right)\right]\left[erfi\left(\frac{c}{2}\sqrt{\frac{i}{a}}\right)\right]$$

$$F_s^\alpha\{1\}(u,v) = \left(\frac{ABi\pi}{4a}e^{\frac{-i(b^2+c^2)}{4a}}\right)\frac{2}{\sqrt{\pi}}\sum_{k=0}^{\infty}\frac{\left(\frac{b}{2}\sqrt{\frac{i}{a}}\right)^{2K+1}}{k!(2k+1)}\frac{2}{\sqrt{\pi}}\sum_{l=0}^{\infty}\frac{\left(\frac{c}{2}\sqrt{\frac{i}{a}}\right)^{2l+1}}{l!(2l+1)}$$

$$F_s^\alpha\{1\}(u,v) = \left(\frac{ABi}{a}e^{\frac{-i(b^2+c^2)}{4a}}\right)\sum_{k=0}^{\infty}\frac{\left(\frac{b}{2}\sqrt{\frac{i}{a}}\right)^{2K+1}}{k!(2k+1)}\sum_{l=0}^{\infty}\frac{\left(\frac{c}{2}\sqrt{\frac{i}{a}}\right)^{2l+1}}{l!(2l+1)}$$

$$F_s^\alpha\{1\}(u,v) =$$
$$\left(\frac{\sqrt{\frac{1-icot\alpha}{2\pi}}e^{i\left(\theta-\frac{\pi}{2}\right)}ie^{\frac{i(u^2+v^2)cot\alpha}{2}}}{\frac{cot\alpha}{2}}e^{\frac{-i(csc^2\alpha u^2+csc^2\alpha v^2)}{4\frac{cot\alpha}{2}}}\right)\sum_{k=0}^{\infty}\frac{\left(\frac{csc\alpha u}{2}\sqrt{\frac{i}{\frac{cot\alpha}{2}}}\right)^{2k+1}}{k!(2k+1)}\sum_{l=0}^{\infty}\frac{\left(\frac{csc\alpha v}{2}\sqrt{\frac{i}{\frac{cot\alpha}{2}}}\right)^{2l+1}}{l!(2l+1)}$$

$$F_s^\alpha\{1\}(u,v) =$$
$$\left(\frac{2\sqrt{\frac{1-icot\alpha}{2\pi}}ie^{i\left((u^2+v^2)\frac{cot\alpha}{2}+\left(\theta-\frac{\pi}{2}\right)\right)}}{cot\alpha}e^{\frac{-i(csc^2\alpha u^2+csc^2\alpha v^2)}{2cot\alpha}}\right)\sum_{k=0}^{\infty}\frac{\left(csc\alpha u\sqrt{\frac{i}{2cot\alpha}}\right)^{2k+1}}{k!(2k+1)}\sum_{l=0}^{\infty}\frac{\left(csc\alpha v\sqrt{\frac{i}{2cot\alpha}}\right)^{2l+1}}{l!(2l+1)}$$

$$F_s^\alpha\{1\}(u,v) =$$
$$\left(\frac{2\sqrt{\frac{1-icot\alpha}{2\pi}}ie^{i\left((u^2+v^2)\frac{cot\alpha}{2}+\left(\theta-\frac{\pi}{2}\right)\right)}}{cot\alpha}e^{-i\left(csc2\alpha(u^2+v^2)\right)}\right)\sum_{k=0}^{\infty}\frac{\left(csc\alpha u\sqrt{\frac{i}{2cot\alpha}}\right)^{2k+1}}{k!(2k+1)}\sum_{l=0}^{\infty}\frac{\left(csc\alpha v\sqrt{\frac{i}{2cot\alpha}}\right)^{2l+1}}{l!(2l+1)}$$

$$F_s^\alpha\{1\}(u,v) =$$
$$i\sqrt{\frac{1-icot\alpha}{2\pi cot^2\alpha}}e^{i\left(\left(\frac{cot\alpha}{2}-csc2\alpha\right)(u^2+v^2)+\left(\alpha-\frac{\pi}{2}\right)\right)}\sum_{k=0}^{\infty}\frac{\left(csc\alpha u\sqrt{\frac{i}{2cot\alpha}}\right)^{2k+1}}{k!(2k+1)}\sum_{l=0}^{\infty}\frac{\left(csc\alpha v\sqrt{\frac{i}{2cot\alpha}}\right)^{2l+1}}{l!(2l+1)}$$

3.2

If$F_c^\alpha\{f(x,y)\}(u,v)$ denotes generalized two dimensional fractional Sine transform of f(x, y) then $F_s^\alpha\{(\delta(x-a).\delta(y-b))\} = K_s^\alpha(a,b,u,v)$

Solution:

$$F_s^\alpha\{(\delta(x-a).\delta(y-b))\} =$$
$$\int_0^\infty\int_0^\infty \delta(x-a)\delta(y-b)\quad \sqrt{\frac{1-icot\alpha}{2\pi}}e^{\frac{i(x^2+y^2+u^2+v^2)cot\alpha}{2}}\quad dy$$
$$e^{i\left(\theta-\frac{\pi}{2}\right)}\sin(cosec\alpha.ux).\sin(cosec\alpha.vy)\,dx$$

$$F_s^{\alpha}\{(\delta(x-a).\delta(y-b))\} = \sqrt{\frac{1-icot\alpha}{2\pi}}\, e^{\frac{i(u^2+v^2)cot\alpha}{2}} e^{i\left(\theta-\frac{\pi}{2}\right)}$$
$$\int_0^{\infty}\int_0^{\infty} (\delta(x-a)\delta(y-b))\, e^{\frac{i(x^2+y^2)cot\alpha}{2}} \sin(cosec\alpha.ux).\sin(cosec\alpha.vy)\, dx\, dy$$

Let A= $\sqrt{\frac{1-icot\alpha}{2\pi}}\, e^{i\left(\theta-\frac{\pi}{2}\right)}$ $\boldsymbol{B} = e^{\frac{i(u^2+v^2)cot\alpha}{2}}$

$$F_s^{\alpha}\{(\delta(x-a).\delta(y-b))\} = AB\int_0^{\infty} \delta(x-a)\, e^{\frac{i(x^2)cot\alpha}{2}} \sin(cosec\alpha.ux)\, dx$$
$$\int_0^{\infty} \delta(y-b)\, e^{\frac{i(y^2)cot\alpha}{2}} \sin(cosec\alpha.vy)\, dy$$
$$F_s^{\alpha}\{(\delta(x-a).\delta(y-b))\} = ABe^{\frac{i(a^2)cot\alpha}{2}} \sin(cosec\alpha.ua)\,.e^{\frac{i(b^2)cot\alpha}{2}} \sin(cosec\alpha.vb)$$

We know that $\int_0^{\omega} \delta(t-a)\,\varphi(t)dt = \varphi(a)$

$$F_s^{\alpha}\{(\delta(x-a).\delta(y-b))\} = ABe^{\frac{i(a^2+b^2)cot\alpha}{2}} \sin(cosec\alpha.ua)\,.\sin(cosec\alpha.vb)$$

$$F_C^{\alpha}\{(\delta(x-a).\delta(y-b))\} = K_s^{\alpha}(a,b,u,v)$$

3.3.

If $F_s^{\alpha}\{f(x,y)\}(u,v)$ denotes generalized two dimensional fractional sine transform of f(x,y) then
$$F_s^{\alpha}\{sinx.siny\}(u,v) = -\sqrt{\frac{(1-icot\alpha)\pi}{8cot^2\alpha}}\, e^{i\left(\frac{(u^2+v^2)cot\alpha}{2}+\left(\theta-\frac{\pi}{2}\right)-\left(\tan+\frac{u^2+v^2}{\sin 2\alpha}\right)\right)} sin(sec\alpha.u)\, sin(sec\alpha.v)$$

Solution:

$F_s^{\alpha}\{sinx.siny\}(u,v) =$
$$\int_0^{\infty}\int_0^{\infty} sinx.siny\sqrt{\frac{1-icot\alpha}{2\pi}}\, e^{i\left(\theta-\frac{\pi}{2}\right)} e^{\frac{i(x^2+y^2+u^2+v^2)cot\alpha}{2}} \sin(cosec\alpha.ux)\,.\sin(cosec\alpha.vy)\, dx\, dy$$

$F_s^{\alpha}\{sinx.siny\}(u,v) =$
$$AB\int_0^{\infty}\int_0^{\infty} sinx.siny\, e^{\frac{i(x^2+y^2)cot\alpha}{2}} \sin(cosec\alpha.ux)\,.\sin(cosec\alpha.vy)\, dx\, dy$$

Let A= $\sqrt{\frac{1-icot\alpha}{2\pi}}\, e^{i\left(\theta-\frac{\pi}{2}\right)}$, $\boldsymbol{B} = e^{\frac{i(u^2+v^2)cot\alpha}{2}}$

$F_s^{\alpha}\{sinx.siny\}(u,v)$
$$= AB\int_0^{\infty}\int_0^{\infty} e^{\frac{i(x^2+y^2)cot\alpha}{2}} \frac{-1}{2}(\cos(csc.u+1)x$$
$$-\cos(csc.u-1)x)\frac{-1}{2}(\cos(csc.v+1)y - \cos(csc.v-1)\,y).dx\, dy$$
$$F_s^{\alpha}\{sinx.siny\}(u,v) = AB\frac{1}{4}\int_0^{\infty}\int_0^{\infty} e^{\frac{i(x^2+y^2)cot\alpha}{2}}$$
$$(\cos(csc.u+1)x - \cos(csc.u-1)x)\,(\cos(csc.v+1)y - \cos(csc.v-1)\,y).dx\, dy$$

Let $b_1 = (csc.u+1)$,$b_2 = (csc.u-1)$, $c_1 = (csc.v+1)$, $c_2 = (csc.v-1)$,$\frac{cot\alpha}{2} = a$

$$F_s^\alpha\{sinx.siny\}(u,v) = AB\frac{1}{4}\int_0^\infty e^{iax^2}(\cos b_1 x - \cos b_2 x)dx \int_0^\infty e^{iay^2}(\cos c_1 y - \cos c_2 y)\,dy$$

$$F_s^\alpha\{sinx.siny\}(u,v)$$
$$= AB\frac{1}{4}\left[\left[\int_0^\infty e^{iax^2}(\cos b_1 x)dx - \int_0^\infty e^{iax^2}\cos b_2 x\right]\left[\int_0^\infty e^{iay^2}(\cos c_1 y)dy - \left(\int_0^\infty e^{iay^2}\cos c_2 y\right)dy\right]$$

$$F_s^\alpha\{sinx.siny\}(u,v) = AB\frac{-\pi i}{64a}\left|\left[\left(e^{\frac{-ib_1^2}{4a}}\left(erfi\left(\frac{\sqrt{i}}{2\sqrt{a}}(2ax-b_1)\right)+erfi\left(\frac{\sqrt{i}}{2\sqrt{a}}(2ax+b_1)\right)\right)-e^{\frac{-ib_2^2}{4a}}\left(erfi\left(\frac{\sqrt{i}}{2\sqrt{a}}(2ax-b_2)\right)+erfi\left(\frac{\sqrt{i}}{2\sqrt{a}}(2ax+b_2)\right)\right)\right)\left(e^{\frac{-ic_1^2}{4a}}\left(erfi\left(\frac{\sqrt{i}}{2\sqrt{a}}(2ay-c_1)\right)+erfi\left(\frac{\sqrt{i}}{2\sqrt{a}}(2ay+c_1)\right)\right)-e^{\frac{-ic_2^2}{4a}}\left(erfi\left(\frac{\sqrt{i}}{2\sqrt{a}}(2ay-c_2)\right)+erfi\left(\frac{\sqrt{i}}{2\sqrt{a}}(2ay+c_2)\right)\right)\right)\right]\right|_0^\infty$$

$$F_s^\alpha\{sinx.siny\}(u,v)$$
$$= AB\frac{-\pi i}{64a}\left[\left(e^{\frac{-ib_1^2}{4a}}(\sqrt{i}+\sqrt{i}) - e^{\frac{-ib_2^2}{4a}}\left((\sqrt{i}+\sqrt{i})\right)\right)\left(e^{\frac{-ic_1^2}{4a}}\left((\sqrt{i}+\sqrt{i})\right) - e^{\frac{-ic_2^2}{4a}}\left((\sqrt{i}+\sqrt{i})\right)\right)\right]$$

$$F_s^\alpha\{sinx.siny\}(u,v) = AB\frac{\pi}{16a}\left[\left(e^{\frac{-ib_1^2}{4a}} - e^{\frac{-ib_2^2}{4a}}\right)\left(e^{\frac{-ic_1^2}{4a}} - e^{\frac{-ic_2^2}{4a}}\right)\right]$$

$$F_s^\alpha\{sinx.siny\}(u,v)$$
$$= AB\frac{\pi}{8\cot\alpha}\left[\left(e^{\frac{-i(csc\alpha.u+1)^2}{2\cot\alpha}} - e^{\frac{-i(csc.u-1)^2}{2\cot\alpha}}\right)\left(e^{\frac{-i(csc\alpha.v+1)^2}{2\cot\alpha}} - e^{\frac{-i(csc\alpha.v-1)^2}{2\cot\alpha}}\right)\right]$$

$$F_s^\alpha\{sinx.siny\}(u,v) =$$
$$AB\frac{\pi}{8\cot\alpha}e^{\frac{-itan\alpha(csc^2\alpha.u^2+1)}{2}}e^{\frac{-itan\alpha(csc^2\alpha.v^2+1)}{2}}\left[\left(e^{-isec\alpha u} - e^{isec\alpha.u}\right)\left(e^{-isec\alpha.v} - e^{isec\alpha}\right)\right]$$

$$F_s^\alpha\{sinx.siny\}(u,v) =$$
$$\sqrt{\frac{(1-icot\alpha)\pi}{2}}e^{i\left(\theta-\frac{\pi}{2}\right)}e^{\frac{i(u^2+v^2)cot\alpha}{2}}\frac{1}{8\cot\alpha}e^{\frac{-itan\alpha(csc^2\alpha(u^2+v^2)+2)}{2}}2isin(sec\alpha.u)\;2isin(sec\alpha.v)$$

$$F_s^\alpha\{sinx.siny\}(u,v) = -\sqrt{\frac{(1-icot\alpha)\pi}{8cot^2\alpha}}e^{i\left(\frac{(u^2+v^2)cot\alpha}{2}+\left(\theta-\frac{\pi}{2}\right)-\left(\tan+\frac{u^2+v^2}{\sin 2\alpha}\right)\right)}sin(sec\alpha.u)\,sin(sec\alpha.v)$$

3.4.

If$F_c^\alpha\{f(x,y)\}(u,v)$ denotes generalized two dimensional fractional Sine transform of f(x,y) then

$$F_s^\alpha\{e^{i(ax^2+by^2)}\}(u,v) = \frac{\sqrt{\frac{2(1-icot\alpha)}{\pi}}ie^{i\left[\theta-\frac{\pi}{2}+\frac{cot\alpha}{2}(u^2+v^2)-\left(\frac{u^2}{1+2atan\alpha}+\frac{u^2}{1+2btan\alpha}\right)csc2\alpha\right]}}{cot\alpha\sqrt{(1+2atan\alpha)(1+2btan\alpha)}}$$

$$\sum_{k=0}^{\infty}\frac{\left(coseca.u\sqrt{\frac{i}{2(1+2atana)cota}}\right)^{2k+1}}{k!\,(2k+1)}\sum_{l=0}^{\infty}\frac{\left((coseca.v)\sqrt{\frac{i}{2(1+2btana)cota}}\right)^{2l+1}}{l!\,(2l+1)}$$

Solution:

$$F_s^{\alpha}\left\{e^{i(ax^2+by^2)}\right\}(u,v)=$$
$$\sqrt{\frac{1-icot\alpha}{2\pi}}\,e^{\frac{i(u^2+v^2)cot\alpha}{2}}e^{i\left(\theta-\frac{\pi}{2}\right)}\int_0^{\infty}\int_0^{\infty}e^{i(ax^2+by^2)}\,e^{\frac{i(x^2+y^2)cot\alpha}{2}}\sin(coseca.ux).\sin(coseca.vy)\,dx$$

Let, A= $\sqrt{\frac{1-icot\alpha}{2\pi}}\,e^{i\left(\theta-\frac{\pi}{2}\right)}$ B= $e^{\frac{i(u^2+v^2)cot\alpha}{2}}$

$$F_s^{\alpha}\left\{e^{i(ax^2+by^2)}\right\}(u,v)$$
$$=AB\int_0^{\infty}\int_0^{\infty}e^{i(ax^2+by^2)}\,e^{\frac{i(x^2+y^2)cot\alpha}{2}}\sin(coseca.ux).\sin(coseca.vy)\,dx\,dy$$

$$F_s^{\alpha}\left\{e^{i(ax^2+by^2)}\right\}(u,v)$$
$$=AB\int_0^{\infty}\int_0^{\infty}e^{\frac{i}{2}\left(x^2+y^2+\frac{2ax^2}{cot\alpha}+\frac{2by^2}{cot\alpha}\right)cot\alpha}\sin(coseca.ux).\sin(coseca.vy)\,dx\,dy$$

$$F_s^{\alpha}\left\{e^{i(ax^2+by^2)}\right\}(u,v)$$
$$=AB\int_0^{\infty}\int_0^{\infty}e^{\frac{i}{2}((1+2atana)x^2+(1+2btana)y^2)cot\alpha}\sin(coseca.ux).\sin(coseca.vy)\,dx\,dy$$

$$F_s^{\alpha}\left\{e^{i(ax^2+by^2)}\right\}(u,v)=AB\int_0^{\infty}e^{i(px^2)}\sin(cx)\,dx\int_0^{\infty}e^{i(qx^2)}.\sin(dy)\,dy$$

Let $p=\frac{(1+2atana)cot\alpha}{2}\, q=\frac{(1+2btana)cot\alpha}{2}$

$$F_s^{\alpha}\left\{e^{i(ax^2+by^2)}\right\}(u,v)=\frac{AB\pi i}{16\sqrt{pq}}e^{\frac{-i}{4}\left(\frac{c^2}{p}+\frac{d^2}{q}\right)}\left[-2erfi\left(\frac{\sqrt{i}c}{2\sqrt{p}}\right)\right]\left[-2erfi\left(\frac{\sqrt{i}d}{2\sqrt{q}}\right)\right]$$

$$F_s^{\alpha}\left\{e^{i(ax^2+by^2)}\right\}(u,v)=\frac{AB\pi i}{4\sqrt{pq}}e^{\frac{-i}{4}\left(\frac{c^2}{p}+\frac{d^2}{q}\right)}\left[erfi\left(\frac{c}{2}\sqrt{\frac{i}{p}}\right)\right]\left[erfi\left(\frac{d}{2}\sqrt{\frac{i}{q}}\right)\right]$$

$$F_s^{\alpha}\left\{e^{i(ax^2+by^2)}\right\}(u,v)=\frac{AB\pi i}{4\sqrt{pq}}e^{\frac{-i}{4}\left(\frac{(coseca.u)^2}{p}+\frac{(coseca.v)^2}{q}\right)}\frac{2}{\sqrt{\pi}}\sum_{k=0}^{\infty}\frac{\left(\frac{c}{2}\sqrt{\frac{i}{p}}\right)^{2k+1}}{k!(2k+1)}\frac{2}{\sqrt{\pi}}\sum_{l=0}^{\infty}\frac{\left(\frac{d}{2}\sqrt{\frac{i}{q}}\right)^{2l+1}}{l!(2l+1)}$$

$$F_s^{\alpha}\left\{e^{i(ax^2+by^2)}\right\}(u,v)=\frac{\sqrt{\frac{1-icot\alpha}{2\pi}}\,e^{i\left(\theta-\frac{\pi}{2}\right)}e^{\frac{i(u^2+v^2)cot\alpha}{2}}\,i}{\sqrt{\frac{(1+2atana)cot\alpha}{2}\frac{(1+2btana)cot\alpha}{2}}}e^{\frac{-i}{4}\left(\frac{(coseca.u)^2}{\frac{(1+2atana)cot\alpha}{2}}+\frac{(coseca.v)^2}{\frac{(1+2btana)cot\alpha}{2}}\right)}$$

$$\sum_{k=0}^{\infty}\frac{\left(\frac{coseca.u}{2}\sqrt{\frac{i}{\frac{(1+2atana)cot\alpha}{2}}}\right)^{2k+1}}{k!\,(2k+1)}\sum_{l=0}^{\infty}\frac{\left(\frac{coseca.v}{2}\sqrt{\frac{i}{\frac{(1+2btana)cot\alpha}{2}}}\right)^{2l+1}}{l!\,(2l+1)}$$

$$F_s^{\alpha}\left\{e^{i(ax^2+by^2)}\right\}(u,v)=\frac{\sqrt{\frac{2(1-icot\alpha)}{\pi}}\,ie^{i\left[\theta-\frac{\pi}{2}+\frac{cot\alpha}{2}(u^2+v^2)-\left(\frac{u^2}{1+2atana}+\frac{u^2}{1+2btana}\right)csc2\alpha\right]}}{cot\alpha\sqrt{(1+2atana)(1+2btana)}}$$

$$\sum_{k=0}^{\infty} \frac{\left(coseca.u\sqrt{\frac{i}{2(1+2atan\alpha)cot\alpha}}\right)^{2k+1}}{k!\,(2k+1)} \sum_{l=0}^{\infty} \frac{\left((coseca.v)\sqrt{\frac{i}{2(1+2btan\alpha)cot\alpha}}\right)^{2l+1}}{l!\,(2l+1)}$$

4. Conclusion

This paper presents the Generalization of two dimensional fractional sine transform in the distributional sense. Also some examples of two dimensional fractional sine transform in the range 0 to ∞ are proved.

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