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# About the Strictly Convex and Uniformly Convex Normed and 2-Normed Spaces

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**Abstract:** *In* [1] *A. Khan introduces the notion of uniformly convex 2-normed space and prove some properties of the uniformly convex 2-normed spaces. In this work, further properties of the uniformly convex 2-normed spaces are given and the question of the convexity of a normed space in which the norm is induced by 2-norm is analysed.*

**Keywords:** 2-*normed space*, 2-*pre-Hibert space, convergent sequence, strictly convex space, uniformly convex space*

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## 1. Introduction

The concept of a uniformly convex 2-normed space is introduced by A. Khan. For our further investigation, we will introduce the definition of the uniformly convex 2-normed space in its equivalent form, as follows.

**Definition 1 ([1]).** A 2-normed space $(L,\|\cdot,\cdot\|)$ is *uniformly convex* if for every $\varepsilon>0$ there exists $\delta(\varepsilon)>0$ such that $\|x,z\|=\|y,z\|=1$, $\|x-y,z\|\geq\varepsilon$ and $z\notin V(x,y)$ implies

$$\|x+y,z\|\leq 2(1-\delta(\varepsilon)),$$

where $V(x,y)$ is the subspace generated by the vectors $x$ and $y$.

**Example 1 ([1]).** A 2-pre-Hibert space is a 2-normed space in which the norm is introduced by $\|x,y\|=(x,x\,|\,y)^2$ and the parallelepiped law is satisfied

$$\|x+y,z\|^2+\|x-y,z\|^2=2(\|x,z\|^2+\|y,z\|^2). \qquad (1)$$

If $\varepsilon>0$ is given and $\|x,z\|=\|y,z\|=1$, $\|x-y,z\|\geq\varepsilon$ and $z\notin V(x,y)$, then from the equality (1) it follows that for $\delta(\varepsilon)=1-\sqrt{1-(\frac{\varepsilon}{2})^2}>0$ the following

$$\|x+y,z\|=(4-\|x-y,z\|^2)^{1/2}\leq(4-\varepsilon^2)^{1/2}=2(1-\delta(\varepsilon))$$

holds. It means, that $(L,(\cdot,\cdot\,|\,\cdot))$ is uniformly convex space.

Let $z$ be a fixed nonzero element in $L$, $V(z)$ be the subspace of $L$ generated by $z$ and let $L_z$ be the quotient space $L/V(z)$. For $x\in L$ by $x_z$ we denote the class of equivalence of $x$ over $V(z)$. Clearly, $L_z$ is a linear space with the operations of adding the two vectors and multiplying a vector by scalar given respectively with $x_z + y_z = (x+y)_z$ and $\alpha x_z = (\alpha x)_z$. In [2] it is proved that by $\| x_z \|_z = \| x, z \|$ a norm on $L_z$ is defined. For the 2-normed space $(L, \|\cdot,\cdot\|)$ and the normed space $(L_z, \|\cdot\|_z)$ the following result holds.

**Theorem 1 ([1]).** Let $(L, \|\cdot,\cdot\|)$ be a 2-normed space. Then $L$ is uniformly convex if and only if for every j $z \in L \setminus \{0\}$ the space $(L_z, \|\cdot\|_z)$ is uniformly convex.

**Definition 2 ([2]).** Let $x, y \in L$ be non-zero elements and let $V(x,y)$ be the subspace of $L$ generated by the vectors $x$ and $y$. The linear 2-normed space $(L, \|\cdot,\cdot\|)$ is *strictly convex* if $\| x, z \| = \| y, z \| = \| \frac{x+y}{2}, z \| = 1$ and $z \notin V(x,y)$, for $x, y, z \in L$, implies that $x = y$.

More characterizations of the strictly convex 2-normed spaces can be found in [3] – [12], and some of them are given in the next theorem.

**Theorem 2.** Let $(L, \|\cdot,\cdot\|)$ be a 2-normed space. The following statements are equivalent:

1) $(L, \|\cdot,\cdot\|)$ is a strictly convex space.
2) For every nonzero element $z \in L$ the space $(L_z, \|\cdot\|_z)$ is strictly convex space..
3) If $\| x+y, z \| = \| x, z \| + \| y, z \|$ and $z \notin V(x,y)$, for $x, y, z \in L$ than $y = \alpha x$ for some $\alpha > 0$.
4) If $\| x-u, z \| = \alpha \| x-y, z \|$, $\| y-u, z \| = (1-\alpha) \| x-y, z \|$, $\alpha \in (0,1)$ and $z \notin V(x-u, y-u)$, then $u = (1-\alpha)x + \alpha y$.
5) If $\| x, z \| = \| y, z \| = 1$, $x \neq y$ and $z \notin V(x,y)$, for $x, y, z \in L$, then $\| \frac{x+y}{2}, z \| < 1$.

**Example 2.** Let $(Y, M)$ be measurable space and $\mu$ is a positive measure on $M$, then $X = L^p(\mu)$, $p > 1$ is the following space

$$X = \{ f : f : Y \to \mathbf{C}, \int_Y | f |^p d\mu < +\infty \}.$$

In [13] it is proved that the function $\|\cdot,\cdot\| : L^p(\mu) \times L^p(\mu) \to \mathbf{R}$ given by:

$$\| f, g \| = \{ \int_{Y \times Y} | \begin{vmatrix} f(x) & f(y) \\ g(x) & g(y) \end{vmatrix} |^p d(\mu \times \mu) \}^{\frac{1}{p}},$$

is a 2-norm on $X = L^p(\mu)$. Let $\| f, h \| = \| g, h \| = 1$, $f \neq g$ and $h \notin V(f,g)$. Then because of the Minkowski's inequality it follows that

$$\begin{aligned} \| f+g, h \| &= ( \int_{Y \times Y} | \begin{vmatrix} f(x) & f(y) \\ h(x) & h(y) \end{vmatrix} + \begin{vmatrix} g(x) & g(y) \\ h(x) & h(y) \end{vmatrix} |^p \, d(\mu \times \mu))^{\frac{1}{p}} \\ &\leq ( \int_{Y \times Y} | \begin{vmatrix} f(x) & f(y) \\ h(x) & h(y) \end{vmatrix} |^p \, d(\mu \times \mu))^{\frac{1}{p}} + ( \int_{Y \times Y} | \begin{vmatrix} g(x) & g(y) \\ h(x) & h(y) \end{vmatrix} |^p \, d(\mu \times \mu))^{\frac{1}{p}} \\ &= \| f, h \| + \| g, h \| = 1 + 1 = 2, \end{aligned}$$

And the equality holds if and only if there exists $\alpha > 0$ such that

$$\begin{vmatrix} f(x) & f(y) \\ h(x) & h(y) \end{vmatrix} = \alpha \begin{vmatrix} g(x) & g(y) \\ h(x) & h(y) \end{vmatrix},$$

almost everywhere. But, because $\| f, h \| = \| g, h \| = 1$ we get

$$1=\| f,h \|=\{ \int\limits_{Y\times Y} |\begin{vmatrix} f(x) & f(y) \\ h(x) & h(y) \end{vmatrix}|^p d(\mu\times\mu)\}^{\frac{1}{p}}=\{ \int\limits_{Y\times Y} |\alpha\begin{vmatrix} g(x) & g(y) \\ h(x) & h(y) \end{vmatrix}|^p d(\mu\times\mu)\}^{\frac{1}{p}}$$

$$=\alpha\{ \int\limits_{Y\times Y} |\begin{vmatrix} g(x) & g(y) \\ h(x) & h(y) \end{vmatrix}|^p d(\mu\times\mu)\}^{\frac{1}{p}}=\alpha \| g,h \|=\alpha\cdot 1=\alpha.$$

The last one contradics the $f\neq g$, so $\| f+g,h \|<2$, that is $\| \frac{f+g}{2},h \|<1$, and by Theorem 2 means that 2-normed space $X=L^p(\mu)$ is strictly convex space.

## 2. The Main Results

Let $(L,\|\cdot,\cdot\|)$ be a 2-normed space and $\{a,b\}$ be linearly independent subspace of $L$. In Theorem 1 and Theorem 2, [14] it is proved that

$$\| x \|=(\| x,a \|^p+\| x,b \|^p)^{1/p},\ x\in L,\ p\geq 1, \tag{2}$$
$$\| x \|=\max\{\| x,a \|,\| x,b \|\},\ x\in L \tag{3}$$

are norms on $L$, which are denoted by $\|\cdot\|_{a,b,p}$ and $\|\cdot\|_{a,b,\infty}$, respectively. Naturally, the following question arise: If the strict convexity of the space $(L,\|\cdot,\cdot\|)$ implies strong convexity of the spaces $(L,\|\cdot\|_{a,b,p})$, $p\geq 1$ and $(L,\|\cdot\|_{a,b,\infty})$. In the following, we will give the answer.

**Theorem 3.** Let $(L,\|\cdot,\cdot\|)$ be is strictly convex 2-normed space, $p>1$ and let $\{a,b\}$ be a linearly independent subset of $L$. Then, the normed space $(L,\|\cdot\|_{a,b,p})$ is a strictly convex space.

**Proof.** Let $(L,\|\cdot,\cdot\|)$ be a strictly convex space, $p>1$ and let $\{a,b\}$ be a linearly independent subspace of $L$. If the following holds

$$\| x \|_{a,b,p}+\| y \|_{a,b,p}=\| x+y \|_{a,b,p},\ x,y\neq 0.$$

then from (2), the parallelepiped law for 2-norm and the Minkowski's inequalities are also satisfied

$$\begin{aligned}(\| x,a \|^p+\| x,b \|^p)^{1/p}+(\| y,a \|^p+\| y,b \|^p)^{1/p} &=(\| x+y,a \|^p+\| x+y,b \|^p)^{1/p}\\ &\leq[(\| x,a \|+\| y,a \|)^p+(\| x,b \|+\| y,b \|)^p]^{1/p}\\ &\leq(\| x,a \|^p+\| x,b \|^p)^{1/p}+(\| y,a \|^p+\| y,b \|^p)^{1/p}.\end{aligned}$$

Because of that, in the above sequence of inequalities acctually the equality holds, which means that in the parallelepiped law and in Minkowski's inequality equality holds, that is

$$\| x+y,a \|=\| x,a \|+\| y,a \|,\ \| x+y,b \|=\| x,b \|+\| y,b \|, \tag{4}$$
$$\| x,a \|\cdot\| y,b \|=\| x,b \|\cdot\| y,a \|. \tag{5}$$

There are two cases:

1) $a\notin V(x,y)$ or $b\notin V(x,y)$ and
2) $a,b\in V(x,y)$.

Let $a\notin V(x,y)$ or $b\notin V(x,y)$. But, $(L,\|\cdot,\cdot\|)$ is strictly convex space, so Theorem 2 and (4) gives that $y=\alpha x$ for some $\alpha>0$, which means that $(L,\|\cdot\|_{a,b,p})$ is strictly convex space.

The second case, $a,b\in V(x,y)$ contradicts the linear independence of the set $\{a,b\}$. That is, if $a=mx+ny,\ b=rx+qy$, for some $m,n,r,q\in\mathbf{R}$, then

$$\begin{aligned}&\| x,a \|=| n |\cdot\| x,y \|,\ \| y,a \|=| m |\cdot\| x,y \|,\ \| x+y,a \|=| n-m |\cdot\| x,y \|,\\ &\| x,b \|=| q |\cdot\| x,y \|,\ \| y,b \|=| r |\cdot\| x,y \|,\ \ \| x+y,b \|=| q-r |\cdot\| x,y \|,\end{aligned} \tag{6}$$

and from the equalities (4) and (5) the following holds

$$\begin{aligned}(|n|+|m|)\,\|x,y\| &= |n-m|\cdot\|x,y\|,\\ (|q|+|r|)\,\|x,y\| &= |q-r|\cdot\|x,y\|,\end{aligned} \tag{7}$$

$$|nr|\cdot\|x,y\|^2 = |mq|\cdot\|x,y\|^2\,. \tag{8}$$

Further, if $\|x,y\|=0$, then $\dim V(x,y)=1$, meaning that the $\{a,b\}$ is linearly dependent, and this is a contradiction. If $\|x,y\|\neq 0$, then from the last three equalities the following is true

$$|n|+|m|=|n-m|,\ |q|+|r|=|q-r|, \tag{9}$$

$$|nr|=|mq|\,. \tag{10}$$

From the equalities (9) it follows that $mn\leq 0$ and $qr\leq 0$, so from the equality (10) $nr=mq$ holds. But, it means that $ra-mb=(nr-mq)y=0$ and if $r\neq 0$ or $m\neq 0$, follows that the set $\{a,b\}$ is linearly dependent, and if $r=m=0$, then $a=ny,\ b=qy$, which again is in the contradiction with the independence of the set $\{a,b\}$.

In the next example we will show that, if $(L,\|\cdot,\cdot\|)$ is a strictly convex 2-normed space and $\{a,b\}$ is linearly independent subset of $L$, then the normed space $(L,\|\cdot\|_{a,b,\infty})$ is not necessarily strictly convex.

**Example 3.** Let $\mathbf{R}^3$ be a Hilbert space with the usual inner product. Then by

$$(x,y\,|\,z)=\begin{vmatrix}(x,y) & (x,z)\\ (y,z) & (z,z)\end{vmatrix},\qquad x,y,z\in L \tag{11}$$

a 2-inner product is defined and by

$$\|x,y\|=\sqrt{\|x\|^2\|y\|^2-(x,y)^2} \tag{12}$$

a 2-norm in $\mathbf{R}^3$ is defined. Also the 2-normed space $(\mathbf{R}^3,\|\cdot,\cdot\|)$ is strictly convex space (see [2]). The vectors $a=(1,1,3)$ and $b=(1,2,0)$ are linearly independent, meaning that by (3), a norm $\|\cdot\|_{a,b,\infty}$ is given on $\mathbf{R}^3$. Let $x=(1,1,1)$ and $y=(1,1,0)$. From (12) it follows that

$$\|x,a\|=2\sqrt{2},\ \|x,b\|=\sqrt{6},\ \|y,a\|=3\sqrt{2},\ \|y,b\|=1,\ \|x+y,a\|=5\sqrt{2},\ \|x+y,b\|=3\,.$$

So, from (3) it follows that

$$\|x\|_{a,b,\infty}=2\sqrt{2},\ \|y\|_{a,b,\infty}=3\sqrt{2},\ \|x+y\|_{a,b,\infty}=5\sqrt{2}\,,$$

which means, that

$$\|x\|_{a,b,\infty}+\|y\|_{a,b,\infty}=\|x+y\|_{a,b,\infty}\,.$$

But, for every $\alpha>0$ the it is true that $y\neq\alpha x$, so the space $(\mathbf{R}^3,\|\cdot\|_{a,b,\infty})$ is not strictly convex spaced. On the other side, according to the Theorem 2.4.4, pp. 53, [15] every uniformly convex space is strictly convex, and because $(\mathbf{R}^3,\|\cdot\|_{a,b,\infty})$ is not a strictly convex space, we can come to a conclusion that it is not a uniformly convex space although the 2-normed space $(\mathbf{R}^3,\|\cdot,\cdot\|)$ is uniformly convex.

**Theorem 4.** Let $(L,\|\cdot,\cdot\|)$ be a 2-normed space. If $L$ is uniformly convex space then it is strictly convex one.

**Proof.** Let $(L,\|\cdot,\cdot\|)$ be a uniformly convex space and let for $x,y,z\in L$, $z\notin V(x,y)$ $\|x,z\|=\|y,z\|=1$ and $x\neq y$ holds. Then, for $\varepsilon=\frac{\|x-y,z\|}{2}$, follows that $\varepsilon>0$ and because $L$ uniformly convex space it follows that there exist $\delta(\varepsilon)>0$ such that from $\|x,z\|=\|y,z\|=1$, $\|x-y,z\|\geq\varepsilon$ and $z\notin V(x,y)$ the following

$$\|x+y,z\|\leq 2(1-\delta(\varepsilon))<2,$$

that is $\|\frac{x+y}{2},z\|<1$ holds. Finally, from Theorem 2 the result that $L$ is strictly convex space follows.

**Example 4.** In [16] it is proved that in the set consisting of all bounded sequences of real numbers $l^\infty$ by

$$\|x,y\|=\sup_{\substack{i,j\in\mathbf{N}\\ i<j}}\left|\begin{vmatrix} x_i & x_j\\ y_i & y_j\end{vmatrix}\right|,\ x=(x_i)_{i=1}^\infty,\ y=(y_i)_{i=1}^\infty\in l^\infty$$

a 2-norm is defined, which means that $(l^\infty,\|\cdot,\cdot\|)$ is a real 2-normed space and also it is proved that $l^\infty$ is not strictly convex 2-normed space. From the Theorem 4 it follows that $l^\infty$ is not a uniformly convex space.

The notion of convergent sequence in a 2-normed space is introduced by A. White, who proved some results concerning this. Namely, the sequence $\{x_n\}_{n=1}^\infty$ in the linear 2-normed space is *convergent* if there exists $x\in L$ such that

$$\lim_{n\to\infty}\|x_n-x,y\|=0,\ \text{for every } y\in L.$$

The vector $x\in L$ is the limit of the sequence $\{x_n\}_{n=1}^\infty$ and we denote $\lim_{n\to\infty}x_n=x$ or $x_n\to x$, $n\to\infty$, ([17]).

**Theorem 5.** A 2-normed space $(L,\|\cdot,\cdot\|)$ is uniformly convex if and only if for every two sequences $\{x_n\}_{n=1}^\infty,\{y_n\}_{n=1}^\infty$ such that

1) $\|x_n,z\|=\|y_n,z\|=1$ and $z\notin V(x_n,y_n)$
2) $\lim_{n\to\infty}\|x_n+y_n,z\|=2$ and $z\notin V(x_n,y_n)$

the following holds $\lim_{n\to\infty}(x_n-y_n)=0$.

**Proof.** Let the conditions 1) and 2) be satisfied, and let we assume that the sequence $\{x_n-y_n\}_{n=1}^\infty$ doesn't converge to 0. Then there exists $\varepsilon_0>0$, $z\in L$ and a sequence of natural numbers $\{n_k\}_{k=1}^\infty$ such that $\|x_{n_k}-y_{n_k},z\|\geq\varepsilon_0$, $z\notin V(x_{n_k},y_{n_k})$. But, $L$ is uniformly convex space, so for this $\varepsilon_0$ there exists $\delta(\varepsilon_0)>0$ such that

$$\|x_{n_k}+y_{n_k},z\|\leq 2(1-\delta(\varepsilon_0)),\ z\notin V(x_{n_k},y_{n_k}),$$

which, is a contradiction with 2). Finally, it follows that $\lim_{n\to\infty}(x_n-y_n)=0$.

Let $L$ is such that for some sequences $\{x_n\}_{n=1}^\infty,\{y_n\}_{n=1}^\infty$ satisfying conditions 1) and 2), $\lim_{n\to\infty}(x_n-y_n)=0$ holds, but let $L$ is not a uniformly convex space. Then, for some $\varepsilon>0$ and for $\delta=\frac{1}{n}$ there exists $x_n,y_n\in L$ such that *i)* $\| x_n, z \| = \| y_n, z \| = 1$ and $z \notin V(x_n, y_n)$,

*ii)* $\| x_n + y_n, z \| \geq 2(1 - \frac{1}{n})$ and $z \notin V(x_n, y_n)$

*iii)* $\| x_n - y_n, z \| \geq \varepsilon$.

But, *iii*) is in contradiction with the assumptions, because from *ii*) it follows that $\lim_{n\to\infty} \| x_n + y_n, z \| = 2$ and $z \notin V(x_n, y_n)$. Finally, from the above contradiction, it follows that $L$ is uniformly convex space

Theorem 4 has the following equivalent form.

**Theorem 5'.** A 2-normed space $(L, \| \cdot, \cdot \|)$ is uniformly convexif and only if for some sequences $\{x_n\}_{n=1}^{\infty}, \{y_n\}_{n=1}^{\infty}$ satisfying

1) $\lim_{n\to\infty} \| x_n, z \| = \lim_{n\to\infty} \| y_n, z \| = 1$ and $z \notin V(x_n, y_n)$

2) $\lim_{n\to\infty} \| x_n + y_n, z \| = 2$ and $z \notin V(x_n, y_n)$

the following holds $\lim_{n\to\infty} (x_n - y_n) = 0$.

**Theorem 6.** If a 2-normed space $(L, \| \cdot, \cdot \|)$ is uniformly convex and $\varphi$ is strictly convex and strictly increasing function on $(0,1]$ such that $\varphi(1) = 1$. Then for the function

$$h(t) = \inf\{\varphi(\| x + ty, z \|) + \varphi(\| x - ty, z \|) - 2,\ \| x, z \| = \| y, z \| = 1, z \notin V(x, y)\}$$

holds $h(t) > 0$, for every $t \in (0,1]$.

**Proof.** Let $L$ is a uniformly convex space and let $\varphi$ is strictly convex and strictly increasing function on $(0,1]$ such that $\varphi(1) = 1$. Also let there is some $t_0 \in (0,1]$ such that $h(t_0) = 0$. From the definition of the function $h(t)$ it follows that there are sequences $\{x_n\}_{n=1}^{\infty}$ and $\{y_n\}_{n=1}^{\infty}$ such that $\| x_n, z \| = \| y_n, z \| = 1$, $z \notin V(x_n, y_n)$ and

$$\lim_{n\to\infty} (\varphi(\| x_n + t_0 y_n, z \|) + \varphi(\| x_n - t_0 y_n, z \|) = 2. \quad (2)$$

But, the function $\varphi$ is strictly increasing with $\varphi(1) = 1$, meaning that it is bounded. So because $\varphi$ is convex it follows that it is a continuous function. So, there exists inverse function $\varphi^{-1}$, also continuous and strictly increasing. Now, taking into consideration the definition of $h(t)$, the properties of the function $\varphi$ and the equality (2) it follows that

$$2 \leq 2\varphi(\tfrac{\| x_n + t_0 y_n, z \| + \| x_n - t_0 y_n, z \|}{2}) \leq \varphi(\| x_n + t_0 y_n, z \|) + \varphi(\| x_n - t_0 y_n, z \|) \to 2,\ n \to \infty,$$

or

$$1 \leq \varphi(\tfrac{\| x_n + t_0 y_n, z \| + \| x_n - t_0 y_n, z \|}{2}) \to 1,\ n \to \infty. \quad (3)$$

Further on, because the function $\varphi$ is strictly convex and because of the previous consideration it follows that

$$\lim_{n\to\infty} | \| x_n + t_0 y_n, z \| - \| x_n - t_0 y_n, z \| | = 0. \quad (4)$$

But, the function $\varphi^{-1}$ is continuous, strictly increasing and $\varphi^{-1}(1) = 1$, so from (3) it follows that

$$\lim_{n\to\infty} (\| x_n + t_0 y_n, z \| + \| x_n - t_0 y_n, z \|) = 2. \quad (5)$$

Finally, from (4) and (5) it follows that

$\lim_{n\to\infty} \| x_n + t_0 y_n, z \| = \lim_{n\to\infty} \| x_n - t_0 y_n, z \| = 1$ and $z \notin V(x_n, y_n)$ and

$\lim_{n\to\infty} \| x_n + t_0 y_n + (x_n - t_0 y_n), z \| = \lim_{n\to\infty} 2\| x_n, z \| = 2$ and $z \notin V(x_n, y_n)$,

and because $(L, \|\cdot,\cdot\|)$ is uniformly convex from Theorem 5' the following is true

$$2t_0 = \lim_{n\to\infty} 2t_0 \| y_n, z \| = \lim_{n\to\infty} \| x_n + t_0 y_n - (x_n - t_0 y_n), z \| = 0,$$

which is a contradiction, so $h(t) > 0$, for every $t \in (0,1]$.

## 3. Conclusion

In Theorem 3 we have proved that, if $(L, \|\cdot,\cdot\|)$ is strictly convex 2-normed space, then for $p > 1$ and $\{a, b\}$ linearly independent subset of $L$, the normed space $(L, \|\cdot\|_{a,b,p})$ is strictly convex, and in Example 3, it is proven that the normed space $(\mathbf{R}^3, \|\cdot\|_{a,b,\infty})$ is not strictly convex space. The question, if the normed space $(L, \|\cdot\|_{a,b,1})$ is strictly convex, arises. Also, it is natural to ask if the opposite of Theorem 6 holds and what kind of other results for uniformly convex normed spaces can be generalized for 2-normed spaces.

## References


[1] Khan, A., Uniformly Convex in 2-normed Spaces, Mathematics Seminar Notes, Vol. 10, (1982).

[2] Diminnie, C., Gähler, S. and White, A., 2-Inner Product Spaces, Demonstratio Mathematica, Vol. VI, No 1, 525-536, (1973).

[3] Cho, Y. J., Ha, K. S. and Kim, S. W., Strictly Convex Linear 2-Normed Spaces, Math. Japonica, 26, 475-478, (1981).

[4] Diminnie, Ch. and Gahler, S., Some geometric results concerning strictly 2-convex 2-normed spaces, Math. Seminar Notes, Kobe Univ., 6, 245-253, (1978).

[5] Diminnie, Ch., Gahler, S. and White, A., Strictly Convex Linear 2-Normed Spaces, Math. Nachr. 59, 319-324, (1974).

[6] Diminnie, Ch., Gahler, S. and White, A., Remarks on Strictly Convex and Strictly 2-Convex 2-Normed Spaces, Math.Nachr. 88, 363-372, (1979).

[7] Ha, K. S., Cho, Y. J. and White, A., Strictly convex and strictly 2-convex linear 2- normed spaces, Math.Japon. 33, 375-384, (1988).

[8] Iseki, K., On non-expansive mappings in strictly convex 2-normed spaces, Math. Seminar Notes, Kobe Univ., 3, 125-129, (1975).

[9] Iseki, K., Mathematics in 2-normed spaces, Math. Seminar Notes, Kobe Univ., 6, 161-176, (1976).

[10] Khan, A. and Siddiqi, A. (1978). Characterization of Strictly Convex Linear 2-normed Spaces in Therms of Duality Bimappings, Math. Japonica 23, No. 1,

[11] Mabizela, S., A characterization of strictly convex linear 2-normed spaces, Questiones Math. 12, 201-204, (1989).

[12] Malčeski, R., Strong convex n-normed spaces, Macedonian Academy of Sciences and Arts, Contributions, XVIII 1-2, 39-57, (1997).

[13] Malčeski, A. and Malčeski, R., $L^p(\mu)$ as a 2-normed space, Matematički bilten, V. 29 (XXIX), 71-76, (2005).

[14] Malčeski, R. and Anevska, K., Families of norms generated by 2-norm, AJER, e-ISSN 2320-0847 p-ISSN 2320-0936, Volume-03, Issue-05, pp-315-320, (2014).

[15] I. V. Istrățescu, Strict convexity and complex strict convexity (theory and applications), Marcel Dekker, New York – Basel, (1984).

[16] Malčeski, A., l∞ as a n-normed space, Matematički bilten, V. 21 (XXI), 103-110, (1997).

[17] White A., 2-Banach Spaces, Math. Nachr. 42, 43-60, (1969).

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