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The Transient Solution of $M^{[X]}/G/1$ Queueing System Subject to Catastrophes, Server Failure and Repair

Dr. G. Ayyappan
Mathematics
Pondicherry Engineering college
Pondicherry, India Country
ayyappan@pec.edu

S.Shyamala
Mathematics
Anna University
Thiruvannaamalai , India
subramaniyanshyamala@gmail.com

Abstract: *In this paper, we consider a single server non-Markovian queueing system which plays a vital role in modeling real life phenomena.In this model,the arrival units are in batches following compound Poisson process where as the service time follows general(arbitrary)distribution. In addition to a Poisson stream of positive arrivals, we assume that there is also a Poisson stream of negative arrivals,which are called catastrophes. These catastrophes may occur at any time of instant and leads to annihilation of all customers in the system. Also catastrophes makes system to be inactive. The broken down system should be repaired in order to move on to the operational state. In this paper we have obtained the time dependent solution of our model.*

Keywords: *Catastrophes, Poisson process, Transient Solution, Steady State Analysis.*

1. INTRODUCTION

The determination of transient solution is very much essential in analyzing the behavior of the system. There are methods that have been derived for obtaining transient solution of queues. Generating method by Bailey[2], combinatorial method by Chambernowne[3], difference equation by Conolly[4]and continued fraction method which could be found in in Jones and Thron[8]. One of the feature related with queueing model, which has been widely studied in the literature is queueing system subject to catastrophes. These catastrophes may come either from outside of system or from another service station and their occurrence leads to the system failure, for example if a job is infected with a virus arrives, it transmit the virus to other processor and inactivate them in computer networks with a virus by queueing networks with catastrophes which has been studied by Chao[5]. Jain and Kumar[7] introduced the concept of restoration in catastrophic queues. Jain and and Kumar[6] studied M/G/1 queue in presence of catastrophes. Krishnakumar[10] studied the transient solution M/M/1 queue with catastrophes, failures and repairs. Thangaraj[18]studied the transient analysis of M/M/1 feedback queue with catastrophes using continued fraction methods.

The rest of the paper has been organized as follows: in section 2, the mathematical description of our model has been found, in section 3,the transient solution of the system has been derived, in section 4, the steady state analysis has been discussed.

2. MATHEMATICAL DESCRIPTION OF THE MODEL

This model has been modeled by assuming the following assumptions.

◊ Let $\lambda c_i dt; i = 1,2,3\ldots$be the first order probability of arrival of 'i' customers in batches in the system during a short period of time(t,t+dt)where $0 \le c_i \le 1, \sum_{i=1}^{\infty} c_i = 1, \lambda > 0$ is the mean arrival rate of batches and they are provided service one by one on a first come $-$ first served basis.

◊ There is a single server which provides service following a general(arbitrary)distribution with distribution function B(v)and density function b(v). Let $\mu(x)dx$ be the conditional probability density function of service completion during the interval (x,x+dx] given that the elapsed service time is x, so that

$$\mu(x) = \frac{b(x)}{1 - B(x)} \tag{1}$$

and therefore

$$b(v) = \mu(v) e^{-\int_0^v \mu(x)} dx \tag{2}$$

◊ The catastrophes may occur at the service facility, when it is not empty according to Poisson stream with mean rate ξ. The catastrophes annihilates all the customers in the system instantaneously and after the the system has been repaired, the system start to work.

◊ During the repair process no customer is allowed.The repair time follows exponential distribution with mean time is $\frac{1}{\gamma}$.

◊ The various stochastic processes involved are independent to each other.

3. Transient State Solution of the Queueing Model

(i) $P_n(x,t)$ =probability that at time t, there are n customers in the queue excluding one customer being served and the elapsed service time of this customer is x.

(ii) $P_{00}(t)$ = the probability that at time 't' there are no customer in the system without the occurrence of catastrophe.

(iii) $Q_{00}(t)$ =the probability that at time 't' there are no customer in the system with the the occurrence of catastrophe.

The differential-difference equations governing the queueing model are

$$\frac{\partial}{\partial t} P_n(x,t) + \frac{\partial}{\partial x} P_n(x,t) + (\lambda + \mu(x) + \xi) P_n(x,t) = \lambda \sum_{i=1}^{n-1} P_{n-i}(x,t) n \geq 1 \tag{3}$$

$$\frac{\partial}{\partial t} P_0(x,t) + \frac{\partial}{\partial x} P_0(x,t) + (\lambda + \mu(x) + \xi) P_0(x,t) = 0 \tag{4}$$

$$\frac{d}{dt} P_{00}(t) = -\lambda P_{00}(t) + \int_0^\infty P_0(x,t) \mu(x) dx + \gamma Q_{00}(t)) \tag{5}$$

$$\frac{d}{dt} Q_{00}(t) = -\gamma Q_{00}(t) - \xi P_{00}(t) + \xi \tag{6}$$

These equations are to be solved with the following boundary conditions

$$P_n(0,t) = \int_0^\infty P_{n+1}(0,t) \mu(x) dx + \lambda c_{n+1} p_{00}(t); n \geq 1 \tag{7}$$

$$P_0(0,t) = \int_0^\infty P_1(0,t) \mu(x) dx + \lambda C_1 P_{00}(t) \tag{8}$$

Define Laplace transform

$$\bar{f}(s) = \int_0^\infty e^{-st} f(t) dt \tag{9}$$

Taking Laplace transforms for the equations (3) to (9)

$$\frac{\partial}{\partial x}\overline{P}_n(x,s)+(s+\lambda+\mu(x)+\xi)\overline{P}_n(x,s)=\lambda\sum_{i=1}^{n-1}\overline{P}_{n-i}(x,s);n\geq 1 \tag{10}$$

$$\frac{\partial}{\partial x}\overline{P}_0(x,s)+(s+\lambda+\mu(x)+\xi)\overline{P}_0(x,s)=0 \tag{11}$$

$$(s+\lambda)\overline{P}_{00}(s)=1+\int_0^\infty\overline{P}_0(x,s)\mu(x)dx+\gamma\overline{Q}_{00}(s) \tag{12}$$

$$(s+\gamma)\overline{Q}_{00}(s)=\frac{\xi}{s}-\xi\overline{P}_{00}(s) \tag{13}$$

$$\overline{P}_n(0,s)=\int_0^\infty\overline{P}_{n+1}(0,s)\mu(x)dx+\lambda c_{n+1}\overline{P}_{00}(s);n\geq 1 \tag{14}$$

$$\overline{P}_0(0,s)=\int_0^\infty\overline{P}_1(0,s)\mu(x)dx+\lambda c_1\overline{P}_{00}(s) \tag{15}$$

Define probability generating function as follows

$$P_q(x,z,t)=\sum_{n=0}^\infty P_n(x,t)z^n \tag{16}$$

$$P_q(z,t)=\sum_{n=0}^\infty P_n(t)z^n \tag{17}$$

$$C(z)=\sum_{n=1}^\infty c_n z^n \tag{18}$$

which are convergent inside the circle given by $|z|\leq 1$

Multiply (10) by z^n and add with (11), we get

$$\frac{\partial}{\partial x}\overline{P}_q(x,z,s)+(s+\lambda-\lambda C(z)+\mu(x)+\xi)\overline{P}_q(x,z,s)=0 \tag{19}$$

For the boundary conditions multiply (14) by z^{n+1} and multiply(15) by z adding them and using equation (12)we get

$$z\overline{P}_q(0,z,s)=(1-s\overline{P}_{00}(s))+\lambda(C(z)-1)\overline{P}_{00}(s)+\gamma\overline{Q}_{00}(s)+\int_0^\infty\overline{P}_q(0,z,s)\mu(x)dx \tag{20}$$

Integrating (19) from o to x yields

$$\overline{P}_q(x,z,s)=\overline{P}_q(0,z,s)e^{-(s+\lambda-\lambda C(z)+\xi)x-\int_0^x\mu(t)dt} \tag{21}$$

where $\overline{P}_q(0,z,s)$ is given by (20).Again integrating equation (21) by parts with respect to x

$$\overline{P}_q(z,s)=\overline{P}_q(0,z,s)\left[\frac{1-\overline{B}(s+\lambda-\lambda C(z)+\xi)}{s+\lambda-\lambda C(z)+\xi}\right] \tag{22}$$

where $\overline{B}(s+\lambda-\lambda C(z)+\xi)$ is given by $\int_0^\infty e^{-(s+\lambda-\lambda C(z)+\xi)x}dB(x)$ is the Laplace - Stieltjes transform of the service time B(x).

Now multiplying both sides of equation (21) by $\mu(x)$ and integrating over x we get

$$\int_0^\infty \overline{P}_q(x,z,s)\mu(x)dx = \overline{P}_q(0,z,s)\overline{B}(s+\lambda-\lambda C(z)+\xi) \tag{23}$$

Using (23) in (20)

$$\overline{P}_q(0,z,s) = \frac{(1-s\overline{P}_{00}(s))+\lambda(C(z)-1)\overline{P}_{00}(s)+\gamma\overline{Q}_{00}(s)}{z-\overline{B}(s+\lambda(C(z)-1)+\xi)} \tag{24}$$

Using (24) in (22)

$$\overline{P}_q(z,s) = \frac{(1-s\overline{P}_{00}(s))+\lambda(C(z)-1)\overline{P}_{00}(s)+\gamma\overline{Q}_{00}(s)\left[\dfrac{1-\overline{B}(s+\lambda(1-C(z))+\xi)}{s+\lambda(1-C(z))+\xi}\right]}{z-\overline{B}(s+\lambda(C(z)-1)+\xi)} \tag{25}$$

4. STEADY STATE SOLUTION OF THE MODEL

In this section we shall derive the steady state probability distribution for our queueing model.To define the steady state probabilities we suppress the argument ’t’ wherever it appears in the time dependent solution analysis, by using the well known ’Tauberian" property.

$$Lt_{s\to 0}s\bar{f}(s) = Lt_{t\to\infty}f(t) \tag{26}$$

Multiplying both sides of equation (25) by s,taking limit $s\to 0$ and applying property from equation (26) we get

$$P_q(z) = \left[\frac{\lambda(C(z)-1)P_{00}+\gamma Q_{00}}{z-\overline{B}(\lambda(1-C(z))+\xi)}\right]\frac{1-\overline{B}(\lambda(1-C(z))+\xi)}{\lambda(1-C(z))+\xi} \tag{27}$$

Let $W_q(z)$ be denote the probability generating function of the queue size irrespective of the state of the system.

$$W_q(z) = P_q(z)+P_{00}+Q_{00} \tag{28}$$

$$W_q(z) = \left[\frac{\lambda(C(z)-1)P_{00}+\gamma Q_{00}}{z-\overline{B}(\lambda(1-C(z))+\xi)}\right]\left[\frac{1-\overline{B}(\lambda(1-C(z))+\xi)}{\lambda(1-C(z))+\xi}\right]+P_{00}+Q_{00} \tag{29}$$

applying equation(26) in (13)

$$Q_{00} = \frac{\xi(1-P_{00})}{\gamma} \tag{30}$$

Now P_{00} can be obtained by using the normalizing condition from

$$|W(z)|_{z=1}=1$$

$$P_{00} = \frac{[\lambda E(I)\overline{B}(\xi)+1]+\lambda\xi\overline{B}(\xi)\left[\dfrac{[\lambda E(I)\overline{B}(\xi)+1]}{\lambda E(I)[1-\overline{B}(\xi)-\xi\overline{B}(\xi)]-\xi}\right]-\dfrac{\xi}{\gamma}}{[1-\dfrac{\xi}{\gamma}]} \tag{31}$$

Equation(31) gives the probability that the server is idle. Substitute P_{00} and Q_{00} from equations (31) and (30) respectively in equation (28), $W_q(z)$ have been completely and explicitly determined which is the the probability generating function of the queue size .

The Average Queue Size:

Let L_q denote the mean number of customers in the queue under the steady state, then it can be obtained as follows.

$$L_q = \frac{d}{dz} W_q(z)\,|_{z=1}$$

5. CONCLUSION

We have proposed a single server subject to catastrophes which leads to server failure. This model can applied in computer and communication networks. We have the probability generating function of transient solutions explicitly and along with this the steady state has also been analysed.

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