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More Functions Associated with Semi-Star-Open Sets

S. Pious Missier
P.G. Department of Mathematics,
V.O.Chidambaram College, Thoothukudi, India
spmissier@gmail.com

A. Robert
Department of Mathematics,
Aditanar College, Tiruchendur,India
arunachalam.robert@gmail.com

Abstract: *In this paper we define semi*-irresolute, contra-semi*-irresolute, totally semi*-continuous, semi*-totally continuous, semi*-totally open functions and semi*-homeomorphisms and investigate their properties.*

Keywords: *semi*-irresolute, contra-semi*-irresolute, totally semi*-continuous, semi*-totally continuous, semi*-totally open, semi*-homeomorphism.*

1. Introduction

In 1963, Levine [1] introduced the concepts of semi-open sets and semi-continuity in topological spaces. Crossley and Hildebrand [2] defined and studied irresolute functions and semi-homeomorphisms. Caldas [3] introduced contra-irresolute and ap-irresolute maps and investigated their properties. In 1980, Jain [4] introduced the concept of totally continuous functions. Nour [5] defined totally semi-continuous and strongly semi-continuous functions. Benchalli et al.[6] introduced and studied semi-totally continuous and semi-totally open functions.

Quite recently, the authors [7, 8, 9] introduced and studied some new concepts, namely semi*-open sets, semi*-closed sets, semi*-Derived set and semi*-Frontier of a set. We have also defined semi*-continuous, semi*-open, semi*-closed, pre-semi*-open and pre-semi*-closed functions and their contra versions [10]. In this paper we define the semi*-irresolute, contra-semi*-irresolute, totally semi*-continuous, semi*-totally continuous, semi*-totally-open functions and semi*-homeomorphisms and investigate their properties.

2. Preliminaries

Throughout this paper X, Y and Z will always denote topological spaces on which no separation axioms are assumed, unless explicitly stated. If A is a subset of a space X, $Cl(A)$, $Int(A)$ and $D[A]$ respectively denote the closure, the interior and the derived set of A in X .

Definition 2.1[11]**:** A subset A of a topological space (X, τ) is called

(i) generalized closed (briefly g-closed) if $Cl(A)\subseteq U$ whenever $A\subseteq U$ and U is open in X.

(ii) generalized open (briefly g-open) if $X\setminus A$ is g-closed in X.

Definition 2.2: Let A be a subset of X. Then

(i) generalized closure [12] of A is defined as the intersection of all g-closed sets containing A and is denoted by $Cl^*(A)$.

(ii) generalized interior of A is defined as the union of all g-open subsets of A and is denoted by $Int^*(A)$.

Definition 2.3: A subset A of a topological space (X, τ) is called

(i) semi-open [1] (resp. semi*-open[7]) if $A\subseteq Cl(Int(A))$ (resp. $A\subseteq Cl^*(Int(A))$).

(ii) semi-closed [13] (resp. semi*-closed[8]) if $Int(Cl(A))\subseteq A$ (resp. $Int^*(Cl(A))\subseteq A$).

(iii) semi*-regular [8] if it is both semi*-open and semi*-closed.

The class of all semi*-open (resp. semi*-closed) sets is denoted by S*O(X, τ) (resp. S*C(X,τ))

Definition 2.4: Let A be a subset of X. Then

(i) The semi*-interior [7] of A is defined as the union of all semi*-open subsets of A and is denoted by $s*Int(A)$.

(ii) The semi*-closure [8] of A is defined as the intersection of all semi*-closed sets containing A and is denoted by $s*Cl(A)$.

Definition 2.5: A function $f:X\longrightarrow Y$ is said to be

(i) semi*-continuous [10] if $f^{-1}(V)$ is semi*-open in X for every open set V in Y.

(ii) pre-semi*-open [10] (resp. pre-semi*-closed [10]) if $f(U)$ is semi*-open (resp. semi*- closed) in Y for every semi*-open (resp. semi*-closed) set U in X.

(iii) totally continuous [4] (resp. totally semi-continuous [5]) if $f^{-1}(V)$ is clopen (resp. semi regular) in X for every open set V in Y.

(iv) semi-totally continuous [6] if $f^{-1}(V)$ is clopen in X for every semi-open set V in Y.

(v) semi-totally open if $f(V)$ is clopen in Y for every semi-open set V in X.

(vi) strongly-continuous if $f^{-1}(V)$ is clopen in X for every subset V of Y.

Theorem 2.6[7]**:** (i) Every open set is semi*-open.

(ii) Every semi*-open set is semi-open.

Theorem 2.7 [7]**:**

(i) If $\{A_\alpha\}$ is a collection of semi*-open sets in X, then $\cup A_\alpha$ is also semi*-open in X.

(ii) If A is semi*-open in X and B is open in X, then $A\cap B$ is semi*-open in X.

Theorem 2.8: A subset A of a space X is

(i) semi*-open if and only if $s*Int(A)=A$.[7]

(ii) semi*-closed if and only if $s*Cl(A)=A$.[8]

Lemma 2.9 [10]**:** Let A be a subset of a space X. Then

(i) A is semi*-open in X if and only if $Cl^*(Int(A))=Cl^*(A)$.

(ii) A is semi*-closed in X if and only if $Int^*(Cl(A))=Int^*(A)$.

Definition 2.10: If A is a subset of X, the semi*-frontier [9] of A is defined by

s*Fr(A)=$s*Cl(A)\backslash s*Int(A)$**.**

Theorem 2.11[9]**:** If A is a subset of X, then s*Fr(A)=$s*Cl(A)\cap s*Cl(X\backslash A)$.

Definition 2.12: Let A be a subset of X. A point x in X is a semi*-limit point [8] of A if every semi*-open set containing x intersects A in a point different from x.

Definition 2.13: The set of all semi*-limit points of A is called the semi*-Derived set [8] of A and is denoted by $D_{s*}[A]$.

Definition 2.14: A topological space X is said to be

(i) $T_{1/2}$ if every g-closed set in X is closed.[11]

(ii) Locally indiscrete if every open set is closed.[14]

Definition 2.15: A space X is said to be

(i) T_1 [14] if for every pair of distinct points x and y in X there exist open sets U and V such that $x\in U$ but $y\notin U$ and $y\in V$ but $x\notin V$.

(ii) an Alexandroff space if for every point x in X has a smallest neighborhood.

Remark 2.16: A topological space *X* is

(i) T_1 if and only if singletons are closed in *X*.[14]

(ii) Alexandroff space if and only if arbitrary intersection of open sets in *X* is open or equivalently if arbitrary union of closed sets is closed.

3. SEMI*-IRRESOLUTE FUNCTIONS

Definition 3.1: A function $f:X\longrightarrow Y$ is said to be ***semi*-irresolute at*** $x\in X$ if for each semi*-open set *V* of *Y* containing $f(x)$, there is a semi*-open set *U* in *X* such that $x\in U$ and $f(U)\subseteq V$.

Definition 3.2: A function $f:X\longrightarrow Y$ is said to be ***semi*-irresolute*** if $f^{-1}(V)$ is semi*-open in *X* for every semi*-open set *V* in *Y*.

Definition 3.3: A function $f:X\longrightarrow Y$ is said to be ***contra-semi*-irresolute*** if $f^{-1}(V)$ is semi*-closed in *X* for every semi*-open set *V* in *Y*.

Definition 3.4: A bijection $f:X\longrightarrow Y$ is said to be a ***semi*-homeomorphism*** if *f* is semi*-irresolute and pre-semi*-open.

Definition 3.5: A function $f:X\longrightarrow Y$ is said to be ***totally semi*-continuous*** if $f^{-1}(V)$ is semi*-regular in *X* for every open set *V* in *Y*.

Definition 3.6: A function $f:X\longrightarrow Y$ is said to be ***semi*-totally continuous*** if $f^{-1}(V)$ is clopen in *X* for every semi*-open set *V* in *Y*.

Definition 3.7: A function $f:X\longrightarrow Y$ is said to be ***semi*-totally open*** if $f(V)$ is clopen in *Y* for every semi*-open set *V* in *X*.

Definition 3.8: A function $f:X\longrightarrow Y$ is said to be ***strongly-semi*-continuous*** if $f^{-1}(V)$ is semi*-regular in *X* for every subset *V* of *Y*.

Theorem 3.9: Every semi*-irresolute function is semi*-continuous.

Proof: Let $f:X\longrightarrow Y$ be semi*-irresolute. Let *V* be open in *Y*. Then by Theorem 2.6(i), V is semi*-open. Since *f* is semi*-irresolute, $f^{-1}(V)$ is semi*-open in *X*. Thus *f* is semi*-continuous.

Remark 3.10: (i) It is not true that every semi*-continuous function is semi*-irresolute as shown by the following example.

(ii) The converse of the above theorem is true if the co-domain is a locally indiscrete space.

Example 3.11: Let X={a, b, c, d} and $\tau_1=\tau_2=\{\phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, X\}$.

Let $f:(X, \tau_1)\longrightarrow(X, \tau_2)$ be defined by *f*(a)=a, *f*(b)=b, *f*(c)=d, *f*(d)=b. Then *f* is semi*-continuous. Here {a, d} is semi*-open in (X, τ_2) but $f^{-1}(\{a, d\})=\{a, c\}$ is not semi*-open in (X, τ_1). Therefore *f* is not semi*-irresolute.

Theorem 3.12: Every constant function is semi*-irresolute.

Proof: Let $f:X\longrightarrow Y$ be a constant function defined by $f(x)=y_0$ for all *x* in *X*, where y_0 is a fixed point in *Y*. Let *V* be a semi*-open set in *Y*. Then $f^{-1}(V)=X$ or ϕ according as $y_0\in V$ or $y_0\notin V$. Thus

$f^{-1}(V)$ is semi*-open in *X*. Hence *f* is semi*-irresolute.

Theorem 3.13: Let $f:X\longrightarrow Y$ be a function. Then the following are equivalent:

(i) *f* is semi*-irresolute.

(ii) *f* is semi*-irresolute at each point of *X*.

(iii) $f^{-1}(F)$ is semi*-closed in *X* for every semi*-closed set *F* in *Y*.

(iv) $f(s^*Cl(A))\subseteq s^*Cl(f(A))$ for every subset *A* of *X*.

(v) $s^*Cl(f^{-1}(B))\subseteq f^{-1}(s^*Cl(B))$ for every subset *B* of *Y*.

(vi) $Int^*(Cl(f^{-1}(F)))=Int^*(f^{-1}(F))$ for every semi*-closed set *F* in *Y*.

(vii) $Cl^*(Int(f^{-1}(V)))=Cl^*(f^{-1}(V))$ for every semi*-open set *V* in *Y*.

(viii) $f^{-1}(s^*Int(B)) \subseteq s^*Int(f^{-1}(B))$ for every subset B of Y.

Proof: (i)⇒(ii): Let $f: X \longrightarrow Y$ be semi*-irresolute. Let $x \in X$ and V be a semi*-open set in Y containing $f(x)$. Then $x \in f^{-1}(V)$. Since f is semi*-irresolute, $U = f^{-1}(V)$ is a semi*-open set containing x such that $f(U) \subseteq V$. This proves (ii).

(ii)⇒(iii): Let F be a semi*-closed set in Y. Then $V = Y \setminus F$ is semi*-open in Y. Let $x \in f^{-1}(V)$, then $f(x) \in V$. By assumption, there is a semi*-open set U_x in X containing x such that $f(x) \in f(U_x) \subseteq V$. This implies that $U_x \subseteq f^{-1}(V)$. Hence $f^{-1}(V) = \cup\{U_x : x \in f^{-1}(V)\}$. By Theorem 2.7(i), $f^{-1}(V)$ is semi*-open in X. Therefore $f^{-1}(F) = f^{-1}(Y \setminus V) = X \setminus f^{-1}(V)$ is semi*-closed. This proves (iii).

(iii)⇒(iv): Let A⊆X. Let F be a semi*-closed set containing f(A). Then by (iii), f^{-1}(F) is a semi*-closed set that contains A. This implies that $s^*Cl(A) \subseteq f^{-1}(F) \Rightarrow f(s^*Cl(A)) \subseteq F$. This implies that $f(s^*Cl(A)) \subseteq s^*Cl(f(A))$.

(iv)⇒(v): Let B⊆Y and let A= f^{-1}(B). By assumption, $f(s^*Cl(A)) \subseteq s^*Cl(f(A)) \subseteq s^*Cl(B)$. This implies that $s^*Cl(A) \subseteq f^{-1}(s^*Cl(B))$. Hence $s^*Cl(f^{-1}(B)) \subseteq f^{-1}(s^*Cl(B))$.

(v)⇒(vi): Let F be a semi*-closed set in Y. Then by Theorem 2.8(ii), $s^*Cl(F)=F$ and hence by (v), $s^*Cl(f^{-1}(F)) \subseteq f^{-1}(s^*Cl(F)) = f^{-1}(F)$. But always $f^{-1}(F) \subseteq s^*Cl(f^{-1}(F))$. Therefore $s^*Cl(f^{-1}(F)) = f^{-1}(F)$. Hence by Theorem 2.8(ii), f^{-1}(F) is semi*-closed. By invoking Lemma 2.9(ii), $Int^*(Cl(f^{-1}(F))) = Int^*(f^{-1}(F))$.

(vi)⇒(vii): Let V be a semi*-open set in Y. Then Y\V is semi*-closed in Y. By assumption, $Int^*(Cl(f^{-1}(Y \setminus V))) = Int^*(f^{-1}(Y \setminus V))$. This implies that, $Cl^*(Int(f^{-1}(V))) = Cl^*(f^{-1}(V))$.

(vii)⇒(i): Let V be any semi*-open set in Y. Then by assumption, $Cl^*(Int(f^{-1}(V))) = Cl^*(f^{-1}(V))$. Now by invoking Lemma 2.9(i), f^{-1}(V) is semi*-open in X. Hence f is semi*-irresolute.

(i)⇒(viii): Let B be a subset of Y. Then by Theorem 2.8(i), s^*Int(B) is semi*-open in Y. By semi*-irresoluteness of f, $f^{-1}(s^*Int(B))$ is semi*-open in X and it is contained in f^{-1}(B). Therefore $f^{-1}(s^*Int(B)) \subseteq s^*Int(f^{-1}(B))$.

(viii)⇒(i): Let V be any semi*-open set in Y. Then $s^*Int(V)=V$. By our assumption, $f^{-1}(V) \subseteq s^*Int(f^{-1}(V))$ and hence $f^{-1}(V) = s^*Int(f^{-1}(V))$. Therefore by Theorem 2.8(i), f^{-1}(V) is semi*-open in X. Thus f is semi*-irresolute.

Theorem 3.14: Let $f: X \longrightarrow Y$ be a bijection. Then the following are equivalent:

(i) f is semi*-irresolute.

(ii) f^{-1} is pre-semi*-open.

(iii) f^{-1} is pre-semi*-closed.

Proof: Follows from the definitions and Theorem 3.13.

Theorem 3.15: Let $f: X \longrightarrow Y$ be a function. Then f is not semi*-irresolute at a point x in X if and only if x belongs to the semi*-frontier of the inverse image of some semi*-open set in Y containing f(x).

Proof: Suppose f is not semi*-irresolute at x. Then there is a semi*-open set V in Y containing f(x) such that f(U) is not a subset of V for every semi*-open set U in X containing x. Hence $U \cap (X \setminus f^{-1}(V)) \neq \phi$ for every semi*-open set U containing x. Therefore $x \in s^*Cl(X \setminus f^{-1}(V))$. We also have $x \in f^{-1}(V) \subseteq s^*Cl(f^{-1}(V))$. Thus $x \in s^*Cl(f^{-1}(V)) \cap s^*Cl(X \setminus f^{-1}(V))$. Hence by Theorem 2.11, $x \in s^*Fr(f^{-1}(V))$. On the other hand, let f be semi*-irresolute at x. Let V be a semi*-open set in Y containing f(x). Then f^{-1}(V) is a semi*-open set in X containing x. Hence $x \in s^*Int(f^{-1}(V))$. Therefore $x \notin s^*Fr(f^{-1}(V))$ for every open set V containing f(x).This proves the theorem.

Theorem 3.16: Let X be a topological space and Y be a space in which every semi*-open set is open. If f :X⟶Y is a semi*-continuous map, then f is semi*-irresolute.

Proof: Let V be a semi*-open set in Y. Then by assumption, V is open. By semi*-continuity of f, $f^{-1}(V)$ is semi*-open in Y. Hence f is semi*-irresolute.

Theorem 3.17: Every contra-semi*-irresolute function is contra-semi*-continuous.

Proof: Follows directly from definitions and Theorem 2.6(i).

Remark 3.18: The converse of the above theorem is not true as shown in the following example.

Example 3.19: Let X={a, b, c, d}; $\tau_1=\tau_2=\{\phi, \{a, b\}, \{a, b, c\}, X\}$. Let $f :(X, \tau_1)\longrightarrow(X, \tau_2)$ be defined by f(a)=f(b)=d, f(c)=a, f(d)=c. Then f is contra-semi*-continuous. But $f^{-1}(\{a, b, d\})=$

{a, b, c} which is not semi*-closed. Therefore f is not contra-semi*-irresolute.

Theorem 3.20: For a function f :X⟶Y, the following are equivalent:

(a) f is contra-semi*-irresolute.

(b) the inverse image of each semi*-closed set in Y is semi*-open in X.

(c) for each x∈X and each semi*-closed set F in Y with $f(x)\in F$, there exists a semi*-open set U in X such that x∈U and $f(U)\subseteq F$.

(d) $Cl^*(Int(f^{-1}(F)))=Cl^*(f^{-1}(F))$ for every semi*-closed set F in Y.

(e) $Int^*(Cl(f^{-1}(V)))=Int^*(f^{-1}(V))$ for every semi*-open set V in Y.

Proof: (a)⇒(b): Let F be a semi*-closed set in Y. Then Y\F is semi*-open in Y. Since f is contra - semi*-irresolute, $X\setminus f^{-1}(F)=f^{-1}(Y\setminus F)$ is semi*-closed in X. Hence $f^{-1}(F)$ is semi*-open in X.

(b)⇒(c): Let F be a semi*-closed set in Y containing f(x).Then U= $f^{-1}(F)$ is a semi*-open set. containing x such that $f(U)\subseteq F$. This proves (c).

(c)⇒(d): Let F be a semi*-closed set in Y and x∈ $f^{-1}(F)$, then $f(x)\in F$. By assumption, there is a semi*-open set U_x in X containing x such that $f(x)\in f(U_x)\subseteq F\Rightarrow U_x\subseteq f^{-1}(F)$. This follows that

$f^{-1}(F)=\cup\{U_x : x\in f^{-1}(F)\}$. By Theorem 2.7(i), $f^{-1}(F)$ is semi*-open in X. By Lemma 2.9(i), $Cl^*(Int(f^{-1}(F)))=Cl^*(f^{-1}(F))$. This proves (d).

(d)⇒(e): Let V be a semi*-open set in Y. Then Y\V is semi*-closed in Y. By assumption,

$Cl^*(Int(f^{-1}(Y\setminus V)))=Cl^*(f^{-1}(Y\setminus V))$. This implies that, $Int^*(Cl(f^{-1}(V)))=Int^*(f^{-1}(V))$.

(e)⇒(a): Let V be any semi*-open set in Y. Then by assumption, $Int^*(Cl(f^{-1}(V)))=Int^*(f^{-1}(V))$. Now by invoking Lemma 2.9(ii), $f^{-1}(V)$ is semi*-closed in X. Hence f is contra-semi*- irresolute.

Theorem 3.21: Let f :X⟶Y be semi*-irresolute and h:Y⟶Z be semi*-continuous. Then

$h\circ f$:X⟶Z is semi*-continuous.

Proof: Let V be an open set in Z. Since h is semi*-continuous, $h^{-1}(V)$ is semi*-open in Y. Since f is semi*-irresolute, $(h\circ f)^{-1}(V)=f^{-1}(h^{-1}(V))$ is semi*-open in X. Hence $h\circ f$ is semi*-continuous.

Theorem 3.22: If f :X⟶Y and h:Y⟶Z are semi*-irresolute, then so is $h\circ f$: X⟶Z.

Proof: Let V be a semi*-open set in Z. Since h is semi*-irresolute, $h^{-1}(V)$ is semi*-open in Y. Since f is semi*-irresolute, $(h\circ f)^{-1}(V)=f^{-1}(h^{-1}(V))$ is semi*-open in X. Hence $h\circ f$ is semi*-irresolute.

Theorem 3.23: Let f :X⟶Y be semi*-irresolute and h:Y⟶Z be contra-semi*-continuous. Then $h\circ f$: X⟶Z is contra-semi*-continuous.

Proof: Let V be an open set in Z. Since h is contra-semi*-continuous, $h^{-1}(V)$ is semi*-closed in Y. Since f is semi*-irresolute, by invoking Theorem 3.13, $(h\circ f)^{-1}(V)=f^{-1}(h^{-1}(V))$ is semi*-closed in X. Hence $h\circ f$ is contra-semi*-continuous.

Theorem 3.24: Let f :X⟶Y be semi*-irresolute and h:Y⟶Z be contra-semi*-irresolute. Then $h\circ f$: X⟶Z is contra-semi*-irresolute.

Proof: Let V be a semi*-open set in Z. Since h is contra-semi*-irresolute, $h^{-1}(V)$ is semi*-closed in Y. Since f is semi*-irresolute, by invoking Theorem 3.13, $(h\circ f)^{-1}(V)=f^{-1}(h^{-1}(V))$ is semi*-closed in X. Hence $h\circ f$ is contra-semi*-irresolute

Theorem 3.25: Let f :X⟶Y be contra-semi*-irresolute and h:Y⟶Z be semi*-irresolute. Then $h\circ f$: X⟶Z is contra-semi*-irresolute.

Proof: Let V be a semi*-open set in Z. Since h is semi*-irresolute, $h^{-1}(V)$ is semi*-open in Y. Since f is contra-semi*-irresolute, $(h\circ f)^{-1}(V)=f^{-1}(h^{-1}(V))$ is semi*-closed in X. Hence $h\circ f$ is contra-semi*-irresolute.

Theorem 3.26: Let f :X⟶Y be contra-semi*-irresolute and h:Y⟶Z be contra-semi*-irresolute. Then $h\circ f$: X⟶Z is semi*-irresolute.

Proof: Let V be a semi*-open set in Z. Since h is contra-semi*-irresolute, $h^{-1}(V)$ is semi*-closed in Y. Since f is contra-semi*-irresolute, by Theorem 3.20, $(h\circ f)^{-1}(V)=f^{-1}(h^{-1}(V))$ is semi*-open in X. Hence $h\circ f$ is semi*-irresolute.

Theorem 3.27: Let f :X⟶Y be a semi*-irresolute injection and A⊆X. If $x\in D_{s*}[A]$, then

$f(x)\in D_{s*}[f(A)]$.

Proof: Let x be a semi*-limit point of A. Let V be a semi*-open set in Y containing f(x). Then

$f^{-1}(V)$ is a semi*-open set in X containing x. Since x is a semi*-limit point of A, $f^{-1}(V)$ intersects A in a point other than x. Hence V intersects f(A) in a point other than f(x). Therefore f(x) is a semi*-limit point of f(A).

Theorem 3.28: Let f :X⟶Y be a function where X is an Alexandroff space and Y is any topological space. Then the following are equivalent:

(i) f is semi*-totally continuous.

(ii) for each x∈X and each semi*-open set V in Y with f(x)∈V, there exists a clopen set U in X such that x∈U and f(U)⊆V.

Proof: (i)⇒(ii): Suppose f :X⟶Y is semi*-totally continuous. Let x∈X and let V be a semi*-open set containing f(x). Then $U=f^{-1}(V)$ is a clopen set in X containing x and hence f(U)⊆V.

(ii)⇒(i): Let V be a semi*-open set in Y. Let $x\in f^{-1}(V)$. Then V is a semi*-open set containing f(x). By hypothesis there exist a clopen set U_x containing x such that $f(U_x)\subseteq V$ which implies that

$U_x\subseteq f^{-1}(V)$. Therefore we have $f^{-1}(V)=\cup\{U_x : x\in f^{-1}(V)\}$. Since each U_x is open, $f^{-1}(V)$ is open. Since each U_x is a closed set in the Alexandroff space X, $f^{-1}(V)$ is closed in X. Hence $f^{-1}(V)$ is clopen in X. Thus f is semi*-totally continuous.

Theorem 3.29: Every semi*-totally continuous function into a T_1 space which is either finite or an Alexandroff space is strongly-continuous.

Proof: Suppose f :X⟶Y is semi*-totally continuous. If Y is a T_1 space which is either finite or an Alexandroff space, then Y is a discrete space. Let B be a subset of Y. Then B is open and hence semi*-open in Y. Since f is semi*-totally continuous, $f^{-1}(B)$ is clopen in X. Thus f is strongly-continuous.

Theorem 3.30: A function f :X⟶Y is semi*-totally continuous if and only if $f^{-1}(F)$ is clopen in X for every semi*-closed set F in Y.

Proof: Let F be a semi*-closed set in Y. Then Y\F is semi*-open in Y. Since f is semi*-totally continuous, $f^{-1}(Y\setminus F)=X\setminus f^{-1}(F)$ is clopen in X. Hence $f^{-1}(F)$ is clopen in X. Conversely, let V be a semi*-open set in Y. Then Y\V is semi*-closed in Y. By hypothesis, f^{-1}(Y\V)=X\ f^{-1}(V) is clopen in X. Hence f^{-1}(V) is clopen in X. Therefore f is semi*-totally continuous.

Theorem 3.31: (i) Every semi*-totally-continuous function is totally continuous.

(ii) Every strongly continuous function is semi*-totally continuous.

(iii) Every semi*-totally continuous function is totally semi*-continuous.

(iv) Every semi*-totally continuous function is semi*-continuous.

(v) Every strongly semi*-continuous function is totally semi*-continuous.

(vi) Every totally semi*-continuous function is semi*-continuous.

(vii) Every semi-totally continuous function is semi*-totally continuous.

(viii) Every totally semi*-continuous function is totally semi-continuous.

Proof: (i) Suppose f :X⟶Y is semi*-totally continuous. Let V be an open set in Y. Then by Theorem 2.6(i), V is semi*-open in Y. Since f is semi*-totally continuous, f^{-1}(V) is clopen in X. Therefore f is totally continuous.

(ii) Suppose f :X⟶Y is strongly continuous. Let V be a semi*-open set in Y. Since f is strongly continuous, f^{-1}(V) is clopen in X. Therefore f is semi*-totally continuous.

(iii) Suppose f :X⟶Y is semi*-totally-continuous. Let V be an open set in Y. Then by invoking Theorem 2.6(i), V is semi*-open in Y. Since f is semi*-totally continuous, f^{-1}(V) is clopen in X and hence f^{-1}(V) is semi*-regular. Therefore f is totally semi*-continuous.

(iv) Suppose f :X⟶Y is semi*-totally continuous. Let V be an open set in Y. Then by Theorem 2.6(i), V is semi*-open in Y. Since f is semi*-totally continuous, f^{-1}(V) is clopen in X. Therefore

f^{-1}(V) is open and hence semi*-open. Thus f is semi*-continuous.

(v) Suppose f :X⟶Y is strongly semi*-continuous. Let V be an open set in Y. Since f is strongly semi*-continuous, f^{-1}(V) is semi*-regular in X. Therefore f is totally semi*-continuous.

(vi) Suppose f :X⟶Y is totally-semi*-continuous. Let V be an open set in Y. Since f is totally-semi*-continuous, f^{-1}(V) is semi*-regular in X and hence semi*-open in X. Therefore f is semi*-continuous.

(vii) Suppose f :X⟶Y is semi-totally continuous. Let V be a semi*-open set in Y. Then by Theorem 2.6(ii), V is semi-open. Since f is semi-totally continuous, f^{-1}(V) is clopen in X. Therefore f is semi*-totally continuous.

(viii) Suppose f :X⟶Y is totally semi*-continuous. Let V be an open set in Y. Since f is totally semi*-continuous, f^{-1}(V) is semi*-regular in X and hence semi-regular in X. Therefore f is totally semi-continuous.

Theorem 3.32: Every totally semi*-continuous function into a T_1 space which is either finite or an Alexandroff space is strongly-semi*-continuous.

Proof: Suppose f :X⟶Y is totally semi*-continuous. If Y is a T_1 space which is either finite or an Alexandroff space, then Y is a discrete space. Let B be a subset of Y. Then B is open in Y. Since f is totally semi*-continuous, f^{-1}(B) is semi*-regular in X. Thus f is strongly-semi*-continuous.

Theorem 3.33: Let f :X⟶Y be semi*-totally continuous and A is a clopen subset of Y. Then the restriction $f_{|A}$: A⟶Y is semi*-totally continuous.

Proof: Let V be a semi*-open set in Y. Then f^{-1}(V) is clopen in X and hence $(f_{|A})^{-1}$(V)=

A∩f^{-1}(V) is clopen in A. Therefore $f_{|A}$ is semi*-totally continuous.

Theorem 3.34: The composition of two semi*-totally continuous functions is semi*-totally-continuous.

Proof: Let f :X⟶Y and h:Y⟶Z be semi*-totally continuous functions. Let V be a semi*-open set in Z. Since h is semi*-totally continuous, h^{-1}(V) is clopen in Y and hence semi*-open in Y.

Since f is semi*-totally continuous, $(h\circ f)^{-1}(V)=f^{-1}(h^{-1}(V))$ is clopen in X. Hence $h\circ f$ is semi*-totally-continuous.

Theorem 3.35: Let f :X⟶Y be semi*-totally continuous and h:Y⟶Z be semi*-irresolute. Then $h\circ f$: X⟶Z is semi*-totally continuous.

Proof: Let V be a semi*-open set in Z. Since h is semi*-irresolute, $h^{-1}(V)$ is semi*-open in Y. Since f is semi*-totally-continuous, $(h\circ f)^{-1}(V)=f^{-1}(h^{-1}(V))$ is clopen in X. Hence $h\circ f$ is semi*-totally-continuous.

Theorem 3.36: Let f :X⟶Y be semi*-totally continuous and h:Y⟶Z be semi*-continuous. Then $h\circ f$: X⟶Z is totally continuous.

Proof: Let V be an open set in Z. By semi*-continuity of h, $h^{-1}(V)$ is semi*-open in Y. Since f is semi*-totally-continuous, $(h\circ f)^{-1}(V)=f^{-1}(h^{-1}(V))$ is clopen in X. Hence $h\circ f$ is totally continuous.

Theorem 3.37: Let f: X⟶Y be continuous and h:Y⟶Z be semi*-totally continuous. Then

$h\circ f$: X⟶Z is semi*-totally continuous.

Proof: Let V be a semi*-open set in Z. Since h is semi*-totally continuous, $h^{-1}(V)$ is clopen in Y. Since f is continuous, $(h\circ f)^{-1}(V)=f^{-1}(h^{-1}(V))$ is clopen in X. Hence $h\circ f$ is semi*-totally continuous.

Theorem 3.38: A bijective function f :X⟶Y is semi*-totally open if and only if f^{-1}:Y⟶X is semi*-totally continuous.

Proof: Suppose f^{-1}:Y⟶X is semi*-totally continuous. Let U be semi*-open in X. Then by semi*-totally continuity of f^{-1}, $f(U)=(f^{-1})^{-1}(U)$ is clopen in Y. Therefore f :X⟶Y is semi*-totally open. Conversely, suppose f :X⟶Y is semi*-totally open. Let U be semi*-open in X. Then

$(f^{-1})^{-1}(U)=f(U)$ is clopen in Y. Therefore f^{-1}:Y⟶X is semi*-totally continuous.

Theorem 3.39: The composition of two semi*-totally open functions is semi*-totally open.

Proof: Let f:X⟶Y and h:Y⟶Z be semi*-totally open. Let V be a semi*-open set in X. Then $f(V)$ is clopen in Y and hence semi*-open in Y. Since h is semi*-totally-open, $(h\circ f)(V)=h(f(V))$ is clopen in Z. Hence $h\circ f$ is semi*-totally open.

Theorem 3.40: Let f: X⟶Y be pre-semi*-open and h:Y⟶Z be semi*-totally open. Then

$h\circ f$: X⟶Z is semi*-totally open.

Proof: Let V be a semi*-open set in X. Since f is pre-semi*-open, $f(V)$ is semi*-open in Y. Since h is semi*-totally open, $(h\circ f)(V)=h(f(V))$ is clopen in Z. Hence $h\circ f$ is semi*-totally open.

Theorem 3.41: If f: X⟶Y and h:Y⟶Z are functions such that $h\circ f$: X⟶Z is semi*-totally open. Then (i) if f is semi*-irresolute and onto, then h is semi*-totally open.

(ii) if h is continuous and one to one, then f is semi*-totally open.

Proof: (i) Let V be semi*-open in Y. Since f is semi*-irresolute, $U=f^{-1}(V)$ is semi*-open in X. This implies that $(h\circ f)(U)=h(f(U))=h(V)$ is clopen in Z. Therefore h is semi*-totally open.

(ii) Let U be semi*-open in X. $(h\circ f)(U)$ is clopen in Z. Since h is continuous, $h^{-1}((h\circ f)(U))=f(U)$ is clopen in Y. Therefore f is semi*-totally open.

Theorem 3.42: Let f :X⟶Y and h:Y⟶Z be functions such that $h\circ f$:X⟶Z is semi*-open. Then

f is semi*-open if h is a semi*-irresolute injection.

Proof: Let U be an open set in X. Since $h\circ f$ is semi*-open, $h\circ f(U)$ is semi*-open in Z. Since h is semi*-irresolute, $f(U)=h^{-1}(h\circ f(U))$ is semi*-open in Y. Hence f is semi*-open.

4. Conclusion

In this paper we have introduced various functions associated with semi*α-open sets and investigated their properties in the light of concepts and results available in the literature.

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AUTHORS' BIOGRAPHY

Dr.S.Pious Missier: He is presently working as associative professor in the PG and Research Department of Mathematics in V.O .Chidambaram College, Tuticorin, Tamilnadu, India. He has been teaching Mathematics for both UG and PG students for the last 31 years. So far eight scholars have been awarded Ph.D under his guidance in Topology. He is actively engaged in research for the last 15 years. He is an approved guide of M.S. University, Tirunelveli. He also has published more than 70 research articles in reputed National and International Journals. Now seven scholars are pursuing Ph.D. under his guidance. He has attended many national and international conferences. He has been rendering voluntary service for many institutions and working for the uplift of the poor and downtrodden.

A.Robert: He is presently working as Associative Professor in the Department of Mathematics in Aditanar College, Tiruchendur, Tamilnadu, India. He has been teaching Mathematics for both UG and PG students for the last 30 years. He has been actively involved in research for the past 10 years. He has published more than 10 research articles in reputed National and International Journals. He has attended many national and international conferences.