Cylindrically Symmetric Massive Scalar Field In Bimetric Relativity

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Abstract: In this paper, cylindrically symmetric space-time is studied with massive scalar field in the context of Rosen’s Bimetric Theory of Relativity. Here it is shown that only vacuum model can be constructed.

Keywords: Cylindrically symmetric, Massive scalar field, Bimetric Relativity

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1. INTRODUCTION

The bimetric theory of gravitation was proposed by Rosen [1][2](1940,1973) to modify the Einstein’s general theory of relativity, by assuming two metric tensors. In this theory he has proposed a new formulation of the general relativity by introducing a background Euclidean metric tensor $\gamma_{ij}$ in addition to the usual Riemannian metric tensor $g_{ij}$ at each point of the four dimensional space-time. With the flat background metric $\gamma_{ij}$ the physical content of the theory is the same as that of the general relativity.

Thus, now the corresponding two line elements in a coordinate system $x^i$ are –

$$ds^2 = g_{ij}dx^i dx^j$$ (1.1)

and $$d\sigma^2 = \gamma_{ij}dx^i dx^j$$ (1.2)

where $ds$ is the interval between two neighboring events as measured by means of a clock and a measuring rod. The interval $d\sigma$ is an abstract or geometrical quantity not directly measurable. One can regard it as describing the geometry that would exist if no matter were present. Cylindrically symmetric perfect fluid distributions both static and non static have been discussed by many investigators for various matters in the context of general relativity. Heckman,O., and Schuck,E. [3], Mardar[4], Benergee,A.,and Benergee,S.[5],Krori,K.D.and Barua,J.[6],Roy and Narain [7][8],H.Baysal [9] have studied various aspects of cylindrically symmetric space-times.In addition to this cylindrically symmetric space-time is studied in bimetric relativity by Deo S.D. [10] with the matter cosmic strings and domain wall. In continuation of this study, here we have investigated cylindrically symmetric space-time with perfect fluid and Massive scalar field in bimetric relativity.

2. FIELD EQUATIONS IN BIMETRIC RELATIVITY

Rosen N. has proposed the field equations of Bimetric Relativity from variation principle as

$$K^i_j = N^i_j - \frac{1}{2} Ng^{ij} = -8\pi\kappa T^i_j$$ (2.1)
Where \( N_{ij} \) = \( \frac{1}{2} \gamma^{\alpha\beta} \left[ g^{hi} g_{hi|\alpha\beta} \right] \) \( \gamma^\beta \) \( (2.2) \)
\[ N = N^\alpha_\alpha, \quad \kappa = \sqrt{g} \] \( (2.3) \)
and \( g = \left| g_{ij} \right|, \quad \gamma = \left| \gamma_{ij} \right| \) \( (2.4) \)

Where a vertical bar (|) denotes a covariant differentiation with respect to \( \gamma_{ij} \)

### 3. CYLINDRICALLY SYMMETRIC SPACE TIME WITH PERFECT FLUID

We consider here the spherically symmetric line element of the form
\[ ds^2 = A^2 \left( dx^2 - dt^2 \right) + B^2 dy^2 + C^2 dz^2 \] \( (3.1) \)
where \( A, B \) and \( C \) are functions of \( x \) and \( t \) only.

Corresponding to equation (3.1), we consider the line element for background metric \( \gamma_{ij} \) as
\[ ds^2 = -dt^2 + dx^2 + dy^2 + dz^2 \] \( (3.2) \)

Since \( \gamma_{ij} \) is the Lorentz metric i.e. \( (1,1,1,-1) \), therefore \( \gamma \)-covariant derivative becomes the ordinary partial derivative.

and \( T_{ij} \) the energy momentum tensor for macro matter field represented by
\[ T_{ij} = T_i^j = \left( \rho + p \right) u_i u_j - p g_{ij} \] \( (3.3) \)

Together with \( g_{ij} u_i u_j = 1 \) where \( u_i \) is the four-velocity of the fluid having \( p \) and \( \rho \) as proper pressure and energy density respectively.

Using equations (2.1) to (2.4) with (3.1) and (3.3)

We get the field equations
\[ \frac{1}{2} \left[ \left( \frac{B^2}{B^2 - B} - \frac{B}{B^2} \right) - \left( \frac{B^2}{B^2 - B} - \frac{B}{B^2} \right) \left( \frac{C^2}{C^2 - C} - \frac{C}{C^2} \right) \right] = 8\pi \kappa \rho \] \( (3.4) \)
\[ \left( \frac{A^2}{A^2} - \frac{A}{A^2} \right) - \left( \frac{A^2}{A^2} - \frac{A}{A^2} \right) \left[ \left( \frac{B^2}{B^2 - B} - \frac{B}{B^2} \right) \left( \frac{C^2}{C^2 - C} - \frac{C}{C^2} \right) \right] = 8\pi \kappa \rho \] \( (3.5) \)
\[ \left( \frac{A^2}{A^2} - \frac{A}{A^2} \right) - \left( \frac{A^2}{A^2} - \frac{A}{A^2} \right) \left[ \left( \frac{B^2}{B^2 - B} - \frac{B}{B^2} \right) \left( \frac{C^2}{C^2 - C} - \frac{C}{C^2} \right) \right] = 8\pi \kappa \rho \] \( (3.6) \)
\[ \frac{1}{2} \left[ \left( \frac{B^2}{B^2 - B} - \frac{B}{B^2} \right) + \left( \frac{C^2}{C^2 - C} - \frac{C}{C^2} \right) \right] = -8\pi \kappa \rho \] \( (3.7) \)
Using equation (3.4) to (3.7), we get
\[ p + \rho = 0 \] (3.8)
This equation of state is known as false vacuum. In view of reality conditions \( p > 0, \rho > 0 \)
Equation (3.8) immediately implies that \( p = 0, \rho = 0 \) that is perfect fluid does not exist in
cylindrically symmetric space-time in bimetric relativity.

4. CYLINDRICALLY SYMMETRIC SPACE TIME WITH MASSIVE SCALAR FIELD

We consider here the spherically symmetric line element of the form
\[ ds^2 = A^2 \left( dx^2 - dt^2 \right) + B^2 dy^2 + C^2 dz^2 \] (4.1)
where \( A, B \) and \( C \) are functions of \( x \) and \( t \) only.

Corresponding to equation (4.1), we consider the line element for background metric \( \gamma_{ij} \) as
\[ ds^2 = -dt^2 + dx^2 + dy^2 + dz^2 \] (4.2)
Since \( \gamma_{ij} \) is the Lorentz metric i.e. \((1,1,1,-1)\), therefore \( \gamma \)-covariant derivative becomes the
ordinary partial derivative.

Also \( T_{ij} \) the energy momentum tensor for massive meson is given by
\[ T_{ij} = T_{ij}^{\text{meson}} = V_i V^j - \frac{1}{2} g_i^j \left( g_{kl} V^k V^l - m^2 V^2 \right) \] (4.3)

\[ 4 \pi \kappa \left( \frac{v_i^2}{2} - m^2 v^2 \right) \]

\[ \frac{1}{2} \left( \frac{B^2}{B^2} - \frac{B}{B} \right) - \left( \frac{B^2}{B^2} - \frac{B}{B} \right) + \left( \frac{C^2}{C^2} - \frac{C}{C} \right) - \left( \frac{C^2}{C^2} - \frac{C}{C} \right) = 4 \pi \kappa \left( \frac{v_i^2}{2} - m^2 v^2 \right) \] (4.4)

\[ \frac{A^2}{A^2} - \frac{A}{A} - \frac{A}{A} + \frac{1}{2} \left( \frac{B^2}{B^2} - \frac{B}{B} \right) - \left( \frac{B^2}{B^2} - \frac{B}{B} \right) + \left( \frac{C^2}{C^2} - \frac{C}{C} \right) - \left( \frac{C^2}{C^2} - \frac{C}{C} \right) = 4 \pi \kappa \left( \frac{v_i^2}{2} - m^2 v^2 \right) \] (4.5)

\[ \frac{A^2}{A^2} - \frac{A}{A} + \frac{1}{2} \left( \frac{B^2}{B^2} - \frac{B}{B} \right) - \left( \frac{B^2}{B^2} - \frac{B}{B} \right) + \left( \frac{C^2}{C^2} - \frac{C}{C} \right) - \left( \frac{C^2}{C^2} - \frac{C}{C} \right) = 4 \pi \kappa \left( \frac{v_i^2}{2} - m^2 v^2 \right) \] (4.6)

And overhead primes and dots stand for ordinary differentiation with respect to coordinate \( x \) and \( t \) respectively.

Using (4.3) - (4.7), we get
\[ 8 \pi \kappa V_4^2 = 0 \]
Thus for the space-time (3.1) the massive scalar field with or without mass parameter does not survive in Bimetric theory of relativity. In both cases source density becomes constant.

5. COUPLING OF PERFECT FLUID AND MASSIVE SCALAR FIELD

The energy momentum tensor for a mixture of macro and micro fields representing perfect fluid and scalar meson field together is given by

\[ T^j_i = T^j_i + T^j_{meson} \quad (5.1) \]

Where \( T^j_i \) and \( T^j_{meson} \) are already defined.

By the use of co-moving co-ordinate system, the field equation (2.1) to (2.4) for the metric (3.1) and (3.2) corresponding to the energy momentum tensor (5.1) can be written as

\[ p + \rho + 8\pi\kappa V^2 = 0 \quad (5.6) \]

In view of the reality conditions i.e. \( p > 0, \rho > 0, \) the above equation implies that

\[ p = 0, \rho = 0 \quad \text{and } V = \text{constant}. \]

6. CONCLUSION

In cylindrically symmetric space-time, there is nil contribution of perfect fluid and massive scalar field in bimetric theory of relativity respectively. It is observed that the matter fields like perfect fluid and massive scalar field cannot be a source of gravitational field in the Rosen’s bimetric theory but only vacuum model exists.

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