International Journal of Scientific and Innovative Mathematical Research (IJSIMR)
Volume 2, Issue 5, May 2014, PP 435-440
ISSN 2347-307X (Print) & ISSN 2347-3142 (Online)
www.arcjournals.org

Strong Efficient Domination Number of Inflated Graphs of Some Standard Graphs

N. Meena

Assistant Professor, Department of Mathematics,
The M.D.T. Hindu College, Tirunelveli-627 010.
Tamilnadu, India.
meenavidya72@gmail.com

Abstract: *Let G = (V,E) be a graph. A subset S of V(G) is called a strong (weak) efficient dominating set of G if for every $v \in V(G)$, $|N_s[v]\cap S| = 1$ ($|N_w[v]\cap S| = 1$), where $N_s(v) = \{u \in V(G): uv \in E(G), \deg u \geq \deg v\}$ and $N_w(v) = \{u \in V(G) : uv \in E(G), \deg v \geq \deg u\}$, $N_s[v] = N_s(v) \cup \{v\}$, ($N_w[v] = N_w(v) \cup \{v\}$). The minimum cardinality of a strong (weak) efficient dominating set is called the strong (weak) efficient domination number of G and is denoted by $\gamma_{se}(G)$ ($\gamma_{we}(G)$). Given a graph G with $\delta(G) \geq 1$, a graph denoted by G_I is obtained as follows. To each $u \in V(G)$, a clique A_u of order $\deg_G u$ is obtained and a bijection $\phi_u : N(u) \to A_u$ is constructed. $\phi_u(v)$ is denoted by v' for all $v \in N(u)$. $V(G_I) = \bigcup_{u\in V(G)} A_u$ and $E(G_I) = \cup E(A_u) \cup \{u'v' : uv \in E(G)$ and $v' \in A_u, u' \in A_v\}$. Then $|V(G_I)| = 2|E(G)|$ and $|E(G_I)| = \frac{1}{2}\bigcup_{u\in V}(\deg u)^2$. The graph G_I is known as the inflated graph of G. In this paper, strong efficient domination number of some standard graphs are obtained.*

Keywords: *Domination, Inflated graph, Strong efficient domination.*

1. Introduction

By a graph G = (V,E) we mean a finite, undirected graph without loops or multiple edges. The degree of any vertex $u \in V(G)$ is the number of edges incident with u and is denoted by deg u. The minimum and the maximum degree of a vertex is denoted by $\delta(G)$ and $\Delta(G)$ respectively. A vertex of degree 0 in G is called an isolated vertex and a vertex of degree 1 is called a pendant vertex. A subset S of V(G) of a graph G is called a dominating set if every vertex in V – S is adjacent to a vertex in S[6]. The domination number $\gamma(G)$ is the minimum cardinality of a dominating set of G. A set of vertices of G is said to be independent if no two of them are adjacent. The maximum number of vertices in any independent set of G is called the independence number of G and is denoted by $\beta_0(G)$. A subset S of V(G) is called an efficient dominating set of G if $|N[v] \cap S| = 1$for all vertices $v\in V(G)$ [1].E.Sampathkumar and L.Pushpalatha introduced the concepts of strong and weak domination in graphs [4]. A subset S of V(G) is called a strong dominating set of G if for every $v \in V – S$ there exists $u \in S$ such that u and v are adjacent and deg u ≥ deg v. The strong domination number $\gamma_s(G)$ is the minimum cardinality of a strong dominating set of G and the upper strong domination number $\Gamma_s(G)$ is the maximum cardinality of a strong dominating set of G. For all graph theoretic terminologies and notations, we follow Harary [5]. In this paper, strong efficient domination number of inflated graph of some standard graphs is obtained.

2. Strong Efficient Domination in Graphs

2.1 Definition[3] Let G = (V, E) be a simple graph. A subset S of V(G) is called a strong(weak) efficient dominating set of G if for every $v \in V(G)$, $|N_s[v]\cap S| = 1$ ($|N_w[v]\cap S| = 1$) where $N_s(v) = \{u \in V(G) : uv \in E(G), \deg u \geq \deg v\}$ and $N_s[v] = N_s(v)\cup\{v\}$($N_w(v) = \{u \in V(G): uv \in E(G), \deg v \geq \deg u\}$ and $N_w[v] = N_w(v) \cup \{v\}$).

2.2 Remark[3] The minimum cardinality of a strong(weak) efficient dominating set of G is called the strong(weak) efficient domination number of G and is denoted by $\gamma_{se}(G)$($\gamma_{we}(G)$). A graph G is strong efficient if there exists a strong efficient dominating set of G. A graph which is not strong efficient is called a non strong efficient graph.

2.3 Observations[3] 1. $\gamma_{se}(K_n) = 1, n \geq 1$.

2. $\gamma_{se}(K_{1,n}) = 1, n \geq 1$.

3. $\gamma_{se}(W_n) = 1, n \geq 4$.

2.4 Theorem[3] Every strong efficient dominating set is independent.

2.5 Theorem[3] For any path P_m,

$$\gamma_{se}(P_m) = \begin{cases} n \text{ if } m = 3n, n \in N \\ n+1 \text{ if } m = 3n+1, n \in N \\ n+2 \text{ if } m = 3n+2, n \in N \end{cases}$$

2.6 Theorem[3] $\gamma_{se}(C_{3n}) = n$ for all $n \in N$.

2.7 Theorem[3] A graph G does not admit a strong efficient dominating set if the distance between any two maximum degree vertices is exactly two.

2.8 Theorem[3] $K_{m,n}$, $m, n \geq 2$ is not strong efficient.

3. MAIN RESULTS

3.1 Definition[2] Given a graph G with $\delta(G) \geq 1$, a graph denoted by G_I is obtained as follows.

To each $u \in V(G)$, a clique A_u of order $\deg_G u$ is obtained and a bijection $\phi_u : N(u) \to A_u$ is constructed. $\phi_u(v)$ is denoted by v' for all $v \in N(u)$. $V(G_I) = \bigcup_{u \in V(G)} A_u$ and $E(G_I) = \cup E(A_u) \cup \{u'v' : uv \in E(G) \text{ and } v' \in A_u, u' \in A_v\}$. Then $|V(G_I)| = 2\,|E(G)|$ and $|E(G_I)| = \frac{1}{2}\bigcup_{u \in V}(\deg u)^2$. The graph G_I is known as the inflated graph of G.

3.2 Theorem Let $G = P_{3n}$. The inflated graph G_I of G is strong efficient and $\gamma_{se}(G_I) = 2\,\gamma_{se}(G)$.

Proof: Let $V(G) = \{v_1, v_2, \ldots, v_{3n}\}$. By theorem 1.5, $\gamma_{se}(G) = n$. Since $\deg v_i = 2, 2 \leq i \leq 3n-1$ and $\deg v_1 = \deg v_{3n} = 1$, there are $3n-2$ cliques A_{v_i}, $2 \leq i \leq 3n-1$, of order 2 and two cliques A_{v_1} and $A_{v_{3n}}$ of order 1 in G_I. The vertex v_i' in $A_{v_{i+1}}$ is adjacent with v_{i+1}' in A_{v_i}, $1 \leq i \leq 3n-1$. Therefore G_I is a path. By definition $|V(G_I)| = 2|E(G)| = 2(3n-1) = 3(2n-1)+1$. Since any path is strong efficient, G_I is strong efficient and $\gamma_{se}(G_I) = 2n = 2\,\gamma_{se}(G)$.

3.3 Theorem Let $G = P_{3n+1}$ or P_{3n+2}, $n \geq 2$. Then the inflated graph G_I of G is strong efficient and $\gamma_{se}(G_I) = 2(\gamma_{se}(G) - 1)$.

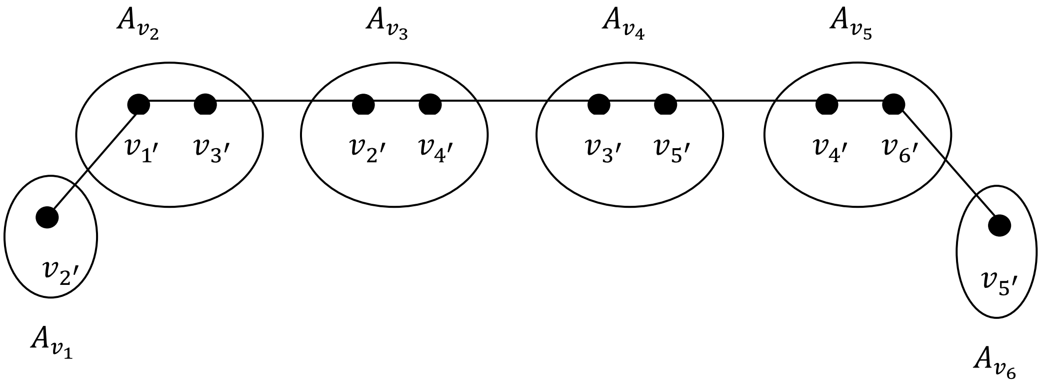
Proof: Case (1): Let $G = P_{3n+1}$, $n \geq 2$. Let $V(G) = \{v_1, v_2, \ldots, v_{3n+1}\}$. By theorem 1.5, $\gamma_{se}(G) = n + 1$. Since $\deg v_i = 2, 2 \leq i \leq 3n$ and $\deg v_1 = \deg v_{3n+1} = 1$, there are $3n-1$ cliques A_{v_i}, $2 \leq i \leq 3n$ of order 2 and two cliques A_{v_1} and $A_{v_{3n+1}}$ of order one in G_I. As in the Proof of Theorem 2.2, G_I is a path. $|V(G_I)| = 2\,|E(G)| = 2(3n) = 3(2n)$. Therefore $\gamma_{se}(G_I) = 2n = 2\,(\gamma_{se}(G) - 1)$.

Case (2): Let $G = P_{3n+2}$, $n \geq 1$. Let $V(P_{3n+2}) = \{v_1, v_2, \ldots, v_{3n+1}, v_{3n+2}\}$. By theorem 1.5, $\gamma_{se}(G) = n + 2$. Since $\deg v_i = 2, 2 \leq i \leq 3n+1$ and $\deg v_1 = \deg v_{3n+2} = 1$, there are $3n$ cliques A_{v_i}, $2 \leq i \leq 3n+1$, of order 2 and two cliques A_{v_1} and $A_{v_{3n+2}}$ of order one in G_I. As in the Proof of Theorem 2.2, G_I is a path. $|V(G_I)| = 2\,|E(G)| = 2(3n+1) = 3(2n)+2$. Therefore $\gamma_{se}(G_I) = 2n + 2 = 2(n+1) = 2\,(\gamma_{se}(G) - 1)$.

3.4 Remark When $G = P_2$, inflated graph G_I is P_2, and hence $\gamma_{se}(G_I) = \gamma_{se}(G)$. When $G = P_4$, G_I is P_6 and hence $\gamma_{se}(G_I) = \gamma_{se}(G)$.

3.5 Illustration Inflated graph G_I of $G = P_6$ is given in the following figure

G_I is a path P_{10}. $\gamma_{se}(G_I) = 4 = 2\gamma_{se}(G)$.



Figure

3.6 Theorem Let G = C_{3n}, n ≥ 1. Then G_I is strong efficient and $\gamma_{se}(G_I) = 2\gamma_{se}(G)$.

Proof: Let V(G) = {$v_1, v_2, \ldots, v_{3n}$}. Since deg v_i = 2, there are 3n cliques A_{v_i}, 1 ≤ i ≤ 3n, of order 2. v_{i+1}' in A_{v_i} is adjacent with v_i in $A_{v_{i+1}}$, 1 ≤ i ≤ 3n – 1. v_n' in A_{v_1} is adjacent with v_1' in A_{v_n}. $|V(G_I)| = 2\,|E(G)| = 6n$. Thus G_I is a cycle C_{6n}. By theorem 1.6, G_I is strong efficient and $\gamma_{se}(G_I)$ = 2n=$2\gamma_{se}(G)$. G_I is a cycle C_6. $\gamma_{se}(G_I) = 2\,(\gamma_{se}(G))$.

3.7 Theorem Let G be a complete graph K_n, n ≥ 2. Inflated graph G_I of K_n is strong efficient and $\gamma_{se}(G_I)$ = n – 1.

Proof: Let G = K_n. Let V(G) = {$v_1, v_2 \ldots v_n$} and $|E(G)| = \frac{n(n-1)}{2}$. Since deg v = n – 1 for all v ∈ V(G), there are cliques A_{v_i}, 1 ≤ i ≤ n of order n – 1, in G_I. $V(A_{v_i})$ = {v_j' / 1 ≤ j ≤ n and j ≠ i}. Therefore v_i' does not belong to A_{v_i} for all i = 1 to n. Each v_{i+j}' in A_{v_i} is adjacent with v_i' in $A_{v_{i+j}}$, 1 ≤ i ≤ n – j, where 1 ≤ j ≤ n – 1. Therefore each v_j' in A_{v_i}, 1 ≤ j ≤ n, j ≠ i is adjacent with all the n – 2 vertices of A_{v_i} and v_i' in A_{v_j} and hence deg v′ = n – 1 for all v′ ∈ $V(G_I)$. Define S_j = {v_j' / v_j' ∈ A_{v_i},1 ≤ i ≤ n}, j ≠ i, 1 ≤ j ≤ n. Then every vertex of G_I is uniquely, strongly dominated by S_j and hence S_j, 1 ≤ j ≤ n is a γ_{se} - set of G_I. Therefore G_I is strong efficient and $\gamma_{se}(G_I)$ = n – 1.

3.8 Theorem Let G = $K_{m,n}$, m, n ≥ 2. The inflated graph G_I of G is strong efficient if and only if m ≠ n. Further $\gamma_{se}(G_I)$ = m + n – 1.

Proof: Let G = $K_{m,n}$, m, n ≥ 2 and m = n. Suppose that the inflated graph G_I of G is strong efficient. Let V(G) = (X(G), Y(G)), where X(G) = {$v_1, v_2, \ldots, v_n$} and Y(G) = {$u_1, u_2, \ldots, u_n$}. deg v_i = deg u_j = n for all i,j = 1 to n. There are cliques A_{v_i} and A_{u_j}, 1 ≤ i, j ≤ n of order n in G_I. For i = 1 to n, $V(A_{v_i})$ = {u_j' / 1 ≤ j ≤ n} and for j = 1 to n, $V(A_{u_j})$ = {v_i' / 1 ≤ i ≤ n}. For j = 1 to n, each u_j' in A_{v_i} is adjacent with v_i' in A_{u_j}, 1 ≤ i ≤ n. deg v_i' = deg u_j' = n for all i,j = 1 to n. Let S be a strong efficient dominating set of G_I. Suppose a vertex u_j', 1 ≤ j ≤ n from A_{v_1} belongs to S. u_j' strongly dominates all the vertices of A_{v_1} and v_1' in A_{u_j}. The vertices v_i' in A_{u_j}, i ≠ 1 do not belong to S, since otherwise v_1' in A_{u_j} is strongly dominated by two vertices of S. To dominate the vertices of v_i' in A_{u_j}, i ≠ 1, the vertices u_j' in A_{v_i}, 2 ≤ i ≤ n, belong to S. Therefore vertices of A_{v_i}, 1 ≤ i ≤ n and A_{u_j} are strongly, efficiently dominated by S. If any vertex v_i', 1 ≤ i ≤ n in A_{u_k}, k ≠ j, belongs to S, then u_k' in A_{v_i} is strongly dominated by two vertices v_i' and u_j' of S.

Therefore no vertex v_i' of A_{u_k}, k ≠ j , 1 ≤ k ≤ n belongs to S. Therefore the vertices of A_{u_k}, k ≠ j, 1 ≤ k ≤ n are not dominated by S, a contradiction. Hence G_I is not strong efficient if m = n.

Conversely, let m ≠ n. Without loss of generality let m < n.

Let X(G) = {$v_1, v_2, \ldots, v_m$} and Y(G) = {$u_1, u_2, \ldots, u_n$}. Since deg v_i = n, 1 ≤ i ≤ m and deg u_j = m, 1 ≤ j ≤ n, there are cliques A_{v_i}, 1 ≤ i ≤ m, of order n and A_{u_j}, 1 ≤ j ≤ n of order m. For 1 ≤ i ≤ m, $V(A_{v_i})$ = {u_j' / 1 ≤ j ≤ n} and for 1 ≤ j ≤ n, $V(A_{u_j})$ = {v_i' / 1 ≤ i ≤ m}. deg u_j' = n, 1 ≤ j ≤ n and deg v_i' = m, $1 \le i \le m$. Since m < n, deg v_i' < deg u_j' for any i and j. Define $S_j = \{u_j' / u_j' \in A_{v_i}, 1 \le i \le m\}$, $1 \le j \le n$. u_j' in each A_{v_i}, $1 \le i \le m$, strongly dominates all the other vertices of A_{v_i} and v_i' in A_{u_j}. Therefore the vertices in A_{v_i}, $1 \le i \le m$ and A_{u_j} are strongly, uniquely dominated by S_j. To dominate the vertices in the remaining A_{u_t}, $t \ne j$, one vertex v_k', $1 \le k \le m$ from each A_{u_t}, $t \ne j$ is chosen and it forms a new set T_k. v_k' from each A_{u_t} strongly dominates the vertices of A_{u_t}, $t \ne j$. Since deg v_k' < deg u_j', v_k' does not strongly dominate u_j'. Therefore $S_j \cup T_k$ is a γ_{se} - set of G_I. This is true for any u_j' belongs to S_j, $1 \le j \le n$. Hence G_I is strong efficient. Also $S_j \cup T_k$ contains one vertex from each A_{v_i}, $1 \le i \le m$ and A_{u_t}, $1 \le t \le n$, $t \ne i$. Hence $|S_j \cup T_k| = m + n - 1$. Therefore $\gamma_{se}(G_I) = m + n - 1$.

3.9 Theorem Let G be a Bistar $D_{r,s}$, r, s ≥ 1. The inflated graph G_I of G is strong efficient and $\gamma_{se}(G_I) = r + s$.

Proof: Let u and v be the central vertices of G. Let $u_1, u_2 \ldots u_r$ and $v_1, v_2 \ldots v_s$ be the vertices adjacent with u and v respectively.

Case (1): Let r ≠ s. Without loss of generality, let r < s. deg u = r + 1 and deg v = s + 1 = Δ(G). Let G_I be the inflated graph of G. There are cliques A_u of order r + 1, A_v of order s +1, A_{u_i}, $1 \le i \le r$ of order 1 and A_{v_j}, $1 \le j \le s$ of order 1 in G_I. $V(A_u) = \{v', u_1', u_2', \ldots, u_r'\}$. $V(A_v) = \{u', v_1', v_2', \ldots, v_s'\}$ and $V(A_{u_i}) = \{u'\}$, $1 \le i \le r$, $V(A_{v_j}) = \{v'\}$, $1 \le j \le s$. v′ in A_u is adjacent with u′ in A_v. u_i' in A_u is adjacent with u′ in A_{u_i}, $1 \le i \le r$ and v_j' in A_v is adjacent with v′ in A_{v_j}, $1 \le j \le s$. The degrees of all vertices in A_v are s+1 and the degrees of all vertices in A_u are r+1. The degree of u′ in A_{u_i}, $1 \le i \le r$ and the degree of v′ in A_{v_j}, $1 \le j \le s$ are 1. Hence the vertices in A_v are maximum degree vertices. Since A_v is a clique, any strong efficient dominating set contains exactly one vertex of A_v. If u′ in A_v is chosen to form a strong efficient dominating set S, then it strongly dominates all the vertices of A_v and v′ in A_u. To dominate u_i' in A_u, $1 \le i \le r$, one among them must be chosen. If any u_i' is chosen, then v′ in A_u is strongly dominated by two vertices u_i' in A_u and u′ in A_v, a contradiction. Therefore u′ in A_v cannot be chosen to form S. The unique way to form a strong efficient dominating set S, any vertex v_j' in A_v, $1 \le j \le s$ is chosen. It strongly dominates all the vertices of A_v and v′ in A_{v_j}. $\{v' / v' \in A_{v_k}, 1 \le k \le s, k \ne j\}$ is independent and not dominated by v_j'. These vertices belong to S. To strongly dominate the vertices of A_u, consider the following sub cases.

Sub case (1a): Suppose v′ in A_u is chosen. It strongly dominates all u_i' in A_u. Since $\{u' / u' \in A_{u_i}, 1 \le i \le r\}$ is independent, all u′ in A_{u_i} belong to any strong efficient dominating set S. For $1 \le j \le s$, define $S_j = \{v_j'\} \cup \{v' / v' \in A_{v_k}, 1 \le k \le s, k \ne j\} \cup \{v'\} \cup \{u' / u' \in A_{u_i}, 1 \le i \le r\}$ where $v_j' \in V(A_v)$ and $v' \in V(A_u)$. Clearly all the vertices of G_I are uniquely strongly dominated by S_j and so S_j is a strong efficient dominating set of G_I. Further, $|S_j| = 1 + s - 1 + 1 + r = r + s + 1$.

Sub case (1b): Suppose any one of u_i' in A_u, $1 \le i \le r$ is chosen. It strongly dominates the vertices of A_u and u′ in A_{u_i}. Therefore $\{u' / u' \in A_{u_t}, 1 \le t \le r, t \ne i\}$ is independent and is not dominated by u_i'. Thus, $1 \le i \le r$, $T_i = \{v_j'\} \cup \{v' / v' \in A_{v_k}, 1 \le k \le s, k \ne j\} \cup \{u_i'\} \cup \{u' / u' \in A_{u_t}, 1 \le t \le r, t \ne i\}$ where $v_j' \in A_v$ and $u_i' \in A_u$, is a strong efficient dominating set of G_I. Further, $|T_i| = 1 + s - 1 + 1 + r - 1 = r + s$. Therefore $\gamma_{se}(G_I) = r + s$ and $\Gamma_{se}(G_I) = r + s + 1$.

Case (2): Let s = r. deg u′ = deg v′ = deg u_i' = deg v_i' = r + 1 = $\Delta(G_I)$, $1 \le i \le r$. If u′ in A_v is chosen to form a strong efficient dominating set of G_I, then no vertex of A_u can be chosen. Otherwise, v′ in A_u is strongly dominated by u_i' in A_u and u′ in A_v. The argument is similar if v′ in A_u is chosen. The unique way is to form a strong efficient dominating set of G_I is, any one of v_i' in A_v, $1 \le i \le r$, or any one of u_i' in A_u is to be chosen. Suppose one of v_i' in A_v is chosen. It strongly dominates the vertices of A_v and v' in A_{v_i}. To dominate the vertices of A_u, any one of u_i' in A_u, $1 \le i \le r$ can be chosen. Therefore $\{u'/ u' \in A_{u_k}, 1 \le k \le r, k \ne i\}$, $\{v' / v' \in A_{v_k}, 1 \le k \le r, k \ne i\}$ are independent and not dominated by v_i' in A_v. Therefore for $1 \le i \le r$, $S_i = \{v_i'\} \cup \{v' / v' \in A_{v_k}, 1 \le$

$k \le r, k \ne i\} \cup \{u_i'\} \cup \{u' / u' \in A_{u_k}, 1 \le k \le r, k \ne i\}$ is a γ_{se} - set of G_I. Further $|S_i| = 1 + r - 1 + 1 + r - 1 = 2r$. Hence $\gamma_{se}(G_I) = 2r$.

3.10 Theorem Let $G = W_n$, $n \ge 4$. Let G_I be the inflated graph of G. Then G_I is strong efficient and $\gamma_{se}(G_I) = n - 1$.

Proof: Let $V(W_n) = \{v, v_1, v_2, \ldots, v_{n-1}\}$, $E(W_n) = \{e_i / e_i = vv_i, 1 \le i \le n-1\} \cup \{v_iv_{i+1} / 1 \le i \le n-1\} \cup \{v_{n-1}v_1\}$. Since deg $v = n - 1$ and deg $v_i = 3$, $1 \le i \le n - 1$, there is a clique A_v of order $n - 1$ and cliques A_{v_i}, $1 \le i \le n - 1$ of order 3 in G_I. $V(A_v) = \{v_i' / 1 \le i \le n - 1\}$. For $2 \le i \le n - 2$, $V(A_{v_i}) = \{v', v_{i-1}', v_{i+1}'\}$, $V(A_{v_1}) = \{v', v_2', v_{n-1}'\}$ and $V(A_{v_{n-1}}) = \{v', v_{n-2}', v_1'\}$. For $1 \le i \le n - 1$, v_i' in A_v is adjacent with v' in A_{v_i}. v_{i+1}' in A_{v_i} is adjacent with v_i' in $A_{v_{i+1}}$, $1 \le i \le n - 2$. v_{n-1}' in A_{v_1} is adjacent with v_1' in $A_{v_{n-1}}$. The degree of each vertex in A_v is $n - 1$ and degree of each vertex in A_{v_i}, $1 \le i \le n - 1$ is 3. Therefore each vertex in A_v is a maximum degree vertex. Consider $S_1 = \{v_1' / v_1'$ in A_v, A_{v_2} and $A_{v_{n-1}}\} \cup \{v' / v' \in A_{v_i}, 3 \le i \le n - 2\}$. The vertices in S_1 strongly uniquely dominate all the vertices of G_I. Clearly S_1 is a γ_{se} - set of G_I and $|S_1| = 3 + n - 4 = n - 1$. Proof is similar if v_{n-1}' in A_v is considered. Suppose any v_i', $2 \le i \le n - 2$ in A_v is chosen. Let $S_i = \{v_i' / v_i' \in A_v, A_{v_{i-1}}, A_{v_{i+1}}\} \cup \{v' / v' \in A_{v_j}, 1 \le j \le n - 1, j \ne i - 1, i, i + 1\}$. The vertices in S_i strongly, uniquely dominate the vertices in G_I. Therefore S_i, $2 \le i \le n - 2$, is a γ_{se} - set of G_I. Therefore G_I is strong efficient.

3.11 Theorem Inflated graph of strong efficient graph need not be strong efficient.

3.12 Example Let $G = S(K_{1,n})$. Let u be the central vertex of $K_{1,n}$. Let $v_1, v_2, \ldots, v_n$ be the vertices adjacent with u. Each edge is subdivided into two edges uu_i and u_iv_i, i = 1 to n. That is $u_1, u_2, \ldots, u_n$ are the new vertices which are adjacent with u. Therefore deg $u = n = \Delta(G)$, deg $u_i = 2$ and deg $v_i = 1$ for all i = 1 to n. Let $S \subseteq V(G)$. If there exists exactly one maximum degree vertex, then any strong efficient dominating set must contain it. Clearly u belongs to S. u strongly dominates $u_1, u_2, \ldots, u_n$ and u does not dominate $v_1, v_2, \ldots, v_n$. Therefore $v_1, v_2, \ldots, v_n$ belong to S. Hence $S = \{u, v_1, v_2, \ldots, v_n\}$ and $|S| = n + 1$. $N_s[u] \cap S = \{u\} = N_s[u_i] \cap S$ for all i = 1 to n. $N_s[v_i] \cap S = \{v_i\}$ for all i = 1 to n. Clearly S is the γ_{se} - set of G. Thus $S(K_{1,n})$ is strong efficient and $\gamma_{se}(S(K_{1,n})) = n + 1$ for all $n \in N$.

Let G_I be the inflated graph of G. Since deg u = n, deg u_i = 2, deg v_i = 1 in G, there are 1clique of order n, n cliques of order 2 and n cliques of order 1in G_I. $V(A_u) = \{u_i' / 1 \le i \le n\}$, $V(A_{u_i}) = \{u', v_i' / 1 \le i \le n\}$ and $V(A_{v_i}) = \{u_i' / 1 \le i \le n\}$. u' in A_{u_i} is adjacent with u_i' in A_u and u_i' in A_{v_i} is adjacent with v_i' in A_{u_i}, $1 \le i \le n$. The vertices in A_u are of maximum degree vertices. Suppose G_I is strong efficient. Let S be a strong efficient dominating set of G_I. One of the vertices of A_u say u_i' must belong to S. u_i' strongly efficiently dominate all the other vertices in the clique A_u and u' in A_{u_i}. To strongly efficiently dominate u_i' in A_{v_i}, the vertex v_i' in A_{u_i} must belong to S. But u' in A_{u_i} is strongly dominated by two vertices u_i' in A_u and v_i' in A_{u_i}, a contradiction. Therefore G_I is not strong efficient.

4. Conclusion

In this paper, the strong efficient domination number of inflated graphs of some standard graphs are found. In future, the characterization of strong efficient domination number and the relationship among strong efficient domination number and other graph theoretic parameters like strong independent domination number, strong domination number will be studied.

Acknowledgement

The author is thankful to the referee for the valuable comments and suggestions.

References

[1] Bange. D.W, Barkauskas. A.E. and Slater. P.J., Efficient dominating sets in graphs, Application of Discrete Mathematics, 189 – 199, SIAM, Philadephia (1988).

[2] Dunbar. J.E., Haynes. T.W, Domination in Inflated Graphs, Congr. Numer, 118, 143 154, (1996).

[3] Meena.N., Subramanian.A.S., Swaminathan.V., Strong Efficient Domination in Graphs, Communicated.

[4] Sampathkumar.E and Pushpa Latha.L. Strong weak domination and domination balance in a graph, Discrete Math., 161: 235 – 242 (1996).

[5] Harary. F., Graph Theory, Addison – Wesley (1969).

[6] Haynes. T W., Stephen T. Hedetniemi, Peter J. Slater. Fundamentals of domination in graphs. Advanced Topics, Marcel Dekker, Inc, New York (1998).