Strong Efficient Domination Number of Inflated Graphs of Some Standard Graphs

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Abstract: Let $G = (V,E)$ be a graph. A subset $S$ of $V(G)$ is called a strong (weak) efficient dominating set of $G$ if for every $v \in V(G)$, $|N_s[v] \cap S| = 1$ ($|N_w[v] \cap S| = 1$), where $N_s(v) = \{u \in V(G): uv \in E(G), \deg u \geq \deg v\}$ and $N_s(v) = \{u \in V(G): uv \in E(G), \deg v \geq \deg u\}$. The minimum cardinality of a strong (weak) efficient dominating set is called the strong (weak) efficient domination number of $G$ and is denoted by $\gamma_{se}(G)$ ($\gamma_{we}(G)$). A graph $G$ with $\delta(G) \geq 1$, a graph denoted by $G_I$ is obtained as follows. To each $u \in V(G)$, a clique $A_u$ of order $\deg G_u$ is obtained and a bijection $\phi_u: N(u) \rightarrow A_u$ is constructed. $\phi_u(v)$ is denoted by $v'$ for all $v \in N(u)$. $V(G_I) = V(G)$ and $E(G_I) = E(A_u) \cup \{u'v': uv \in E(G) and v' \in A_u, u' \in A_v\}$. Then $|V(G_I)| = 2|E(G)|$ and $|E(G_I)| = \frac{1}{2} \sum_{u \in V(G)} (\deg u)^2$. The graph $G_I$ is known as the inflated graph of $G$. In this paper, strong efficient domination number of some standard graphs are obtained.

Keywords: Domination, Inflated graph, Strong efficient domination.

1. INTRODUCTION

By a graph $G = (V,E)$ we mean a finite, undirected graph without loops or multiple edges. The degree of any vertex $u \in V(G)$ is the number of edges incident with $u$ and is denoted by $\deg u$. The minimum and the maximum degree of a vertex is denoted by $\delta(G)$ and $\Delta(G)$ respectively. A vertex of degree 0 in $G$ is called an isolated vertex and a vertex of degree 1 is called a pendant vertex. A subset $S$ of $V(G)$ of a graph $G$ is called a dominating set if every vertex in $V \setminus S$ is adjacent to a vertex in $S$. The domination number $\gamma(G)$ is the minimum cardinality of a dominating set of $G$. A set of vertices of $G$ is said to be independent if no two of them are adjacent. The maximum number of vertices in any independent set of $G$ is called the independence number of $G$ and is denoted by $\alpha(G)$. A subset $S$ of $V(G)$ is called an efficient dominating set of $G$ if for all $v \in V(G)$, $|N[v] \cap S| = 1$. E.Sampathkumar and L.Pushpalatha introduced the concepts of strong and weak domination in graphs [4]. A subset $S$ of $V(G)$ is called a strong dominating set of $G$ if for every $v \in V \setminus S$ there exists $u \in S$ such that $u$ and $v$ are adjacent and $\deg u \geq \deg v$. The strong domination number $\gamma_s(G)$ is the minimum cardinality of a strong dominating set of $G$ and the upper strong domination number $\Gamma_s(G)$ is the maximum cardinality of a strong dominating set of $G$. For all graph theoretic terminologies and notations, we follow Harary [5]. In this paper, strong efficient domination number of inflated graph of some standard graphs is obtained.

2. STRONG EFFICIENT DOMINATION IN GRAPHS

2.1 Definition[3] Let $G = (V,E)$ be a simple graph. A subset $S$ of $V(G)$ is called a strong(weak) efficient dominating set of $G$ if for every $v \in V(G)$, $|N_s[v] \cap S| = 1$ ($|N_w[v] \cap S| = 1$) where $N_s(v) = \{u \in V(G): uv \in E(G), \deg u \geq \deg v\}$ and $N_s(v) = \{u \in V(G): uv \in E(G), \deg v \geq \deg u\}$. The minimum cardinality of a strong(weak) efficient dominating set of $G$ is called the strong(weak) efficient domination number of $G$ and is denoted by $\gamma_{se}(G)$ ($\gamma_{we}(G)$).
graph \( G \) is strong efficient if there exists a strong efficient dominating set of \( G \). A graph which is not strong efficient is called a non strong efficient graph.

2.3 Observations[3]  1. \( \gamma_{se}(K_n) = 1, n \geq 1 \).
2. \( \gamma_{se}(K_{1,n}) = 1, n \geq 1 \).
3. \( \gamma_{se}(W_n) = 1, n \geq 4 \).

2.4 Theorem[3] Every strong efficient dominating set is independent.

2.5 Theorem[3] For any path \( P_m \),
\[
\gamma_{se}(P_m) = \begin{cases} 
n & \text{if } m = 3n, n \in \mathbb{N} 
n + 1 & \text{if } m = 3n + 1, n \in \mathbb{N} 
n + 2 & \text{if } m = 3n + 2, n \in \mathbb{N}
\end{cases}
\]

2.6 Theorem[3] \( \gamma_{se}(C_{3n}) = n \) for all \( n \in \mathbb{N} \).

2.7 Theorem[3] A graph \( G \) does not admit a strong efficient dominating set if the distance between any two maximum degree vertices is exactly two.

2.8 Theorem[3] \( K_{m,n}, m, n \geq 2 \) is not strong efficient.

3. MAIN RESULTS

3.1 Definition[2] Given a graph \( G \) with \( \delta(G) \geq 1 \), a graph denoted by \( G_i \) is obtained as follows.

To each \( u \in V(G) \), a clique \( A_u \) of order \( deg_u \) is obtained and a bijection \( \phi_u : N(u) \rightarrow A_u \) is constructed. \( \phi_u(v) \) is denoted by \( v' \) for all \( v \in N(u) \). \( V(G_i) = \bigcup_{u \in V(G)} A_u \) and \( E(G_i) = \bigcup_{u \in V(G)} E(A_u) \cup \{ u'v' : uv \in E(G) \text{ and } v' \in A_u \} \). Then \( |V(G_i)| = 2|E(G)| \) and \( |E(G_i)| = \frac{1}{2} \sum_{u \in V(G)} (deg_u)^2 \). The graph \( G_i \) is known as the inflated graph of \( G \).

3.2 Theorem Let \( G = P_{3n} \). The inflated graph \( G_i \) of \( G \) is strong efficient and \( \gamma_{se}(G_i) = 2 \gamma_{se}(G) \).

Proof: Let \( V(G) = \{v_1, v_2, \ldots, v_{3n}\} \). By theorem 1.5, \( \gamma_{se}(G) = n \). Since \( deg v_i = 2, 2 \leq i \leq 3n - 1 \) and \( deg v_i = deg v_{3n} = 1 \), there are \( 3n - 2 \) cliques \( A_{v_1}, 2 \leq i \leq 3n - 1 \), of order 2 and two cliques \( A_{v_1} \) and \( A_{v_{3n}} \) of order 1 in \( G_i \). The vertex \( v_i' \) in \( A_{v_{3n+1}} \) is adjacent with \( v_{i+1}' \) in \( A_{v_1} \). Therefore \( G_i \) is a path. By definition \( |V(G_i)| = 2|E(G)| = 2(3n - 1) = 3(2n - 1) + 1 \). Since any path is strong efficient, \( G_i \) is strong efficient and \( \gamma_{se}(G_i) = 2n = 2 \gamma_{se}(G) \).

3.3 Theorem Let \( G = P_{3n+1} \) or \( P_{3n+2}, n \geq 2 \). Then the inflated graph \( G_i \) of \( G \) is strong efficient and \( \gamma_{se}(G_i) = 2(\gamma_{se}(G) - 1) \).

Proof: Case (1): Let \( G = P_{3n+1}, n \geq 2 \). Let \( V(G) = \{v_1, v_2, \ldots, v_{3n+1}\} \). By theorem 1.5, \( \gamma_{se}(G) = n + 1 \). Since \( deg v_i = 2, 2 \leq i \leq 3n \) and \( deg v_i = deg v_{3n+1} = 1 \), there are \( 3n - 1 \) cliques \( A_{v_1}, 2 \leq i \leq 3n \) of order 2 and two cliques \( A_{v_1} \) and \( A_{v_{3n}} \) of order one in \( G_i \). As in the Proof of Theorem 2.2, \( G_i \) is a path. \( |V(G_i)| = 2|E(G)| = 2(3n) = 3(2n) \). Therefore \( \gamma_{se}(G_i) = 2n = 2(\gamma_{se}(G) - 1) \).

Case (2): Let \( G = P_{3n+2}, n \geq 1 \). Let \( V(G_{3n+2}) = \{v_1, v_2, \ldots, v_{3n+1}, v_{3n+2}\} \). By theorem 1.5, \( \gamma_{se}(G) = n + 2 \). Since \( deg v_i = 2, 2 \leq i \leq 3n + 1 \) and \( deg v_i = deg v_{3n+2} = 1 \), there are \( 3n \) cliques \( A_{v_1}, 2 \leq i \leq 3n + 1 \), of order 2 and two cliques \( A_{v_1} \) and \( A_{v_{3n+2}} \) of order one in \( G_i \). As in the Proof of Theorem 2.2, \( G_i \) is a path. \( |V(G_i)| = 2|E(G)| = 2(3n + 1) = 3(2n) + 2 \). Therefore \( \gamma_{se}(G_i) = 2n + 2 = 2(n + 1) = 2(\gamma_{se}(G) - 1) \).

3.4 Remark When \( G = P_2 \), inflated graph \( G_i \) is \( P_2 \), and hence \( \gamma_{se}(G_i) = \gamma_{se}(G) \). When \( G = P_n \), \( G_i \) is \( P_6 \) and hence \( \gamma_{se}(G_i) = \gamma_{se}(G) \).

3.5 Illustration Inflated graph \( G_i \) of \( G = P_6 \) is given in the following figure

\( G_i \) is a path \( P_{10} \). \( \gamma_{se}(G_i) = 4 = 2\gamma_{se}(G) \).
3.6 Theorem Let $G = C_{3n}$, $n \geq 1$. Then $G_1$ is strong efficient and $\gamma_{se}(G_1) = 2\gamma_{se}(G)$.

Proof: Let $V(G) = \{v_1, v_2, \ldots, v_n\}$. Since $\deg v_i = 2$, there are $3n$ cliques $A_{v_j}, 1 \leq i \leq n$, of order 2. $v_{i+j}$ in $A_{v_j}$ is adjacent with $v_i$ in $A_{v_{i+j}}, 1 \leq i \leq 3n - 1$. $v'_n$ in $A_{v_1}$ is adjacent with $v'_1$ in $A_{v_n}$.

$|V(G)| = 2|E(G)| = 6n$. Thus $G_1$ is a cycle $C_{6n}$. By theorem 1.6, $G_1$ is strong efficient and $\gamma_{se}(G_1) = 2n=2\gamma_{se}(G)$. $G_1$ is a cycle $C_{6n}$. Further $\gamma_{se}(G_1) = 2 \gamma_{se}(G)$.

3.7 Theorem Let $G$ be a complete graph $K_n$, $n \geq 2$. Inflated graph $G_1$ of $K_n$ is strong efficient and $\gamma_{se}(G_1) = n - 1$.

Proof: Let $G = K_n$. Let $V(G) = \{v_1, v_2, \ldots, v_n\}$ and $|E(G)| = \frac{n(n-1)}{2}$. Since $\deg v = n - 1$ for all $v \in V(G)$, there are cliques $A_{v_j}, 1 \leq i \leq n$ of order $n - 1$, in $G_1$, $V(A_{v_j}) = \{v'_j / 1 \leq j \leq n \text{ and } j \neq i\}$. Therefore $v'_j$ does not belong to $A_{v_i}$ for all $i = 1$ to $n$. Each $v_{i+j}$ in $A_{v_j}$ is adjacent with $v_i$ in $A_{v_{i+j}}, 1 \leq i \leq n - j$, where $1 \leq j \leq n - 1$. Therefore each $v'_j$ in $A_{v_j}, 1 \leq j \leq n, j \neq i$ is adjacent with all the $n - 2$ vertices of $A_{v_i}$ and $v'_j$ in $A_{v_j}$ and hence $\deg v' = n - 1$ for all $v' \in V(G_1)$. Define $S_j = \{v'_j / v'_j \in A_{v_j}, 1 \leq i \leq n\}, j \neq i, 1 \leq j \leq n$. Then every vertex of $G_1$ is uniquely, strongly dominated by $S_j$ and hence $S_n, 1 \leq j \leq n$ is a $\gamma_{se} -$ set of $G$. Therefore $G_1$ is strong efficient and $\gamma_{se}(G_1) = n - 1$.

3.8 Theorem Let $G = K_{m,n}$, $m, n \geq 2$. The inflated graph $G_1$ of $G$ is strong efficient if and only if $m \neq n$. Further $\gamma_{se}(G_1) = m + n - 1$.

Proof: Let $G = K_{m,n}$. Let $V(G) = \{v_1, v_2, \ldots, v_n\}$ and $|E(G)| = \frac{n(n-1)}{2}$. Suppose that the inflated graph $G_1$ of $G$ is strong efficient. Let $V(G) = \{X(G), Y(G)\}$, where $X(G) = \{v_1, v_2, \ldots, v_n\}$ and $Y(G) = \{u_1, u_2, \ldots, u_m\}$. $\deg v_i = \deg u_j = n$ for all $i, j = 1$ to $n$. There are cliques $A_{v_j}$ and $A_{u_j}, 1 \leq i, j \leq n$ of order $n$ in $G$. For $i = 1$ to $n$, $V(A_{v_j}) = \{u'_j / 1 \leq j \leq n\}$ and for $j = 1$ to $n$, $V(A_{u_j}) = \{v'_j / 1 \leq i \leq n\}$. For $j = 1$ to $n$, each $u'_j$ in $A_{v_j}$ is adjacent with $v'_j$ in $A_{u_j}, 1 \leq i \leq n$. $\deg v'_j = \deg u'_j = n$ for all $i, j = 1$ to $n$. Let $S$ be a strong efficient dominating set of $G_1$. Suppose a vertex $u'_j, 1 \leq j \leq n$ from $A_{v_j}$ belongs to $S$. $u'_j$ strongly dominates all the vertices of $A_{v_i}$ and $v'_j$ in $A_{u_j}$. The vertices $v'_j$ in $A_{u_j}, i \neq j$ do not belong to $S$, since otherwise $v'_j$ in $A_{u_j}$ is strongly dominated by two vertices of $S$. To dominate the vertices of $v'_j$ in $A_{u_j}, i \neq j$, the vertices $u'_j$ in $A_{v_i}, 2 \leq i \leq n$, belong to $S$. Therefore vertices of $A_{u_k}, 1 \leq i \leq n$ and $A_{v_k}$ are strongly, efficiently dominated by $S$. If any vertex $v'_j, 1 \leq i \leq n$ in $A_{u_k}, k \neq j$, belongs to $S$, then $u'_k$ in $A_{v_i}$ is strongly dominated by two vertices $v'_j$ and $u'_j$ of $S$.

Therefore no vertex $v'_j$ of $A_{u_k}, k \neq j$, $1 \leq k \leq n$ belongs to $S$. Therefore the vertices of $A_{u_k}, k \neq j$, $1 \leq k \leq n$ are not dominated by $S$, a contradiction. Hence $G_1$ is not strong efficient if $m = n$.

Conversely, let $m \neq n$. Without loss of generality let $m < n$.

Let $X(G) = \{v_1, v_2, \ldots, v_m\}$ and $Y(G) = \{u_1, u_2, \ldots, u_n\}$. Since $\deg v_i = n, 1 \leq i \leq m$ and $\deg u_j = n, 1 \leq j \leq n$, there are cliques $A_{v_i}, 1 \leq i \leq m$, of order $n$ and $A_{u_j}, 1 \leq j \leq n$ of order $m$. For $1 \leq i \leq m$, $V(A_{v_i}) = \{u'_j / 1 \leq j \leq n\}$ and for $1 \leq j \leq n$, $V(A_{u_j}) = \{v'_j / 1 \leq i \leq m\}$. $\deg u'_j = n, 1 \leq j \leq n$ and...
deg \( v' = m \), \( 1 \leq i \leq m \). Since \( m < n \), deg \( v' < \) deg \( u' \) for any \( i \) and \( j \). Define \( S_I = \{ u' / u' \in A_{vi} \}, 1 \leq i \leq m \}, \ 1 \leq j \leq \). Define \( A_{vi} \), \( 1 \leq i \leq m \), strong dominates all the other vertices of \( A_{vi} \) and \( v' \) in \( A_{uj} \). Therefore the vertices in \( A_{vi} \), \( 1 \leq i \leq m \) and \( A_{uj} \) are strongly, uniquely dominated by \( S_i \). To dominate the vertices in the remaining \( A_{uj} \), \( t \neq j \), one vertex \( v' \), \( 1 \leq k \leq m \) from each \( A_{uj} \), \( t \neq j \) is chosen and it forms a new set \( T_k \). Therefore \( S_j \cup T_k \) is a \( \gamma_{se} \) set of \( G_i \). This is true for any \( u' \) belongs to \( A_{ui} \), \( 1 \leq j \leq n \). Hence \( G_i \) is strong efficient. Also \( S_i \cup T_k \) contains one vertex from each \( A_{vi} \), \( 1 \leq i \leq m \) and \( A_{uj} \), \( 1 \leq t \leq n \), \( t \neq i \). Hence \( |S_i \cup T_k| = m + n - 1 \). Therefore \( \gamma_{se}(G_i) = m + n - 1 \).

3.9 Theorem Let \( G \) be a Bistar \( D_{r,s} \), \( r, s \geq 1 \). The inflated graph \( G_i \) of \( G \) is strong efficient and \( \gamma_{se}(G_i) = r + s \).

Proof: Let \( u \) and \( v \) be the central vertices of \( G \). Let \( u_1, u_2 \ldots u_r \) and \( v_1, v_2 \ldots v_s \) be the vertices adjacent with \( u \) and \( v \) respectively.

Case (1): Let \( r \neq s \). Without loss of generality, let \( r < s \). deg \( u = r + 1 \) and deg \( v = s + 1 = \Delta(G) \).

Let \( G_i \) be the inflated graph of \( G \). There are cliques \( A_u \) of order \( r + 1 \), \( A_v \) of order \( s + 1 \), \( 1 \leq i \leq r \) and \( A_{vi} \), \( 1 \leq j \leq s \) of order 1 in \( G_i \). \( V(A_u) = \{ u', u_1', u_2', \ldots , u_r' \} \). \( V(A_v) = \{ v', v_1', v_2', \ldots , v_s' \} \). \( V(A_{vi}) = \{ v'_i \}, 1 \leq i \leq r \), \( V(A_{uj}) = \{ v'_j \}, 1 \leq j \leq s \). v' in \( A_{vi} \) is adjacent with \( u' \in A_u \), \( u' \) in \( A_u \) is adjacent with \( u' \in A_{uj} \), \( 1 \leq i \leq r \) and \( v'_j \) in \( A_v \) is adjacent with \( v' \in A_{uj} \), \( 1 \leq j \leq s \). The degrees of all vertices in \( A_u \) are \( r + 1 \) and the degrees of all vertices in \( A_v \) are \( s + 1 \). The degree of \( u' \) in \( A_{uj} \), \( 1 \leq i \leq r \) and the degree of \( v' \) in \( A_{uj} \), \( 1 \leq j \leq s \) are 1. Hence the vertices in \( A_u \) are maximum degree vertices. Since \( A_u \) is a clique, any strong efficient dominating set contains exactly one vertex of \( A_u \). If any \( u' \) in \( A_u \) is chosen to form a strong efficient dominating set \( S_i \), then it strongly dominates all the vertices of \( A_u \) and \( v' \) in \( A_v \). To dominate \( u' \in A_u \), \( 1 \leq i \leq r \), one among them must be chosen. If any \( u' \) is chosen, then \( v' \) in \( A_v \) is strongly dominated by two vertices \( u_1' \in A_u \) and \( u_r' \in A_u \), a contradiction. Therefore \( u' \) in \( A_u \) cannot be chosen to form \( S_i \). The unique way to form a strong efficient dominating set \( S_i \), any vertex \( v'_j \) in \( A_v \), \( 1 \leq j \leq s \) is chosen. Therefore \( S_i = \{ v'_j / v'_j \in A_{uj} \}, 1 \leq k \leq s, k \neq j \} \) is independent and not dominated by \( v'_j \). These vertices belong to \( S_i \). To strongly dominate the vertices of \( A_u \), consider the following sub cases.

Sub case (1a): Suppose \( v' \) in \( A_u \) is chosen. It strongly dominates all \( u'_i \) in \( A_u \). Since \( \{ u'/u' \in A_{uj} \} \), \( 1 \leq i \leq r \) is independent, all \( u' \in A_{uj} \) belong to any strong efficient dominating set \( S_i \). For \( 1 \leq j \leq s \), define \( S_j = \{ v'_j \} \cup \{ v'/v' \in A_{vk} \}, 1 \leq k \leq s, k \neq j \} \cup \{ v' \} \cup \{ u'/u' \in A_{uj} \}, 1 \leq i \leq r \} \) where \( v'_j \in V(A_v) \) and \( v' \in V(A_u) \). Clearly all the vertices of \( G_i \) are uniquely strongly dominated by \( S_j \) and \( S_i \) is a strong efficient dominating set of \( G_i \). Further, \( |S_i| = 1 + s - 1 + 1 + r = s + 1 \).

Sub case (1b): Suppose any one of \( u'_i \) in \( A_u \), \( 1 \leq i \leq r \) is chosen. It strongly dominates the vertices of \( A_u \) and \( u' \in A_{uj} \). Therefore \( \{ u'/u' \in A_{uj} \}, 1 \leq t \leq r, t \neq i \} \) is independent and not dominated by \( u'_i \). Thus, \( 1 \leq i \leq r, T_i = \{ v'_j \} \cup \{ v'/v' \in A_{vk} \}, 1 \leq k \leq s, k \neq j \} \cup \{ u'_i \} \cup \{ u'/u' \in A_{uj} \}, 1 \leq t \leq r, t \neq i \} \) where \( v'_j \in A_v \) and \( u'_i \in A_u \) is a strong efficient dominating set of \( G_i \). Further, \( |T_i| = 1 + s - 1 + 1 + r = r + s + 1 \).

Case (2): Let \( s = r \). deg \( u' = \) deg \( v' = \) deg \( u' = \) deg \( v' = r + 1 = \Delta(G_i) \), \( 1 \leq i \leq r \). If \( u'_i \) in \( A_u \) is chosen to form a strong efficient dominating set of \( G_i \), then no vertex of \( A_v \) can be chosen. Otherwise, \( v' \) in \( A_v \) is strongly dominated by \( u'_i \) in \( A_u \) and \( u'_i \) in \( A_v \). The argument is similar if \( v' \) in \( A_v \) is chosen. The unique way to form a strong efficient dominating set of \( G_i \) is, any one of \( v'_j \) in \( A_v \), \( 1 \leq i \leq r \), or any one of \( u'_i \) in \( A_u \) is to be chosen. Suppose one of \( v'_i \) in \( A_v \) is chosen. It strongly dominates the vertices of \( A_u \) and \( v' \in A_{vi} \). To dominate the vertices of \( A_u \), any one of \( u'_i \) in \( A_u \), \( 1 \leq i \leq r \) can be chosen. Therefore \( \{ u'/u' \in A_{uj} \}, 1 \leq k \leq r, k \neq i \} \cup \{ v'/v' \in A_{vk} \}, 1 \leq k \leq r, k \neq i \} \) are independent and not dominated by \( v'_j \) in \( A_v \). Therefore for \( 1 \leq i \leq r, S_i = \{ v'_i \} \cup \{ v'/v' \in A_{vk} \}, 1 \leq k \leq r, k \neq i \} \)
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k ≤ r, k ≠ i} \cup \{u_{i}'\} \cup \{u' \setminus u' \in A_{uk}, 1 \leq k \leq r, k \neq i\} is a γse - set of G_i. Further |S_i| = 1 + r - 1 + 1 + r - 1 = 2r. Hence γse(G_i) = 2r.

3.10 Theorem Let G = G_i, n ≥ 4. Let G_i be the inflated graph of G. Then G_i is strong efficient and γse(G_i) = n - 1.

Proof: Let V(W_n) = {v, v_1, v_2,..., v_n}. E(W_n) = {e_i / e_i = vv_i, 1 ≤ i ≤ n - 1} \cup \{v_i v_{i+1} / 1 ≤ i ≤ n - 1\} \cup \{v_i v_1\}. Since deg v = n - 1 and deg v_i = 3, 1 ≤ i ≤ n - 1, there is a clique A_0 of order n - 1 and cliques A_{v_1}, 1 ≤ i ≤ n - 1 of order 3 in G_i. V(A_i) = {v_i' / 1 ≤ i ≤ n - 1}. For 2 ≤ i ≤ n - 2, V(A_{v_i}) = \{v_i', v_{i+1}', v_{i-1}'\}, V(A_{v_1}) = \{v_1', v_2', v_0'\} and V(A_{v_{n-1}}) = \{v_n', v_{n-1}', v_{n-2}'\}. For 1 ≤ i ≤ n - 1, v_i in A_0 is adjacent with v_i' in A_{v_i}. v_i' in A_{v_i} is adjacent with v_{i+1}' in A_{v_{i+1}}. The degree of each vertex in A_0 is n - 1 and degree of each vertex in A_{v_i}, 1 ≤ i ≤ n - 1 is 3. Therefore each vertex in A_0 is a maximum degree vertex. Consider S_i = \{v_i' / v_i' in A_0, A_{v_i} and A_{v_{i-1}}\} \cup \{v_i' \setminus v_i' \in A_{v_i}, 3 ≤ i ≤ 2\}. The vertices in S_i strongly uniquely dominate all the vertices of G_i. Clearly S_i is a γ_{se} - set of G_i and |S_i| = 3 + n - 4 = n - 1. Proof is similar if v_{i-1}' in A_0 is considered. Suppose any v_i', 2 ≤ i ≤ n - 2 in A_0 is chosen. Let S_i = \{v_i' \setminus v_i' \in A_0, A_{v_{i-1}}, A_{v_{i+1}}\} \cup \{v_i' \setminus v_i' \in A_{v_i}, 1 ≤ i ≤ n - 1, i \neq i - 1, i + 1\}. The vertices in S_i strongly, uniquely dominate the vertices in G_i. Therefore S_n, 2 ≤ i ≤ n - 2, is a γ_{se} - set of G_i. Therefore G_i is strong efficient.

3.11 Theorem Inflated graph of strong efficient graph need not be strong efficient.

3.12 Example Let G = S(K_{1,n}). Let u be the central vertex of K_{1,n}. Let v_1, v_2,..., v_n be the vertices adjacent with u. Each edge is subdivided into two edges uu_i and u_i v_i, i = 1 to n. That is u_1, u_2,..., u_n are the new vertices which are adjacent with u. Therefore deg u = n = Δ(G), deg u_i = 2 and deg v_i = 1 for all i = 1 to n. Let S \subseteq V(G). If there exists exactly one maximum degree vertex, then any strong efficient dominating set must contain it. Clearly u belongs to S. u strongly dominates u_1, u_2,..., u_n and u does not dominate v_1, v_2,..., v_n. Therefore v_1, v_2,..., v_n belong to S. Hence S = \{u, v_1, v_2,..., v_n\} and |S| = n + 1. N_u \cap S = \{u\} = N_u [u] \cap S for all i = 1 to n. N_i [v_i] \cap S = \{v_i\} for all i = 1 to n. Clearly S is the γ_{se} - set of G. Thus S(K_{1,n}) is strong efficient and γ_{se}(S(K_{1,n})) = n + 1 for all n \in N.

Let G_i be the inflated graph of G. Since deg u = n, deg u_i = 2, deg v_i = 1 in G, there are 1 clique of order n, n cliques of order 2 and n cliques of order 1 in G_i. V(A_0) = \{u_i' \setminus 1 ≤ i ≤ n\}, V(A_{u_i}) = \{u_i', v_i' \setminus 1 ≤ i ≤ n\} and V(A_{u_{i+1}}) = \{u_i', v_i' \setminus 1 ≤ i ≤ n\}. u_i' in A_{u_i} is adjacent with u_i in A_0 and u_i' in A_{v_i} is adjacent with v_i' in A_{u_i}. 1 ≤ i ≤ n. The vertices in A_{u_i} are of maximum degree vertices. Suppose G_i is strong efficient. Let S be a strong efficient dominating set of G_i. One of the vertices of A_0 say u_i' must belong to S. u_i' strongly efficiently dominate all the other vertices in the clique A_0 and u_i' in A_{u_i}. To strongly efficiently dominate u_i' in A_{u_i}, the vertex v_i' in A_{u_i} must belong to S. But u_i' in A_{u_i} is strongly dominated by two vertices u_i' in A_0 and v_i' in A_{u_i}, a contradiction. Therefore G_i is not strong efficient.

4. CONCLUSION

In this paper, the strong efficient domination number of inflated graphs of some standard graphs are found. In future, the characterization of strong efficient domination number and the relationship among strong efficient domination number and other graph theoretic parameters like strong independent domination number, strong domination number will be studied.

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