Common Fixed Point Results for Hybrid Pairs of Occasionally Weakly Compatible Mappings Defined on $b$-Metric Space

Priyanka Nigam
Sagar Institute of Science and Technology, Bhopal (M.P.) India.
priyanka_nigam01@yahoo.co.in

S.S. Pagey
Institute for Excellence in Higher Education, Bhopal (M.P.) India.
pagedrss@rediffmail.com

Abstract: The aim of this paper is to obtain some common fixed point theorems for hybrid pairs of single and multi-valued occasionally weakly compatible mappings using a symmetric $\delta$ derived from an ordinary symmetric $d$ in $b$-metric space.

Keywords: Occasionally weakly compatible mappings, single and multi—valued maps, common fixed point theorem, $b$-metric space.

2000 Mathematics Subject Classification: 47H10; 54H25.

1 INTRODUCTION

The study of fixed point theorems, involving four single-valued maps, began with the assumption that all of the maps are commuted. Sessa [8] weakened the condition of commutativity to that of pairwise weakly commuting. Jungck generalized the notion of weak commutativity to that of pairwise compatible [5] and then pairwise weakly compatible maps [6]. Jungck and Rhoades [7] introduced the concept of occasionally weakly compatible maps.

Abbas and Rhoades [1] generalized the concept of weak compatibility in the setting of single and multi-valued maps by introducing the notion of occasionally weakly compatible (owc).

The concept of $b$—metric space was introduced by Czerwik [3]. Several papers deal with fixed point theory for single and multi—valued maps in $b$—metric space.

The aim of this paper is to obtain some common fixed point theorems for owc maps to hybrid pairs of single and multi—valued maps using a symmetric $\delta$ derived from an ordinary symmetric $d$ in $b$—metric space.

2 PRELIMINARY NOTES

Definition 2.1.[2] Let $(X, d)$ denotes a metric space, $x \in X$ and $A \subseteq X, \mathcal{D}(x, A) = \inf \{ d(x, a) : a \in A \}$ and $\mathcal{C}B(X)$ is the class of all nonempty closed and bounded subsets of $X$. For every $A, B \in \mathcal{C}B(X)$

$$\delta(A, B) = \sup \{ d(a, b) : a \in A, b \in B \}.$$  

We appeal to the fact that $\delta(A, B) = 0$ iff $A = B = \{x\}$ for $A, B \in \mathcal{C}B(X)$. If $a \in X$, we write $\delta(a, B)$ for $\delta(\{a\}, B)$.

Definition 2.2.[3] Let $X$ be a nonempty set and $s \geq 1$ a given real number. A function $d : X \times X \rightarrow [0, \infty)$ (nonnegative real numbers) is called a $b$—metric provided that, for all $x, y, z \in X$,

$$(b_i) \ d(x, y) = 0 \iff x = y,$$

$$(b_{ii}) \ d(x, y) = d(y, x),$$
The pair \((X, d)\) is called \(b - \text{metric space}\) with parameter \(s\).

It is clear that the definition of \(b - \text{metric space}\) is an extension of usual metric space. Also, if we consider \(s = 1\) in above definition, then we obtain definition of usual metric space.

**Definition 2.3.**\(^1\) Maps \(f : X \to X\) and \(T : X \to CB(X)\) are said to be occasionally weakly compatible (owc) if and only if there exist some point \(x\) in \(X\) such that \(fx \in Tx\) and \(fTx \subseteq Tfx\).

### 3 MAIN RESULTS

**Theorem 3.1** Let \((X, d)\) be a b-metric. Let \(f, g : X \to X\) and \(F, G : X \to CB(X)\) be single and multi-valued maps, respectively such that the pairs \(\{f, F\}\) and \(\{g, G\}\) are owc and satisfy inequality

\[
\delta(Fx, Gy) \leq \max \left\{d(fx, gy), \delta(gy, Fx) \frac{1 + \delta(fx, Fx)}{1 + \delta(gy, Gy)} \right\} \quad (3.1)
\]

for all \(x, y \in X\). Then \(f, g, F & G\) have a unique common fixed point in \(X\).

**Proof:** By hypothesis there exist points \(x, y \in X\) such that \(fx \in Fx, fFx \subseteq Ffx\) and \(gy \in Gy, gGy \subseteq Ggy\).

As \(fx \in Fx\) so \(fFx \subseteq Ffx\), \(gy \in Gy\) so \(gGy \subseteq Ggy\) and hence \(d(f^2x, g^2y) \leq \delta(Ffx, Ggy)\). First we show that \(gy = fx\). Suppose not. Then condition \((3.1)\) implies that

\[
\delta(Fx, Ggy) 
\leq \max \left\{d(fFx, ggy), \delta(ggy, Ffx) \frac{1 + \delta(fFx, Ffx)}{1 + \delta(ggy, Ggy)} \right\} 
\leq \max \left\{d(f^2x, g^2y), \delta(g^2y, Ffx) \frac{1 + \delta(fFx, Ffx)}{1 + \delta(g^2y, Ggy)} \right\} 
\leq \max \left\{d(f^2x, g^2y), \delta(FFx, Ggy), \delta(FFx, Ggy) \right\} 
= \delta(FFx, Ggy),
\]

a contradiction, and hence \(gy = fx\). Obviously \(d(fx, g^2y) \leq \delta(Fx, Gfx)\). Next we claim that \(x = fx\). If not, then condition \((3.1)\) implies that

\[
\delta(Fx, Gfx) \leq \max \left\{d(fx, gfy), \delta(gfy, Fx) \frac{1 + \delta(fx, Fx)}{1 + \delta(gfy, Fx)} \right\} 
\leq \max \left\{d(fx, ggy), \delta(ggy, Fx) \frac{1 + \delta(fx, Fx)}{1 + \delta(ggy, Ggy)} \right\} 
\leq \max \left\{d(fx, g^2y), \delta(ggy, Fx), 0 \right\} 
\leq \max \left\{\delta(Fx, Gfx), \delta(Fx, Fx) \right\} 
= \delta(Fx, Gfx),
\]

which is again a contradiction. Similarly we can prove \(y = gy\). Thus \(f, g, F, G\) have a common fixed point. Uniqueness follows from \((3.1)\).

**Theorem 3.2** Let \((X, d)\) be a b-metric. Let \(f, g : X \to X\) and \(F, G : X \to CB(X)\) be single and multi-valued maps, respectively such that the pairs \(\{f, F\}\) and \(\{g, G\}\) are owc and satisfy inequality
Common Fixed Point Results for Hybrid Pairs of Occasionally Weakly Compatible Mappings Defined on b-Metric Space

\[
\delta(Fx,Gy) \leq \max \left\{ d(fx,gy), \left( 1 - \frac{\delta(gyGy) + \delta(fx,Gy)}{\delta(fx,Fx) + \delta(gy,Fx)} \right) \right\} \tag{3.2}
\]

for all \( x, y \in X \). Then \( f, g, F & G \) have a unique common fixed point in \( X \).

**Proof:** By hypothesis there exist points \( x, y \in X \) such that \( fx \in Fx, fx \subseteq Ffx \) and \( gy \in Gy, gy \subseteq Ggy \).

As \( fx \in Fx \) so \( ffx \subseteq Ffx \subseteq Ffx \) so \( ggy \subseteq ggy \subseteq Ggy \) and hence \( d(f^2x, g^2y) \leq \delta(Ffx, Ggy) \). First we show that \( gy = fx \). Suppose not. Then condition (3.2) implies that

\[
\delta(Ffx, Ggy) \leq \max \left\{ d(ffx, ggy), \left( 1 - \frac{\delta(ggyggy) + \delta(ffx, ggy)}{\delta(ffx, Fx) + \delta(ggy, Fx)} \right) \right\}
\]

\[
\leq \max \left\{ d(f^2x, g^2y), \left( 1 - \frac{\delta(ggyggy) + \delta(f^2x, ggy)}{\delta(f^2x, Fx) + \delta(g^2y, Fx)} \right) \right\}
\]

\[
\leq \max \left\{ \delta(Ffx, Ggy), \left( 1 - \frac{\delta(Ffx, ggy)}{\delta(ggy, Fx)} \right) \right\}
\]

\[
\leq \delta(Ffx, Ggy),
\]

a contradiction, and hence \( gy = fx \). Obviously \( d(fx, g^2y) \leq \delta(Fx, Gfx) \). Next we claim that \( x = fx \). If not, then condition (3.2) implies that

\[
\delta(Fx, Gfx) \leq \max \left\{ d(fx, gfx), \left( 1 - \frac{\delta(gfx, gfx) + \delta(fx, gfx)}{\delta(fx, Fx) + \delta(gfx, Fx)} \right) \right\}
\]

\[
\leq \max \left\{ d(fx, gfx), \left( 1 - \frac{\delta(ggyggy) + \delta(fx, gfx)}{\delta(fx, Fx) + \delta(ggy, Fx)} \right) \right\}
\]

\[
\leq \max \left\{ d(fx, gfx), \left( 1 - \frac{\delta(fx, gfx)}{\delta(gyy, Fx)} \right) \right\}
\]

\[
\leq \max \left\{ \delta(Fx, gfx), \left( 1 - \frac{\delta(fx, gfx)}{\delta(gxy, Fx)} \right) \right\}
\]

\[
= \delta(Fx, gfx),
\]

which is again a contradiction. Similarly we can prove \( y = gy \). Thus \( f, g, F, G \) have a common fixed point. Uniqueness follows from (3.2).

**Theorem 3.3** Let \( (X, d) \) be a b-metric. Let \( f, g : X \to X \) and \( F, G : X \to CB(X) \) be single and multi-valued maps, respectively such that the pairs \( \{f, F\} \) and \( \{g, G\} \) are owc and satisfy inequality

\[
\delta(Fx, Gy) \leq ad(fx, gy) + \beta \delta(fx, Gy) + \gamma \delta(gy, Fx) \tag{3.3}
\]

for all \( x, y \in X, a, \beta, \gamma > 0 \) and \( (a + \beta + \gamma) = 1 \). Then \( f, g, F & G \) have a unique common fixed point in \( X \).

**Proof:** By hypothesis there exist points \( x, y \in X \) such that \( fx \in Fx, fx \subseteq Ffx \) and \( gy \in Gy, gy \subseteq Ggy \).

As \( fx \in Fx \) so \( ffx \subseteq Ffx \subseteq Ffx \) so \( ggy \subseteq ggy \subseteq Ggy \) and hence \( d(f^2x, g^2y) \leq \delta(Ffx, Ggy) \). First we show that \( gy = fx \). Suppose not. Then condition (3.3) implies that

\[
\delta(Ffx, Ggy) \leq ad(f^2x, g^2y) + \beta \delta(ffx, Ggy) + \gamma \delta(ggy, Ffx)
\]
\begin{align*}
l &\leq a\delta(Ffx, Ggy) + \beta\delta(Ffx, Ggy) + \gamma\delta(Ggy, Ffx) \\
&\leq (\alpha + \beta + \gamma)\delta(Ffx, Ggy) \\
&= \delta(Ffx, Ggy),
\end{align*}

a contradiction, and hence \(g y = f x\). Obviously \(d(f x, g^2 y) \leq \delta(Fx, Gfx)\). Next we claim that \(x = fx\). If not, then condition (3.3) implies that

\begin{align*}
\delta(Fx, Gfx) &\leq ad(fx, gfy) + \beta\delta(Fx, Gfx) + \gamma\delta(gfy, Fx) \\
&\leq ad(fx, gfy) + \beta\delta(Fx, Gfx) + \gamma\delta(gfy, Fx) \\
&\leq ad(fx, g^2 y) + \beta\delta(Fx, Gfx) + \gamma\delta(Ggy, Fx) \\
&\leq (\alpha + \beta + \gamma)\delta(Fx, Gfx) \\
&= \delta(Fx, Gfx)
\end{align*}

which is again a contradiction. Similarly we can prove \(y = gy\). Thus \(f, g, F, G\) have a common fixed point. Uniqueness follows from (3.3).

**Theorem 3.4** Let \((X, d)\) be a \(b\)-metric. Let \(f, g: X \to X\) and \(F, G: X \to CB(X)\) be single and multi-valued maps, respectively such that the pairs \(\{f, F\}\) and \(\{g, G\}\) are owc and satisfy inequality

\begin{equation}
\delta(Fx, Gy) \leq ad(fx, gy) + (1 - \alpha) \left[ \frac{\delta(fx, Fx) + \delta(gy, Gy) + \delta(fx, Gy) + \delta(gy, Fx)}{2} \right]
\end{equation}

for all \(x, y \in X\) & \(\alpha \in [0, 1)\). Then \(f, g, F & G\) have a unique common fixed point in \(X\).

**Proof:** By hypothesis there exist points \(x, y \in X\) such that \(fx \in Fx, fFx \subseteq Ffx\) and \(gy \in Gy, gGy \subseteq GGy\).

As \(fx \in Fx\) so \(ffx \subseteq fFx \subseteq Ffx\), \(gy \in Gy\) so \(ggx \subseteq gGy \subseteq GGx\) and hence \(d(f^2 x, g^2 y) \leq \delta(Ffx, Ggy)\). First we show that \(gy = fx\). Suppose not. Then condition (3.4) implies that

\begin{align*}
\delta(Fxx, Ggy) &\leq ad(ffx, ggy) + (1 - \alpha) \left[ \frac{\delta(ffx, Ffx) + \delta(ggy, Ggy) + \delta(ffx, Ggy) + \delta(ggy, Ffx)}{2} \right] \\
&\leq ad(f^2 x, g^2 y) + (1 - \alpha) \left[ \frac{\delta(ffx, ffx) + \delta(g^2 y, ggy) + \delta(ffx, ggy) + \delta(ggy, ffx)}{2} \right] \\
&= \delta(fxx, Ggy),
\end{align*}

a contradiction, and hence \(gy = fx\). Obviously \(d(fx, g^2 y) \leq \delta(Fx, Gfx)\). Next we claim that \(x = fx\). If not, then condition (3.4) implies that

\begin{align*}
\delta(Fx, Gfx) &\leq ad(fx, gfx) + (1 - \alpha) \left[ \frac{\delta(fx, Fx) + \delta(gfx, Gfx) + \delta(fx, Gfx) + \delta(gfx, Fx)}{2} \right] \\
&= \delta(Fx, Gfx),
\end{align*}

which is again a contradiction. Similarly we can prove \(y = gy\). Thus \(f, g, F, G\) have a common fixed point. Uniqueness follows from (3.4).

**Corollary 3.5** Let \((X, d)\) be a \(b\)-metric. Let \(f, g: X \to X\) and \(F, G: X \to CB(X)\) be single and multi-valued maps, respectively such that the pairs \(\{f, F\}\) and \(\{g, G\}\) are owc and satisfy inequality
Common Fixed Point Results for Hybrid Pairs of Occasionally Weakly Compatible Mappings Defined on b-Metric Space

\[ \delta(Fx, Gy) \leq \alpha \left[ \frac{d(fx, gy) + \delta(gy, Fx) + \delta(fx, Fx) + \delta(gy, Gy) + \delta(fx, gy)}{1 + \delta(fx, Gy)} \right] \]  

(3.5)

for all \( x, y \in X \) and \( \alpha \in [0, 1) \). Then \( f, g, F \) and \( G \) have a unique common fixed point in \( X \).

**Theorem 3.6** Let \((X, d)\) be a b-metric. Let \( f, g: X \rightarrow X \) and \( F, G: X \rightarrow CB(X) \) be single and multi-valued maps, respectively such that the pairs \( \{f, F\} \) and \( \{g, G\} \) are owc and satisfy inequality

\[ \delta(Fx, Gy) \leq \max \left\{ d(fx, gy), \delta(fx, Fx), \delta(gy, Gy), \frac{1}{2} \left[ \delta(fx, Gy) + \delta(gy, Fx) \right] \right\} \]  

(3.6)

for all \( x, y \in X \). Then \( f, g, F \) and \( G \) have a unique common fixed point in \( X \).

**Proof:** By hypothesis there exist points \( x, y \in X \) such that \( fx \in Fx, fFx \subseteq Ffx \) and \( gy \in Gx, gGx \subseteq Ggy \). As \( fx \in Fx \) so \( ffx \subseteq fFx \subseteq Ffx \), \( gy \in Gx \) so \( ggy \subseteq gGx \subseteq Ggy \) and hence \( d(f^2x, g^2y) \leq \delta(Ffx, Ggy) \). First we show that \( gy = Fx \). Suppose not. Then condition (3.6) implies that

\[ \delta(FFx, Ggy) \leq \max \left\{ d(f^2x, g^2y), \delta(f^2x, FFx), \delta(g^2y, Ggy), \frac{1}{2} \left[ \delta(f^2x, Ggy) + \delta(g^2y, FFx) \right] \right\} \]

\[ \leq \max \{ \delta(FFx, Ggy), 0, 0, \delta(FFx, Ggy) \} \]

\[ = \delta(FFx, Ggy), \]

a contradiction, and hence \( gy = Fx \). Obviously \( d(fx, g^2y) \leq \delta(Fx, Gfx) \). Next we claim that \( x = fx \). If not, then condition (3.6) implies that

\[ \delta(Fx, Gfx) \leq \max \left\{ d(fx, g^2y), \delta(fx, Fx), \delta(g^2x, Gfx), \frac{1}{2} \left[ \delta(fx, Gfx) + \delta(g^2x, Fx) \right] \right\} \]

\[ \leq \max \left\{ d(fx, g^2y), \delta(fx, Fx), \delta(g^2x, Gfx), \frac{1}{2} \left[ \delta(g^2x, Ggy) + \delta(g^2y, Fx) \right] \right\} \]

\[ \leq \max \left\{ \delta(Fx, Gfx), 0, 0, \frac{1}{2} \left[ \delta(g^2x, Ggy) + \delta(g^2y, Fx) \right] \right\} \]

\[ \leq \max \left\{ \delta(Fx, Gfx), 0, 0, \frac{1}{2} \left[ \delta(fx, Gfx) + \delta(Fx, Fx) \right] \right\} \]

\[ = \delta(Fx, Gfx), \]

which is again a contradiction. Similarly we can prove \( y = gy \). Thus \( f, g, F, G \) have a common fixed point. Uniqueness follows from (3.6).

**Corollary 3.7** Let \((X, d)\) be a b-metric. Let \( f, g: X \rightarrow X \) and \( F, G: X \rightarrow CB(X) \) be single and multi-valued maps, respectively such that the pairs \( \{f, F\} \) and \( \{g, G\} \) are owc and satisfy inequality

\[ \delta(Fx, Gy) \leq \max \{ d(fx, gy), \delta(fx, Fx), \delta(gy, Gy), \delta(gy, Fx) \} \]  

(3.7)

for all \( x, y \in X \). Then \( f, g, F \) and \( G \) have a unique common fixed point in \( X \).

**Proof:** Clearly the result immediately follows from Theorem 3.1.

**Corollary 3.8** Let \((X, d)\) be a b-metric. Let \( f: X \rightarrow X \) and \( F: X \rightarrow CB(X) \) be single and multi-valued maps, respectively such that the pairs \( \{f, F\} \) and \( \{g, G\} \) are owc and satisfy inequality

\[ \delta(Fx, Gy) \leq \max \left\{ d(fx, gy), \frac{1}{2} \left[ \delta(fx, Fx) + \delta(fy, Fy) \right], \frac{1}{2} \left[ \delta(fx, Fx) + \delta(fy, Fy) \right] \right\} \]  

(3.8)

for all \( x, y \in X \). Then \( f, F \) have a unique common fixed point in \( X \).
Theorem 3.9 Let \((X, d)\) be a b-metric space. Let \(f, g: X \to X\) and \(F, G: X \to CB(X)\) be single and multi-valued maps, respectively such that the pairs \(\{f, F\}\) and \(\{g, G\}\) are owc and satisfy inequality
\[
\delta^p(Fx, Gy) \leq \max\{ad(fx, gy)\delta^{p-1}(fx, Fx), ad(fx, gy)\delta^{p-1}(gy, Gy), a\delta(fx, Fx)\delta^{p-1}(gy, Gy), c\delta^{p-1}(fx, Gy) + \delta(gy, Fx)\}
\]  
(3.9)
for all \(x, y \in X\), with \(a \geq 0, 0 < c < 1\) and \(p \geq 2\). Then \(f, g, F \& G\) have a unique common fixed point in \(X\).

Proof: By hypothesis there exist points \(x, y \in X\) such that \(fx \in Fx, fFx \subseteq Ffx\) and \(gy \in Gy, gGy \subseteq Ggy\).

As \(fx \in Fx\) so \(ffx \subseteq fFx \subseteq Ffx\), \(gy \in Gy\) so \(ggx \subseteq gGy \subseteq Ggy\) and hence \(d(f^2x, g^2y) \leq \delta(Ffx, Ggy)\). First we show that \(gy = fx\). Suppose not. Then condition (3.9) implies that
\[
\delta^p(fx, Ggy)
\]
\[
\leq \max\{ad(fx, ggx)\delta^{p-1}(fx, Fx), ad(fx, ggx)\delta^{p-1}(ggx, Ggy), a\delta(fx, Fx)\delta^{p-1}(ggx, Ggy), c\delta^{p-1}(fx, Ggy) + \delta(ggx, Fx)\}
\]
\[
\leq \delta^p(fx, Ggy).
\]
a contradiction, and hence \(gy = fx\). Obviously \(d(fx, g^2y) \leq \delta(Fx, Gfx)\). Next we claim that \(x = fx\). If not, then condition (3.9) implies that
\[
\delta^p(Fx, Ggy)
\]
\[
\leq \max\{ad(fx, ggx)\delta^{p-1}(fx, Fx), ad(fx, ggx)\delta^{p-1}(ggx, Ggy), a\delta(fx, Fx)\delta^{p-1}(ggx, Ggy), c\delta^{p-1}(fx, Ggy) + \delta(ggx, Fx)\}
\]
\[
\leq \delta^p(Fx, Ggy).
\]
which is again a contradiction. Similarly we can prove \(y = gx\). Thus \(f, g, F, G\) have a common fixed point. Uniqueness follows from (3.9).

Corollary 3.10 Let \((X, d)\) be a b-metric space. Let \(f: X \to X\) and \(F: X \to CB(X)\) be single and multi-valued maps, respectively such that the pair \(\{f, F\}\) is owc and satisfy inequality
\[
\delta^p(Fx, Fy) \leq \max\{ad(fx, fy)\delta^{p-1}(fx, Fx), ad(fx, fy)\delta^{p-1}(fy, Fy), a\delta(fx, Fx)\delta^{p-1}(fy, Fy), c\delta^{p-1}(fx, Fy) + \delta(fy, Fx)\}
\]  
(3.10)
for all \(x, y \in X\), with \(a \geq 0, 0 < c < 1\) and \(p \geq 2\). Then \(f, g, F \& G\) have a unique common fixed point in \(X\).

If we put in above Theorem \(f = g\) and \(F = G\), we obtain the following result.

Corollary 3.11 Let \((X, d)\) be a b-metric space with parameter \(s \geq 1\). Let \(f: X \to X\) and \(F, G: X \to CB(X)\) be single and multi-valued maps, respectively such that the pairs \(\{f, F\}\) and \(\{f, G\}\) are owc and satisfy inequality
\[
\delta^p(Fx, Gy) \leq \max\{ad(fx, fy)\delta^{p-1}(fx, Fx), ad(fx, fy)\delta^{p-1}(fy, Gy), a\delta(fx, Fx)\delta^{p-1}(fy, Gy), c\delta^{p-1}(fx, Gy) + \delta(fy, Fx)\}
\]  
(3.11)
for all \(x, y \in X\), with \(a \geq 0, 0 < c < 1\) and \(p \geq 2\). Then \(f, g, F \& G\) have a unique common fixed point in \(X\).

Now, letting \(f = g\) we get the next corollary.
Common Fixed Point Results for Hybrid Pairs of Occasionally Weakly Compatible Mappings Defined on b-Metric Space

Corollary 3.12 Let \((X, d)\) be a b-metric space with parameter \(s \geq 1\). Let \(f, g : X \rightarrow X\) and \(F, G : X \rightarrow CB(X)\) be single and multi-valued maps, respectively such that the pairs \(\{f, F\}\) and \(\{g, G\}\) are owc and satisfy inequality
\[
\delta^p(Fx, Gy) \leq \alpha d^p(fx, gy) + (1 - \alpha) \max \{\delta^p(fx, gy), \delta^p(gy, Fz), \delta^p(Fz, Gy)\}^p
\]
for all \(x, y \in X\), with \(\alpha \geq 0, 0 < c < 1\) and \(p \geq 2\). Then \(f, g, F, G\) have a unique common fixed point in \(X\).

We can also prove the result with contractive modulus i.e.,
A function \(\varphi : [0, \infty) \rightarrow [0, \infty)\) is said to have a contractive modulus if \(\varphi(0) = 0\) & \(\varphi(t) < t\) for \(t > 0\).

Corollary 3.13 Let \((X, d)\) be a b-metric. Let \(f, g : X \rightarrow X\) and \(F, G : X \rightarrow CB(X)\) be single and multi-valued maps, respectively such that the pairs \(\{f, F\}\) and \(\{g, G\}\) are owc and satisfy inequality
\[
\delta(Fx, Gy) \leq \max \{\varphi(d(fx, gy)), \varphi(\delta(fx, Fz)), \varphi(\delta(fx, Gy)), \varphi(\delta(gy, Gz)), \varphi(\delta(gy, Fz))\}
\]
for all \(x, y \in X\). Then \(f, g, F, G\) have a unique common fixed point in \(X\).

4 Conclusion

We prove some common fixed point results for occasionally weakly compatible mappings in hybrid pairs of single and multi-valued maps using a symmetric \(\delta\) derived from an ordinary symmetric \(d\) in \(b\)–metric space. Our result generalize the result of various authors present in \(b\)–metric space.

References


Ms. Priyanka Nigam is working as Assistant Professor in Mathematics, Department of Science & Humanities, Sagar Institute of Science and Technology, Bhopal (M.P.). She completed her Master Degree in Mathematics from Sarojini Naidu Govt. Girls College, Bhopal in 2008. Also she took her M.Phil degree from Institute for Excellence in Higher Education; Bhopal in 2009. She was the topper in M.Phil. She is pursuing her Doctoral degree under the guidance of Dr. S.S. Pagey, Bhopal (M.P.) in the area of "fixed point theory". She has published more than 5 papers in various National and International journals. Her total teaching experience is 6 years. Her field of interest is Functional Analysis, Operation Research and Real Analysis.