# Star-in-Coloring of Some New Class of Graphs 

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#### Abstract

Jan Mycielski defined the Mycielskian graph as an extension of a graph with certain conditions. Sampathkumar and Walikar by omitting some of the conditions of Mycielski graph obtained the splitting graph of a graph. In this paper, we found the star-in-coloring concept introduced by Sudha, et al for the following graphs: (i) the splitting graph of complete-bipartite graphs (ii) theMycielski's graphs of paths (iii) theMycielski's graphs of cycles (iv) tensor product of complete-bipartite graphs and paths (v) tensor product of complete-bipartite graphs and cycles.

In addition we have given the general coloring pattern of all these graphs and their star-in-chromatic number.


Keywords: star-in-coloring, splitting graph, Mycielski graph, tensor product of two graphs
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## 1. Introduction

In 1973, Grunbaum[1] has defined proper coloring by avoiding 2-colored paths with four vertices and defined it as star-in-coloring. Star-in-coloring has been discussed by Fertin, et al[2] and Nesetril, et al[3]. Jan Mycielski[4] in 1955 has given the construction of Mycielski graph for the graphs. Splitting graph $\mathrm{S}(\mathrm{G})$ was defined by Sampathkumar and Walikar[5]. The tensor product of graphs was defined by Alfred North Whitehead, et al[6] in their Principia Mathematica.

Definition 1.1 A star-coloring of a graph $G$ is a proper coloring of a graph with the condition that no path on four vertices $\left(P_{4}\right)$ is 2-colored.
A $k$-star-coloring of a graph $G$ is a star-coloring of $G$ using atmost $k$ colors.
Definition1.2 An in-coloring of a graph $G$ is a proper coloring of a graph $G$ if there exist any path $P_{3}$ of length 2 with the end vertices having same color, then the edges of $P_{3}$ are oriented towards the central vertex.

By combining these two definitions, Sudha, et al[7,8] defined the star-in-coloring of graphs as follows:

Definition1.3 A graph $G$ is said to admit star-in-coloring orientation if

1. No path on four vertices is bicolored
2. Any path of length 2 with end vertices of same color are directed towards the middle vertex.

The minimum number of colors required to color the graph $G$ satisfying the above conditions for star-in-coloring is called the star-in-chromatic number of $G$ and is denoted by $\chi_{s i}(G)$.


Figure 1. Cycle $C_{4}$
In fig- 1 , the vertices $v_{1}$ and $v_{3}$ are assigned with the color 1 , the vertex $v_{2}$ is assigned with the color 2 and the vertex $v_{4}$ is assigned with the color 3 . This pattern of coloring satisfies both the conditions required for star-in-coloring orientation. In this graph we see that no two adjacent vertices have the same color; no path on four vertices is bicolored; each and every edge in a path of length two in which end vertices have same color are oriented towards the central vertex. Hence it is star-in-colored with orientation.

Definition1.4 For any graph $G$, the splitting graph $S(G)$ is obtained by adding to each vertex $v_{i}$ in $G$ a new vertex $v_{i}^{\prime}$ such that $v_{i}^{\prime}$ is adjacent to the neighbors of $v_{i}$ in $G$.


Figure 2. Cycle $C_{4}$ and its splitting graph $S\left(C_{4}\right)$
Definition1.5 Let $G$ be a graph with $n$ vertices denoted by $v_{1}, v_{2}, \cdots, v_{n}$. The Mycielski graph $\mu(G)$ is obtained by adding to each vertex $v_{i}$ a new vertex $u_{i}$ such that $u_{i}$ is adjacent to the neighbors of $v_{i}$. Finally, add a new vertex $w$ such that $w$ is adjacent to each and every $u_{i}$.


Figure 3. Cycle $C_{4}$ and its Mycielski's graph $\mu\left(C_{4}\right)$
Definition1.6 The tensor product of two graphs $G_{1}$ and $G_{2}$ denoted by $G_{1} \otimes G_{2}$ has the vertex set $V\left(G_{1} \otimes G_{2}\right)=V\left(G_{1}\right) \times V\left(G_{2}\right)$ and the edge set $E\left(G_{1} \otimes G_{2}\right)=\left\{\left(u_{1}, v_{1}\right)\left(u_{2}, v_{2}\right) / u_{1} u_{2} \in E\left(G_{1}\right)\right.$ and $\left.v_{1} v_{2} \in E\left(G_{2}\right)\right\}$.


Figure 4. Tensor product of $P_{4}$ and $P_{3}$

## 2. MAIN Results

Theorem 2.1 The splitting graph of complete-bipartite graphs $S\left(K_{m, n}\right)$ admit star-in- coloring and its star-in-chromatic number is $\chi_{s i}\left(S\left(K_{m, n}\right)\right)=2(\min (m, n))+1$ for all $m$ and $n$.

Proof. The complete-bipartite graph $K_{m, n}$ consists of $m+n$ vertices $\left\{u_{1}, u_{2}, \cdots, u_{m+n}\right\}$ and $m n$ edges. The splitting graph of complete-bipartite graph $K_{m, n}$ consists of $2(m+n)$ vertices and $3 m n$ edges. It is denoted by $S\left(K_{m, n}\right)$.

Define a function $f: V \rightarrow\{1,2,3, \ldots\}$ such that $f(u) \neq f(v)$ if $u v \in E$ where $V$ is the vertex set of $S\left(K_{m, n}\right)$ and $E$ is the edge set of $S\left(K_{m, n}\right)$.

The coloring pattern is as follows:
Case (i): If $m \leq n$

$$
\begin{gathered}
f\left(u_{i}\right)=\left\{\begin{array}{rr}
\quad 1+i, \quad 1 \leq i \leq m \\
1, \quad m+1 \leq i \leq m+n
\end{array}\right. \\
f\left(u_{i}^{\prime}\right)=\left\{\begin{array}{rr}
m+1+i, & 1 \leq i \leq m \\
1, & m+1 \leq i \leq m+n
\end{array}\right.
\end{gathered}
$$

Case (ii): If $m>n$

$$
\begin{gathered}
f\left(u_{i}\right)=\left\{\begin{array}{cc}
1, & 1 \leq i \leq m \\
1-m+i, & m+1 \leq i \leq m+n
\end{array}\right. \\
f\left(u_{i}^{\prime}\right)=\left\{\begin{array}{cc}
1, & 1 \leq i \leq m \\
1-m+n+i, & m+1 \leq i \leq m+n
\end{array}\right.
\end{gathered}
$$

With this pattern we can color the graph $S\left(K_{m, n}\right)$ satisfying star-in-coloring condition.
Illustration 2.1.1 Consider a complete-bipartite graph $K_{2,4}$. As per the definition of splitting graph $S\left(K_{2,4}\right)$ consists of 12 vertices and 24 edges.

According to case(i) of theorem-2.1 the vertices $u_{1}$ and $u_{2}$ are assigned with colors 2 and 3 respectively. The vertices $u_{3}, u_{4}, u_{5}$ and $u_{6}$ take the color 1 . The vertices $u_{1}^{\prime}$ and $u_{2}^{\prime}$ are assigned with colors 4 and 5 respectively. The vertices $u_{3}^{\prime}, u_{4}^{\prime}, u_{5}^{\prime}$ and $u_{6}^{\prime}$ take the color 1 .

The star-in-chromatic number of $S\left(K_{2,4}\right)$ is 5 .


Figure 5. Star-in-coloring of $S\left(K_{2,4}\right)$
Theorem 2.2 Mycielski's graph of path $\mu\left(P_{n}\right)$ for all odd $n \geq 2$ admit star-in-coloring and its star-in-chromatic number is $\chi_{s i}\left(\mu\left(P_{n}\right)\right)=5+j$ with $j=\left(\frac{n-3}{2}\right)$.

## S.Sudha \& V. Kanniga

Proof. Consider a path $P_{n}$ with $n$ vertices and $n-1$ edges. Let the vertices be denoted by $v_{1}, v_{2}, \cdots, v_{n}$. As per the construction of Mycielski's graph a new vertex set say $\left\{u_{1}, u_{2}, \cdots, u_{n}\right\}$ is introduced and each and every vertex say $u_{i}$ is adjacent to the neighbor of $v_{i}$ for all $i$. Then another new vertex say $w$ is introduced and an edgeisadded fromwto each $u_{i}$ for all $i$. This newly constructed graph $\mu\left(P_{n}\right)$ consists of $(2 n+1)$ vertices and $4 n-3$ edges.

Define a function $f: V \rightarrow\{1,2,3, \ldots\}$ such that $f(u) \neq f(v)$ if $u v \in E$ where $V$ is the vertex set of $\mu\left(P_{n}\right)$ and $E$ is the edge set of $\mu\left(P_{n}\right)$ as follows:

$$
\begin{gathered}
f\left(v_{i}\right)=\left\{\begin{array}{cc}
1, & i f i \equiv 1(\bmod 4) \operatorname{and} i \equiv 3(\bmod 4) \\
2, \quad & \quad i f i \equiv 2(\bmod 4) \\
3, & \text { ifi } i \equiv 0(\bmod 4)
\end{array}\right. \\
f\left(u_{i}\right)= \begin{cases}4, & \text { ifi } \equiv 1(\bmod 4) \text { and } i \equiv 3(\bmod 4) \\
4+\left(\frac{i}{2}\right), & i f i \equiv 2(\bmod 4) \operatorname{and} i \equiv 0(\bmod 4)\end{cases} \\
f(w)=1
\end{gathered}
$$

By using the above pattern of coloring the Mycielski graph of paths is star-in- colored.
Illustration 2.2.1 Consider the path graph $P_{11}$. According to the construction of Mycielski graph we obtain the graph $\mu\left(P_{11}\right)$. It consists of 23 vertices and 41 edges.


By using theorem-2.2 the vertices $v_{1}, v_{3}, v_{5}, v_{7}, v_{9}$ and $v_{11}$ take the color 1 . The vertices $v_{2}, v_{6}$ and $v_{10}$ take the color 2 . The vertices $v_{4}$ and $v_{8}$ take the color 3 . The vertices $u_{1}, u_{3}, u_{5}, u_{7}, u_{9}$ and $u_{11}$ take the color 4 . The vertices $u_{2}, u_{4}, u_{6}, u_{8}$ and $u_{10}$ are assigned with colors $5,6,7,8$ and 9 respectively. The vertex $w$ takes the color 1 .
The star-in-chromatic number of $\mu\left(P_{11}\right)$ is 9 .
Remark: For neven $\mu\left(P_{n}\right)$ there isatleast one edge without orientation. Hence Star-in-coloring condition is not satisfied.
Theorem 2.3 Mycielski graph of cycles $\mu\left(C_{n}\right)$ for all even $n \geq 3$ admit star-in-coloring and its star-in-chromatic number is

$$
\chi_{\mathrm{si}}\left(\mu\left(C_{n}\right)\right)=\left\{\begin{array}{cc}
2(2+j), & \text { ifn }=4 j, j=1,2,3, \ldots \\
2(3+j), & \text { ifn }=2+4 j, j=1,2,3, \ldots
\end{array}\right.
$$

Proof: Consider a cycle $C_{n}$ with $n$ vertices and $n$ edges. The vertices are denoted by $v_{1}, v_{2}, \cdots, v_{n}$.As per the construction of Mycielski's graph a new vertex $\operatorname{set}\left\{u_{1}, u_{2}, \cdots, u_{n}\right\}$ is introduced and draw an edge from each vertex $u_{i}$ to the neighbor of $v_{i}$ for all $i$. A new vertex $w$ is
introduced and we add an edge from $w$ to each $u_{i}$. This newly constructed graph $\mu\left(C_{n}\right)$ consists of $(2 n+1)$ vertices and $4 n$ edges.

Define a function $f: V \rightarrow\{1,2,3, \ldots\}$ such that $f(u) \neq f(v)$ if $u v \in E$ where $V$ is the vertex set of $\mu(\mathrm{Cn})$ and $E$ is the edge set of $\mu\left(C_{n}\right)$ as follows:

Case (i): If $n=4 j, j=1,2,3, \ldots$

$$
\begin{gathered}
f\left(v_{i}\right)=\left\{\begin{array}{cc}
1, & i f i \equiv 1(\bmod 4) \operatorname{and} i \equiv 3(\bmod 4) \\
2, & i f i \equiv 2(\bmod 4) \\
3, & i f i \equiv 0(\bmod 4)
\end{array}\right. \\
f\left(u_{i}\right)= \begin{cases}4, & i f i \equiv 1(\bmod 4) \operatorname{and} i \equiv 3(\bmod 4) \\
4+\left(\frac{i}{2}\right), & i f i \equiv 2(\bmod 4) \operatorname{and} i \equiv 0(\bmod 4)\end{cases} \\
f(w)=1
\end{gathered}
$$

Case (ii): If $n=2+4 j, j=1,2,3, \ldots$

$$
\begin{aligned}
& f\left(v_{i}\right)=\left\{\begin{array}{cc}
1, & \text { ifi } \equiv 1(\bmod 4) \text { and } i \equiv 3(\bmod 4) \\
2, \quad i f i \equiv 2(\bmod 4) \text { and } i<n \\
3, \quad i f i \equiv 0(\bmod 4)
\end{array}\right. \\
& f\left(v_{n}\right)=4 \\
& f\left(u_{i}\right)= \begin{cases}5, & \text { ifi } \equiv 1(\bmod 4) \text { and } i \equiv 3(\bmod 4) \\
5+\left(\frac{i}{2}\right), & \text { ifi} \equiv 2(\bmod 4) a n d i \equiv 0(\bmod 4)\end{cases} \\
& f(w)=1
\end{aligned}
$$

By using the above pattern of coloring the Mycielski's graph of cycles $C_{n}$ for all even $n \geq 3$ is star-in-colored.
Illustration 2.3.1 Consider thecycle $C_{8}$. According to the construction of Mycielski graph $\mu\left(C_{8}\right)$ consists of 17 vertices and 32 edges.


Cycle $C_{8}$


Mycielski's graph of $C_{8}$

Figure 8. Star-in-coloring of $\mu\left(C_{8}\right)$
By using case(i) of theorem-2.3, the vertices $v_{1}, v_{3}, v_{5}, v_{7}$ and $w$ take the color 1 . The vertices $v_{2}$ and $v_{6}$ take the color 2 . The vertices $v_{4}$ and $v_{8}$ take the color 3 . The vertices $u_{1}, u_{3}, u_{5}$ and $u_{7}$ take the color 4 . The vertices $u_{2}, u_{4}, u_{6}$ and $u_{8}$ are assigned with colors $5,6,7$ and 8 respectively.

The star-in-chromatic number of $\mu\left(C_{8}\right)$ is 8 .

## S.Sudha \& V. Kanniga

Theorem2.4 The tensor product of complete-bipartite graph and a path admits star-in-coloring and its star-in-chromatic number is

$$
\chi_{\mathrm{si}}\left(\mathrm{~K}_{\mathrm{m}, \mathrm{n}} \otimes \mathrm{P}_{\mathrm{r}}\right)= \begin{cases}\min (m, n)+1, & \text { ifr }=2 \\ 2(\min (m, n))+1, & \text { ifr }>2\end{cases}
$$

Proof. Consider a complete-bipartite graph $K_{m, n}$ which consists of $m+n$ vertices denoted by $u_{1}, u_{2}, \cdots, u_{m+n}$ and $m n$ edges and the path graph $P_{r}$ which consists of $r$ vertices denoted by $v_{1}, v_{2}, \cdots, v_{r}$ and $r-1$ edges. The tensor product of $K_{m, n}$ and $P_{r}$ is obtained as per the definition6. This newly obtained graph consists of $r(m+n)$ vertices and $2 m n(r-1)$ edges.

Define a function $f: V \rightarrow\{1,2,3, \ldots\}$ such that $f(u) \neq f(v)$ if $u v \in E$ where $V$ is the vertex set of $K_{m, n} \otimes P_{r}$ and $E$ is the edge set of $K_{m, n} \otimes P_{r}$ as follows:

Case (i): If $m \leq n$
For $j \equiv 1(\bmod 4)$ and $j \equiv 2(\bmod 4)$

$$
f\left(u_{i} v_{j}\right)=\left\{\begin{array}{rr}
\quad i+1, \quad \text { for } 1 \leq i \leq \operatorname{mand} 1 \leq j \leq r \\
1, & \text { form }+1 \leq i \leq m+\text { nand } 1 \leq j \leq r
\end{array}\right.
$$

For $j \equiv 3(\bmod 4)$ and $j \equiv 0(\bmod 4)$

$$
f\left(u_{i} v_{j}\right)=\left\{\begin{array}{rr}
m+i+1, \quad \text { for } 1 \leq i \leq m a n d ~ & m \leq j \leq r \\
1, & \text { form }+1 \leq i \leq m+\text { nand } 1 \leq j \leq r
\end{array}\right.
$$

Case (ii): If $m>n$
For $j \equiv 1(\bmod 4)$ and $j \equiv 2(\bmod 4)$

$$
f\left(u_{i} v_{j}\right)=\left\{\begin{array}{cc}
1, & \text { for } 1 \leq i \leq \text { mand } 1 \leq j \leq r \\
1-m+i, & \text { form }+1 \leq i \leq m+\text { nand } 1 \leq j \leq r
\end{array}\right.
$$

For $j \equiv 3(\bmod 4)$ and $j \equiv 0(\bmod 4)$

$$
f\left(u_{i} v_{j}\right)=\left\{\begin{array}{cc}
1, & \text { for } 1 \leq i \leq \operatorname{mand} 1 \leq j \leq r \\
n-m+1+i, & \text { form }+1 \leq i \leq m+\text { nand } 1 \leq j \leq r
\end{array}\right.
$$

By using this pattern of coloring the graph is be star-in-colored.
Illustration2.4.1 Consider a complete-bipartite graph $K_{2,3}$ and a path $P_{3}$. The tensor product of $K_{2,3}$ and $P_{3}$ consists of 15 vertices and 24 edges.


By using case(i) of theorem-2.4 the vertices in $K_{2,3} \otimes P_{3}$ are assigned with colors 1,2,3,4 and 5 which satisfy the conditions of star-in-coloring.

Thus the star-in-chromatic number of $K_{2,3} \otimes P_{3}$ is 5 .
Remark: The case $m>n$ is the mirror image of case $m \leq n$.
Theorem 2.5 The tensor product of complete-bipartite graph and a cycle admits star-in-coloring and its star-in-chromatic number is given by

$$
\chi_{s i}\left(K_{m, n} \otimes C_{s}\right)=\left\{\begin{array}{c}
3(\min (m, n))+1, \quad \text { ifs } \equiv 1(\bmod 4), \quad s \equiv 2(\bmod 4), \quad s \equiv 3(\bmod 4) \\
2(\min (m, n))+1, \quad \text { ifs } \equiv 0(\bmod 4)
\end{array}\right.
$$

Proof. Consider a complete-bipartite graph $K_{m, n}$ which consists of $m+n$ vertices denoted by $u_{1}, u_{2}, \cdots, u_{m+n}$ and $m n$ edges and a cycle graph $C_{s}$ which consists of $s$ vertices denoted by $v_{1}, v_{2}, \cdots, v_{s}$ and $s$ edges. The tensor product of $K_{m, n}$ and $C_{s}$ is obtained as per the definition- 6 . This newly obtained graph consists of $s(m+n)$ vertices and $2 m n s$ edges.

Define a function $f: V \rightarrow\{1,2,3, \ldots\}$ such that $f(u) \neq f(v)$ if $u v \in E$ where $V$ is the vertex set of $K_{m, n} \otimes C_{s}$ and $E$ is the edge set of $K_{m, n} \otimes C_{s}$ as follows:

There are two cases one for $m \leq n$ and other for $m>n$.
Case (i): For $m \leq n$
If $s \equiv 1(\bmod 4)$
$f\left(u_{i} v_{j}\right)=\left\{\begin{array}{cl}1+i, & \text { if } 1 \leq i \leq m \text { with } j \equiv 0(\bmod 4) \text { and } j \equiv 1(\bmod 4), j \neq s-1 \\ m+1+i, & \quad \text { if } 1 \leq i \leq m \text { with } j \equiv 2(\bmod 4) \text { and } j \equiv 3(\bmod 4) \\ 2 m+1+i, & \text { if } 1 \leq i \leq m \text { and } j=s-1 \\ 1, & \text { if } m+1 \leq i \leq m+n \text { and for all } j\end{array}\right.$
If $s \equiv 2(\bmod 4)$
$f\left(u_{i} v_{j}\right)=\left\{\begin{array}{cl}1+i, & \text { if } 1 \leq i \leq m \text { with } j \equiv 1(\bmod 4) \text { and } j \equiv 2(\bmod 4), j<s-1 \\ m+1+i, & \quad \text { if } 1 \leq i \leq m \text { with } j \equiv 3(\bmod 4) \text { and } j \equiv 0(\bmod 4) \\ 2 m+1+i, & \text { if } 1 \leq i \leq m \text { and } j=s \text { and } j=s-1 \\ 1, & \text { if } m+1 \leq i \leq m+n \text { and for all } j\end{array}\right.$
If $s \equiv 3(\bmod 4)$

$f\left(u_{i} v_{j}\right)=\left\{\right.$| $1+i$, | if $1 \leq i \leq m$ with $j \equiv 1(\bmod 4)$ and $j \equiv 2(\bmod 4), j \neq s-1$ |
| :---: | :--- |
| $m+1+i$, | if $1 \leq i \leq m$ with $j \equiv 3(\bmod 4)$ and $j \equiv 0(\bmod 4)$ |
| $2 m+1+i$, | if $1 \leq i \leq m$ and $j=s-1$ |
| 1, | if $m+1 \leq i \leq m+n$ and for all $j$ |

If $s \equiv 0(\bmod 4)$
$f\left(u_{i} v_{j}\right)= \begin{cases}1+i, & \text { if } 1 \leq i \leq m \text { with } j \equiv 1(\bmod 4) \text { and } j \equiv 2(\bmod 4) \\ m+1+i, & \text { if } 1 \leq i \leq m \text { with } j \equiv 3(\bmod 4) \text { and } j \equiv 0(\bmod 4) \\ 1, & \text { if } m+1 \leq i \leq m+n \text { and for all } j\end{cases}$
Case (ii): For $m>n$
If $s \equiv 1(\bmod 4)$

$$
f\left(u_{i} v_{j}\right)=\left\{\begin{array}{cc}
1, & \text { if } 1 \leq i \leq m \text { and for all } j \\
1-m+i, \quad \text { if } m+1 \leq i \leq m+n \text { with } j \equiv 1(\bmod 4) \text { and } j \equiv 0(\bmod 4), j \neq s-1 \\
n-m+1+i, & \text { if } m+1 \leq i \leq m+n \text { with } j \equiv 2(\bmod 4) \text { and } j \equiv 3(\bmod 4) \\
2 n-m+1+i, & \text { if } m+1 \leq i \leq m+n \text { and } j=s-1
\end{array}\right.
$$

If $s \equiv 2(\bmod 4)$

$$
f\left(u_{i} v_{j}\right)=\left\{\begin{array}{cc}
1, & \text { if } 1 \leq i \leq m \text { and for all } j \\
1-m+i, \quad \text { if } m+1 \leq i \leq m+n \text { with } j \equiv 1(\bmod 4) \text { and } j \equiv 2(\bmod 4), j<s-1 \\
n-m+1+i, & \text { if } m+1 \leq i \leq m+n \text { with } j \equiv 3(\bmod 4) \text { and } j \equiv 0(\bmod 4) \\
2 n-m+1+i, & \text { if } m+1 \leq i \leq m+n \text { and } j=s \text { and } j=s-1
\end{array}\right.
$$

If $s \equiv 3(\bmod 4)$

$$
f\left(u_{i} v_{j}\right)=\left\{\begin{array}{c}
1, \quad \text { if } 1 \leq i \leq m \text { and for all } j \\
1-m+i, \quad \text { if } m+1 \leq i \leq m+n \text { with } j \equiv 1(\bmod 4) \text { and } j \equiv 2(\bmod 4), j \neq s-1 \\
n-m+1+i, \quad \text { if } m+1 \leq i \leq m+n \text { with } j \equiv 3(\bmod 4) \text { and } j \equiv 0(\bmod 4) \\
2 n-m+1+i,
\end{array} \quad \text { if } m+1 \leq i \leq m+n \text { and } j=s-14\right.
$$

If $s \equiv 0(\bmod 4)$
$f\left(u_{i} v_{j}\right)=\left\{\begin{array}{cc}1, & \text { if } 1 \leq i \leq m \text { and for all } j \\ 1-m+i, & \text { if } m+1 \leq i \leq m+n \text { with } j \equiv 1(\bmod 4) \text { and } j \equiv 2(\bmod 4) \\ n-m+1+i, & \text { if } m+1 \leq i \leq m+n \text { with } j \equiv 3(\bmod 4) \operatorname{and} j \equiv 0(\bmod 4)\end{array}\right.$
With this pattern of coloring, the tensor product of $K_{m, n}$ and $C_{s}$ can be star-in-colored.
Illustration 2.5.1 Consider a complete-bipartite graph $K_{2,3}$ and a cycle $C_{3}$. The tensor product of $K_{2,3}$ and $C_{3}$ consists of 15 vertices and 36 edges.


Figure 12. Star-in-coloring of $K_{2,3} \otimes C_{5}$

By using case(i) of theorem- 2.5 the vertices in $K_{2,3} \otimes C_{3}$ are assigned with colors $1,2,3,4,5,6$ and 7 which satisfy the conditions of star-in-coloring.

The star-in-chromatic number of $K_{2,3} \otimes C_{3}$ is 7 .

## 3. CONCLUSION

In this paper, we have proved that the following graphs are star-in-colored with orientation by giving the general pattern of coloring and their star-in-chromatic number is also found:
(i) the star-in-chromatic number of $S\left(K_{m, n}\right)$ is $2(\min (m, n))+1$
(ii) the star-in-chromatic number of $\mu\left(P_{n}\right)$ is $\frac{1}{2}(n+7)$
(iii) the star-in-chromatic number of $\mu\left(C_{n}\right)$ is $2(2+j)$ if $n=4 j$ and $2(3+j)$ if $n=2+4 j$ where $j=1,2,3, \ldots$
(iv) the star-in-chromatic number of $K_{m, n} \otimes P_{r}$ is $\min (m, n)+1$ if $r=2$ and $2(\min (m, n))+$ 1if $r>2$
(v) the star-in-chromatic number of $K_{m, n} \otimes C_{s}$ is $2(\min (m, n))+1$ if $s \equiv 0(\bmod 4)$ and $3(\min (\operatorname{m}, n))+1$ if $s \equiv 1(\bmod 4)$ or $s \equiv 2(\bmod 4)$ or $s \equiv 3(\bmod 4)$.

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