

Harmonic Progression on Huge Networks – A Scientific Approach with Computational Study on Case (I)

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Abstract: *The Paper is mainly intended to explain whether Harmonic Progression (H.P) in a special case will support a network or not. A huge network is considered in a systematic way with 94 nodes and 124 activities. H.P is employed on a most likely time estimate among the three time estimates namely optimistic, most likely and pessimistic. Systematic Analytical study is carried out on the considered network. Some results are obtained. All float values are also derived. Critical path is identified and project analysis has been carried out with standard normal distribution curves which are illustrated whenever necessary.*

Keywords: *Network, Time estimates, Float, Critical path, Normal distribution.*

AMS Classification: **90-08, 90B10, 90C90**

1. INTRODUCTION

In Operations Research, many problems are not aimed only to get the solution. They are intended to get the solutions with the certainty and consistency of the relative situations. The corresponding situation mainly depends upon the generation of the problem, Collection of the required and possible information, treatment of the problem at various phases and limitations. The perfect implementation of scientific techniques will enable us to maintain the effort correctly to reach the goal. In the research point of view, many problems in OR are very interesting to evolve a proper and precise optimum solution with exciting challenges.

Levin and Kirkpatrick [9] studied about planning and control with the aid of PERT and CPM in 1966. Wiest and Levy [10] opened new eras with management guide to PERT/CPM for research community of operations research. PERT algorithm was discussed with various models by Billy E.Gillett [11] in 1979. S.D Sharma [12] classified clearly the application area of PERT&CPM techniques and also specified advantages and limitations of network techniques .The present authors [1-5] investigated some peculiar cases of Game theory problems earlier. Now, K.V.L.N.Acharyulu et.al. concentrated on Networks which are influenced by various progressions.

In this paper the authors aimed to investigate whether harmonic arithmetic progression in a peculiar case will support a network or not. The harmonic progression is taken on most likely time estimate among the three time estimations. Periodical analysis is also done. Total Float, Free float and Independent Float are calculated for more accuracy of getting critical path. The standard normal distribution curves are illustrated with the help of Mat Lab wherever feasible and necessary.

2. BASIC CONSTRUCTION OF NETWORK

A network is constructed with 94 nodes and 124 activities in a systematic way for studying the influence of **Harmonic Progression**. H.P is employed on most likely time estimate (m) in case (I) among the three estimates. No Dummy activity is considered. Moreover, no error in Network is involved.

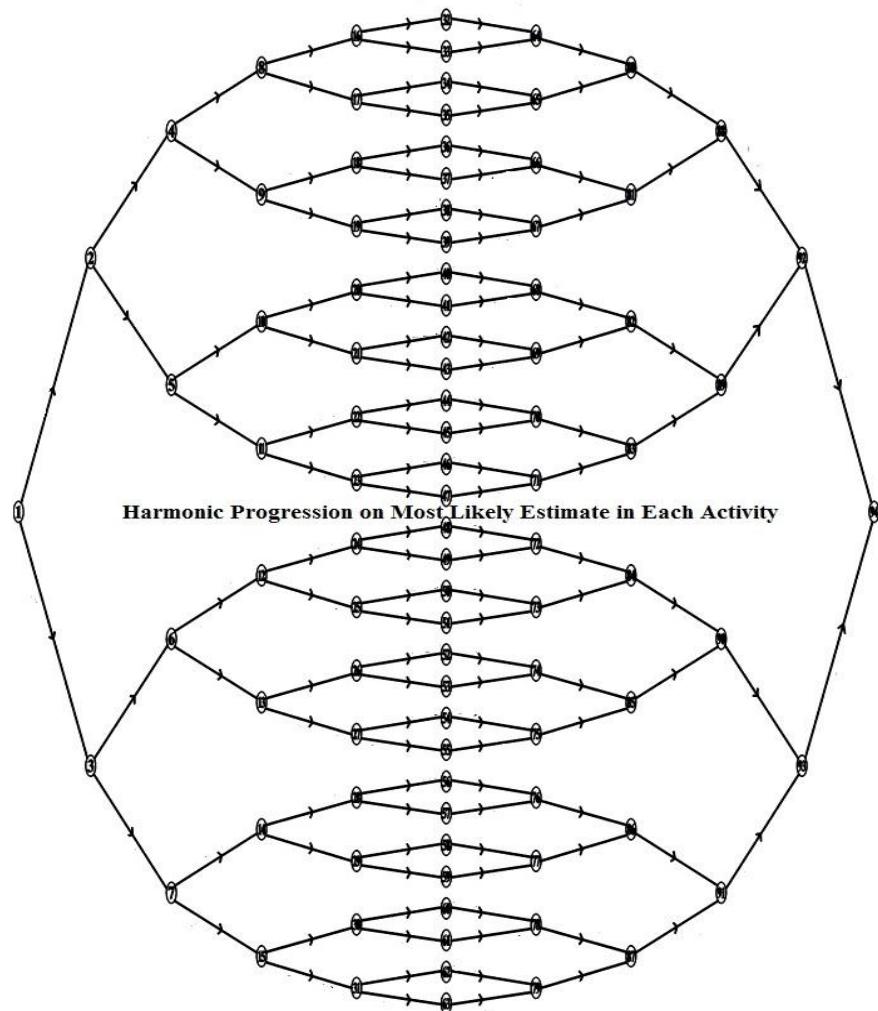


Fig.1. constructed Network with 94 nodes and 124 activities

3. PRELIMINARIES AND NOTATIONS

(i).**TE**= Earliest expected completion time of event (TE)

Def: For the fixed value of $j=TE(j)=\text{Max}[TE(i)+ET(i,j)]$ which ranges over all activities from i-j.

(ii).**TL**= Latest allowable event completion time (TL)

Def: For the fixed value of $i=TL(i)=\text{Min}[TL(j)+ET(i,j)]$ which ranges over all activities from i-j.

(iii).**ET**= Expected completion time of activity (I,J)

(iv). **a**= Optimistic time estimate

(v). **m**= Most likely time estimate

(vi). **b**= Pessimistic time estimate

(vii). **ES** = Earliest start of an activity

(viii). **EF** = Earliest finish of an activity

(ix). **LS** = Latest start of an activity

(x). **LF** = Latest finish of an activity

(xi). **TF** = Total Float

Def: TF of activity i-j = LF_{i,j}-EF_{i,j} (or) LS_{i,j}-ES_{i,j}

(xii).**FF** = Free Float

Def: FF of activity i-j = TF - (TL-TE) of node j

(xiii).**IF** = Independent Float

Def: IF of activity i-j = FF - (TL-TE) of node i

(xiv).**SE**=Slack event time

(xv).**CPI**=Critical Path Indicator

(xvi).**SCT**= Scheduled Time

(xvii). σ = Standard deviation of project length

4. MATERIAL AND METHODS

Step 1: Draw the project network completion time

Step 2: Compute the expected duration of each activity by using the formula $ET = \frac{a + 4m + b}{6}$

from the time estimates a,m and p. Also calculate the expected variance. σ^2 of each activity

Step 3: Calculate TE, TL

Step 4: Find Total Float, Free Float and Independent Float

Step 5: Find the critical path and identify the critical activities

Step 6: Compute project length which is a square root to sum of variance of all the critical activities.

Step 7: From the standard normal variable $z = \frac{SCT - ET}{\sigma}$, Where SCT is scheduled

Completion time of event, σ =standard deviation of project length.Using the standard normal curve, we can estimate the probability of completing project within specified time.

5. RESULTS

By applying CPM and PERT algorithm on the Network, the critical path is obtained from Table-1 which is built with all Activities, Time estimates, ET, Variance, ES, EF, LS, LF and all Float values. The Critical path indicator furnishes the critical Activities which are shown in the Table-1.

Table-1

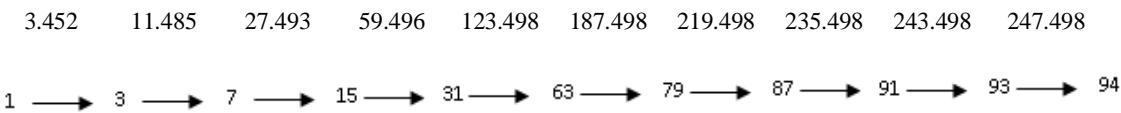
Activity	Time Estimates			σ^2	Earliest[E]		Latest[L]		TF	FF	IF	CPI
	a	m	b		ES	EF	LS	LF				
1--2	1	1.333	2	1.388	0.027	0	1.388	124.074	125.462	124.074	0	0
1-3	3	3.428	4	3.452	0.027	0	3.452	0	3.452	0	0	*
2--4	5	5.454	6	5.469	0.027	1.388	6.857	185.476	190.945	184.088	0	-124.074
2--5	7	7.466	8	7.477	0.027	1.388	8.865	125.462	132.939	124.074	0	-124.074
3--6	9	9.473	10	9.482	0.027	3.452	12.934	63.456	72.938	60.004	0	0
3--7	11	11.478	12	11.485	0.027	3.452	14.937	3.452	14.937	0	0	*
4--8	13	13.481	14	13.487	0.027	6.857	20.344	218.946	232.433	212.089	0	-184.088
4--9	15	15.483	16	15.488	0.027	6.857	22.345	190.945	206.433	184.088	0	-184.088
5--10	17	17.485	18	17.49	0.027	8.865	26.355	160.941	178.431	152.076	0	-124.074
5--11	19	19.487	20	19.491	0.027	8.865	28.356	132.939	152.43	124.074	0	-124.074
6--12	21	21.488	22	21.492	0.027	12.934	34.426	100.938	122.43	88.004	0	-60.004
6--13	23	23.489	24	23.492	0.027	12.934	36.426	72.938	96.43	60.004	0	-60.004

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		8										
41-68	143	143. ⁴⁹ ₈	144	143.498	0.027	143.347	286.845	307.424	450.922	164.077	0	-164.077
42-69	145	145. ⁴⁹ ₈	146	145.498	0.027	147.347	292.845	303.424	448.922	156.077	4	-152.077
43-69	147	147. ⁴⁹ ₈	148	147.498	0.027	149.347	296.845	301.424	448.922	152.077	0	-152.077
44-70	149	149. ⁴⁹ ₈	150	149.498	0.027	155.349	304.847	295.424	444.922	140.075	4	-136.075
45-70	151	151. ⁴⁹ ₈	152	151.498	0.027	157.349	308.847	293.424	444.922	136.075	0	-136.075
46-71	153	153. ⁴⁹ ₈	154	153.498	0.027	161.35	314.848	289.424	442.922	128.074	4	-124.074
47-71	155	155. ⁴⁹ ₈	156	155.498	0.027	163.35	318.848	287.424	442.922	124.074	0	-124.074
48-72	157	157. ⁴⁹ ₈	158	157.498	0.027	173.42	330.918	277.424	434.922	104.004	4	-100.004
49-72	159	159. ⁴⁹ ₈	160	159.498	0.027	175.42	334.918	275.424	434.922	100.004	0	-100.004
50-73	161	161. ⁴⁹ ₈	162	161.498	0.027	179.42	340.918	271.424	432.922	92.004	4	-88.004
51-73	163	163. ⁴⁹ ₈	164	163.498	0.027	181.42	344.918	269.424	432.922	88.004	0	-88.004
52-74	165	165. ⁴⁹ ₈	166	165.498	0.027	187.42	352.918	263.424	428.922	76.004	4	-72.004
53-74	167	167. ⁴⁹ ₈	168	167.498	0.027	189.42	356.918	261.424	428.922	72.004	0	-72.004
54-75	169	169. ⁴⁹ ₈	170	169.498	0.027	193.42	362.918	257.424	426.922	64.004	4	-60.004
55-75	171	171. ⁴⁹ ₈	172	171.498	0.027	195.42	366.918	255.424	426.922	60.004	0	-60.004
56-76	173	173. ⁴⁹ ₈	174	173.498	0.027	203.424	376.922	247.424	420.922	44	4	-40
57-76	175	175. ⁴⁹ ₈	176	175.498	0.027	205.424	380.922	245.424	420.922	40	0	-40
58-77	177	177. ⁴⁹ ₈	178	177.498	0.027	209.424	386.922	241.424	418.922	32	4	-28
59-77	179	179. ⁴⁹ ₈	180	179.498	0.027	211.424	390.922	239.424	418.922	28	0	-28
60-78	181	181. ⁴⁹ ₈	182	181.498	0.027	217.424	398.922	233.424	414.922	16	4	-12
61-78	183	183. ⁴⁹ ₈	184	183.498	0.027	219.424	402.922	231.424	414.922	12	0	-12
62-79	185	185. ⁴⁹ ₈	186	185.498	0.027	223.424	408.922	227.424	412.922	4	0	-4
63-79	187	187. ⁴⁹ ₈	188	187.498	0.027	225.424	412.922	225.424	412.922	0	0	0
64-80	189	189. ⁴⁹ ₈	190	189.498	0.027	240.833	430.331	464.922	654.42	224.089	12	-212.089
65-80	191	191. ⁴⁹ ₈	192	191.498	0.027	250.833	442.331	462.922	654.42	212.089	0	-212.089
66-81	193	193. ⁴⁹ ₈	194	193.498	0.027	262.834	456.332	458.922	652.42	196.088	12	-184.088
67-81	195	195. ⁴⁹ ₈	196	195.498	0.027	272.834	468.332	456.922	652.42	184.088	0	-184.088
68-82	197	197. ⁴⁹ ₈	198	197.498	0.027	286.845	484.343	450.922	648.42	164.077	12	-152.077
69-82	199	199. ⁴⁹ ₈	200	199.498	0.027	296.845	496.343	448.922	648.42	152.077	0	-152.077
70-83	201	201. ⁴⁹ ₈	202	201.498	0.027	308.847	510.345	444.922	646.42	136.075	12.0 01	-124.074
71-83	203	203. ⁴⁹ ₈	204	203.498	0.027	318.848	522.346	442.922	646.42	124.074	0	-124.074
72-84	205	205. ⁴⁹ ₈	206	205.498	0.027	334.918	540.416	434.922	640.42	100.004	12	-88.004
73-84	207	207. ⁴⁹ ₈	208	207.498	0.027	344.918	552.416	432.922	640.42	88.004	0	-88.004
74-85	209	209. ⁴⁹ ₈	210	209.498	0.027	356.918	566.416	428.922	638.42	72.004	12	-60.004
75-85	211	211. ⁴⁹ ₈	212	211.498	0.027	366.918	578.416	426.922	638.42	60.004	0	-60.004
76-86	213	213. ⁴⁹ ₈	214	213.498	0.027	380.922	594.42	420.922	634.42	40	12	-28
77-86	215	215. ⁴⁹ ₈	216	215.498	0.027	390.922	606.42	418.922	634.42	28	0	-28
78-87	217	217. ⁴⁹ ₈	218	217.498	0.027	402.922	620.42	414.922	632.42	12	12	0
79-87	219	219. ⁴⁹ ₈	220	219.498	0.027	412.922	632.42	412.922	632.42	0	0	*
80-88	221	221. ⁴⁹ ₈	222	221.498	0.027	442.331	663.829	654.42	875.918	212.089	28.0 01	-184.088
81-88	223	223. ⁴⁹ ₈	224	223.498	0.027	468.332	691.83	652.42	875.918	184.088	0	-184.088
82-89	225	225. ⁴⁹ ₈	226	225.498	0.027	496.343	721.841	648.42	873.918	152.077	28.0 03	-124.074

83-89	227	227.49 8	228	227.498	0.027	522.346	749.844	646.42	873.918	124.074	0	-124.074	
84-90	229	229.49 8	230	229.498	0.027	552.416	781.914	640.42	869.918	88.004	28	-60.004	
85-90	231	231.49 8	232	231.498	0.027	578.416	809.914	638.42	869.918	60.004	0	-60.004	
86-91	233	233.49 8	234	233.498	0.027	606.42	839.918	634.42	867.918	28	28	0	
87-91	235	235.49 8	236	235.498	0.027	632.42	867.918	632.42	867.918	0	0	0	*
88-92	237	237.49 8	238	237.498	0.027	691.83	929.328	875.918	1113.416	184.088	60.0 14	-124.074	
89-92	239	239.49 8	240	239.498	0.027	749.844	989.342	873.918	1113.416	124.074	0	-124.074	
90-93	241	241.49 8	242	241.498	0.027	809.914	1051.412	869.918	1111.416	60.004	60.0 04	0	
91-93	243	243.49 8	244	243.498	0.027	867.918	1111.416	867.918	1111.416	0	0	0	*
92-94	245	245.49 8	246	245.498	0.027	989.342	1234.84	1113.416	1358.914	124.074	124. 074	0	
93-94	247	247.49 8	248	247.498	0.027	1111.416	1358.914	1111.416	1358.914	0	0	0	*

Critical path is obtained as below



Project Length is defined as $\sqrt{\text{Sum of Variances of each Critical activity}}$

$$\text{i.e Project Length} = \sqrt{0.027 + 0.027 + 0.027 + 0.027 + 0.027 + 0.027 + 0.027} = 0.4647$$

The values of TE, TL and SE corresponding to every node are given in table (2).

The slack event time may be positive, negative or zero.

It is also observed that the values of slack event time vanish at each critical activity.

Slack event time is defined as the amount of time in which the event can be retarded without involving the scheduled completion time for the project. Any activity on the critical path necessitates time in excess of its expected completion time and detains the project completion consequently.

Table 2.

S.No	Node	TE	TL	SE	S.no	Node	TE	TL	SE
1	1	0	0	0	48	48	173.42	277.424	104.004
2	2	1.388	125.462	124.074	49	49	175.42	275.424	100.004
3	3	3.452	3.452	0	50	50	179.42	271.424	92.004
4	4	6.857	190.945	184.088	51	51	181.42	269.424	88.004
5	5	8.865	132.939	124.074	52	52	187.42	263.424	76.004
6	6	12.934	72.938	60.004	53	53	189.42	261.424	72.004
7	7	14.937	14.937	0	54	54	193.42	257.424	64.004
8	8	20.344	232.433	212.089	55	55	195.42	255.424	60.004
9	9	22.345	206.433	184.088	56	56	203.424	247.424	44
10	10	26.355	178.431	152.076	57	57	205.424	245.424	40
11	11	28.356	152.43	124.074	58	58	209.424	241.424	32
12	12	34.426	122.43	88.004	59	59	211.424	239.424	28
13	13	36.426	96.43	60.004	60	60	217.424	233.424	16
14	14	40.43	68.43	28	61	61	219.424	231.424	12
15	15	42.43	42.43	0	62	62	223.424	227.424	4
16	16	49.838	273.927	224.089	63	63	225.424	225.424	0
17	17	51.838	263.927	212.089	64	64	240.833	464.922	224.089

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18	18	55.839	251.927	196.088	65	65	250.833	462.922	212.089
19	19	57.839	241.927	184.088	66	66	262.834	458.922	196.088
20	20	63.85	227.927	164.077	67	67	272.834	456.922	184.088
21	21	65.85	217.926	152.076	68	68	286.845	450.922	164.077
22	22	69.851	205.926	136.075	69	69	296.845	448.922	152.077
23	23	71.852	195.926	124.074	70	70	308.847	444.922	136.075
24	24	79.922	179.926	100.004	71	71	334.918	442.922	108.004
25	25	81.922	169.926	88.004	72	72	334.918	434.922	100.004
26	26	85.922	157.926	72.004	73	73	344.918	432.922	88.004
27	27	87.922	147.926	60.004	74	74	356.918	428.922	72.004
28	28	93.926	133.926	40	75	75	366.918	426.922	60.004
29	29	95.926	123.926	28	76	76	380.922	420.922	40
30	30	99.926	111.926	12	77	77	390.922	418.922	28
31	31	101.926	101.926	0	78	78	402.922	414.922	12
32	32	111.334	339.424	228.09	79	79	412.922	412.922	0
33	33	113.335	337.424	224.089	80	80	442.331	654.42	212.089
34	34	117.335	333.424	216.089	81	81	468.332	652.42	184.088
35	35	119.335	331.424	212.089	82	82	496.343	648.42	152.077
36	36	125.336	325.424	200.088	83	83	522.346	646.42	124.074
37	37	127.336	323.424	196.088	84	84	552.416	640.42	88.004
38	38	131.336	319.424	188.088	85	85	578.416	638.42	60.004
39	39	133.336	317.424	184.088	86	86	606.42	634.42	28
40	40	141.347	309.424	168.077	87	87	632.42	632.42	0
41	41	143.347	307.424	164.077	88	88	691.83	875.918	184.088
42	42	147.347	303.424	156.077	89	89	749.844	873.918	124.074
43	43	149.347	301.424	152.077	90	90	809.914	869.918	60.004
44	44	155.349	295.424	140.075	91	91	867.918	867.918	0
45	45	157.349	293.424	136.075	92	92	989.342	1113.416	124.074
46	46	161.35	289.424	128.074	93	93	1111.416	1111.416	0
47	47	163.35	287.424	124.074	94	94	1358.914	1358.914	0

6. PERIODICAL ANALYSIS

Periodical analysis is carried out with specific schedule times and the standard normal variables are identified in the entire range of probability from 0 to 1. The percentage of possibilities of completion of the Project are derived and given in the following Table-3. Pictorial representations are also given.

Table-3.

SCT	ETC	PL	sct-etc	z	p	per
1356	1358.914	0.5196	-2.914	-5.60816	0	0
1357	1358.914	0.5196	-1.914	-3.6836	0.00012	0.012
1358	1358.914	0.5196	-0.914	-1.75905	0.04006	4.006
1359	1358.914	0.5196	0.086	0.165512	0.56356	56.356
1360	1358.914	0.5196	1.086	2.090069	0.98169	98.169
1361	1358.914	0.5196	2.086	4.014627	1	100

The Obtained Standard Normal Curves are illustrated from Fig.2-Fig.6

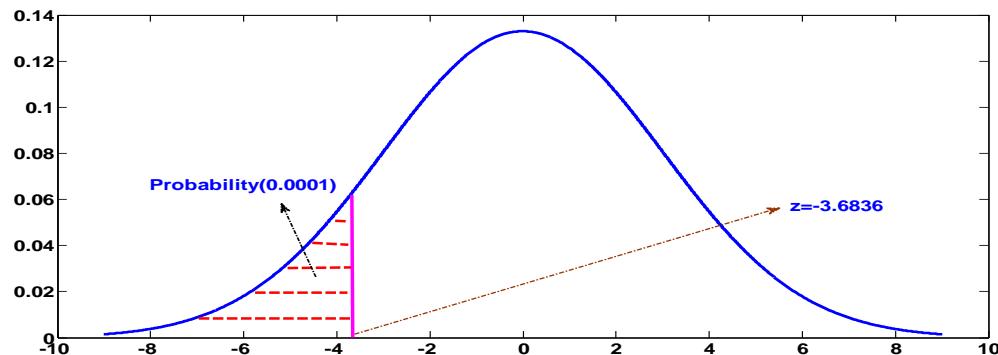


Fig.2

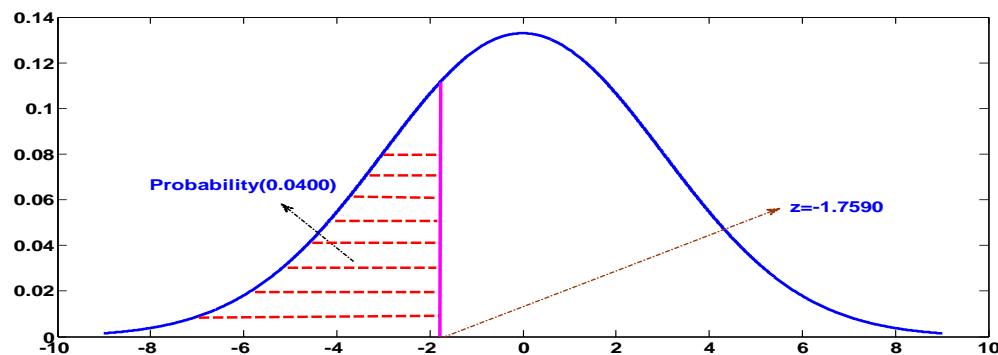


Fig.3

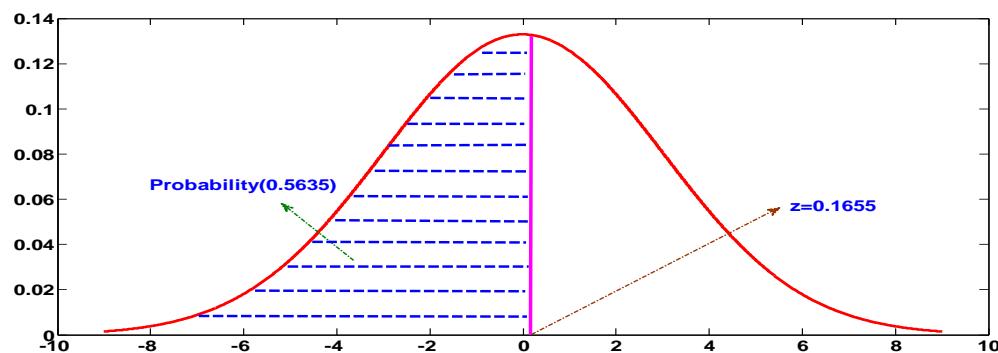


Fig.4

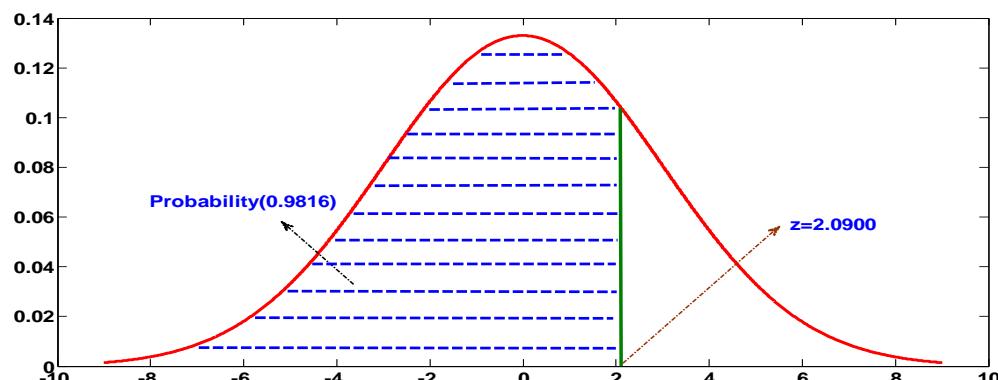


Fig.5

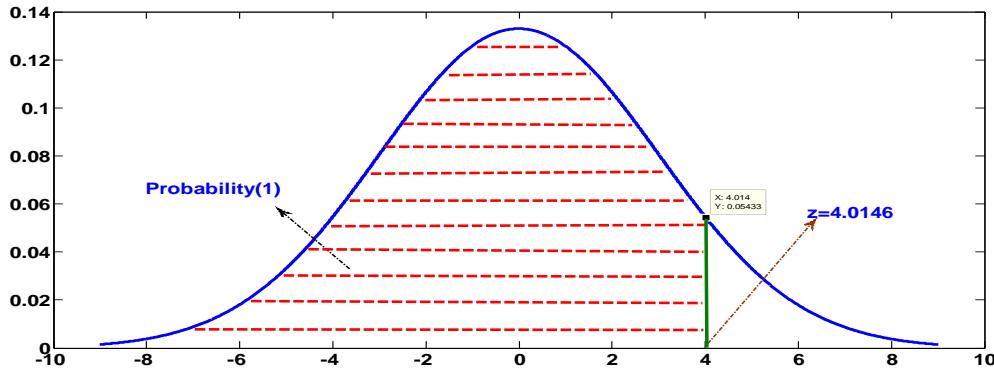


Fig.6

7. CONCLUSIONS

The following results are established from this scientific computational Study.

- (i).H.P supports systematically the Network even though the network has large size.
- (ii).Constant Variances are observed in any activity of the Network.
- (iii).In Critical Path
 - (a).All Total Float values of Critical activities are traced as zeros..
 - (b).The value of Slack event of each node in critical path is Zero
 - (c). TE and TL are same at each node in critical path.
- (iv).In the increased data of Net work in which H.P is employed on most likely time estimate, the expected completion time of successive activity is gradually enhanced.
- (v).The classifications of the impact of H.P in the Network are as follows
 - (a).H.P supports accurately only when SCT is greater than ET.
 - (b).H.P does not support effectively when SCT is less than or equal to ET.
 - (c). Standard Normal Distribution curves illustrate the percentage of possibilities of the Project.

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