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Signal Detection using Geodesic Projecting onto Background Submanifold

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Abstract: *The problem of detection by small-pulse data in signals containing large amounts of noise has been the focus of recent research. Recently, several detectors have been proposed to find the geometric mean over a positive Hermitian matrix manifold and to use the Riemannianian distance between the detection cells and various types of divergence as dissimilarity measures between noise and target. These detectors improve detection performance compared to conventional CFAR methods, but the lack of geometric mean or median reflects the overall noise background limits the detection performance to be degraded when the noise signal level is high or the false alarm rate is low. This paper proposed new detector called Geodetic Mapping (BSMP) detector to the background submanifold that constructs a submanifold reflecting the noise background and uses the geodetic projection distance to this submanifold as a dissimilarity measure between the background and the target, thus further improving the detection performance.*

Keywords: *Geodesic projection, Matrix divergence, Positive definition matrix, Riemannianian metric, target detection*

1. Introduction

The performance of signal detection depends on the use of the distance between two points in the signal model.[8] The canonical constant false alarm rate (CFAR) detector has the limitation that the detection performance decreases with decreasing number of pulse branches and increasing noise intensity.[7,10]

In [4,5,6,13,14], received signals are modeled as a Hermitian positive matrix manifold, the arithmetic mean of the reference signals used in CFAR is replaced by the Riemannianian mean, and the distance between the two signals is proposed by detectors using Riemannian distance instead of Euclidean distance to improve the detection performance.

Improved detectors use the method of finding the Riemannian mean in the received signals modeled as a positive Hermitian matrix manifold and separating targets using either the Riemannian distance between the detection cell or the various types of matrix divergence.[11,12]

This method results in a false target detection when the detection cell is somewhat distant, even if the same noise is greater than the threshold, with a distance of the mean or median greater than the threshold. However, when all the same noises constitute a background submanifold that can be considered as an element of the background manifold, the projection distance is zero because the detection cell is its element when projected onto the submanifold in the case of noise, and in the case of the target, the projection distance is not zero because the point of the background submanifold is not. Thus, a good construction of the submanifold that consists of noisy signals can significantly improve the target detection performance.

A Riemannianian metric structure derived from convex functions on differential manifolds is introduced, and a positive Hermitian matrix manifold with respect to Riemannianian metric by convex functions defined in affine coordinates is found to be a dually flat manifold.[1,2,3]

We construct a background submanifold consisting of noisy signals on a positive Hermitian matrix manifold and propose a new detection idea to perform target detection using the projection onto the background submanifold (BSMP) using Bregman divergence by a matrix convex function that gives a self-dual flat manifold structure.

The results of the comparison of the detection performance of the CFAR detector and the total square loss (TSL) detector[4] demonstrate that the proposed detection method improves the detection performance.

The rest of this paper is organized as follows.

In Section 2, we construct the background submanifold that the noise signals constitute. Section 3 finds the projection points from the target signal to the background manifold by Lagrangian optimization with equality constraints and constructs the detection algorithm. Section 4 presents the simulation experiments on the proposed detector and compares its performance with previous detectors. Section 5 concludes the paper.

2. BSMP DETECTOR MODEL AND BACKGROUND SUBMANIFOLD CONFIGURATION

Conventional detection methods using CFAR have limitations that make the reference set with a small pulse branch, affecting resolution and degrading detection performance.[9]

The full-Bregman divergence (TBD) detectors[4], which use a matrix constant false alarm rate (MCFAR) detector[5] or a variety of types of divergence, reflecting information geometry, do not directly use signal data to calculate the covariance matrix of the reflected signal data and detect the target using either the Riemannian distance between the detection signal and the reference cell average or the detection signal [4,5,6], but have a significant difference from the noisy and target signals. Hence, in case of constructing any submanifold that reflects the noise signal propagation, if the detected signal is noisy, then the projection distance to this submanifold will be zero, and if the target signal is a target, the geodesic projection distance will not be zero. Hence, we propose a new detector that constructs a background submanifold consisting of noisy signals and detects by projecting the detected signal onto this submanifold to calculate the projection point and the geometric distance between the detected signal. Proposed detection model is as follows.

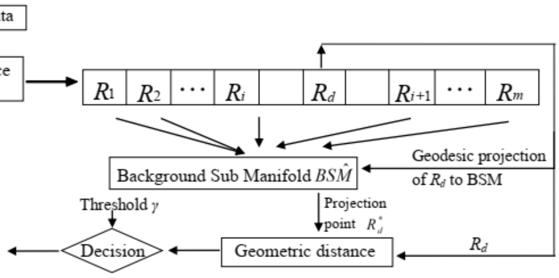

Fig1. A *detector model (BSMP) mapping to a background submanifold*

Let $Z=[z_1,z_2\cdots z_n]^T$ be a sequence of signal samples received from the radar. This sample data $Z=[z_1,z_2\cdots z_n]^T$ is assumed to follow a complex multivariate Gaussian distribution with zero mean. That is, $z\sim CN(0,R)$ and

$$p(z\,|\,R)=\frac{1}{\pi^n\det(R)}\exp\{-z^H R^{-1}z\} \tag{1}$$

Here, the matrix R is a positive Hermitian matrix as follows :

$$R=[zz^H]=\begin{bmatrix} r_0 & \bar{r}_1 & \cdots & \bar{r}_{n-1} \\ r_1 & r_0 & \cdots & \bar{r}_{n-2} \\ \vdots & \ddots & \ddots & \vdots \\ r_{n-1} & \cdots & r_1 & r_0 \end{bmatrix} \quad r_k=[z_i z_{i+k}],0\le k\le n-1,0\le i\le n-1$$

where $\bar{r}_i$ denotes the conjugation of r_i and r_k is the correlation coefficient of the sample data.

If the sample data $\boldsymbol{z}$ lies in a linear space, R becomes an element of a positive Hermitian matrix manifold $PH(n)$.[6] This will allow target detection on the Riemannianian manifold.

Let $\{R_1,R_2,...,R_m\}\subset PH(n)$ be the set obtained from the received signals. Then, we can classify the reference cells around the detection cell into groups that can be considered to be composed only of relatively noisy signals and those that can be discarded if their Frobenius norm $\|R_i\|_F, i=1,...,m$ is found and expressed on the real axis.

[Definition] A class of clusters that can be considered to be composed only of noisy signals is called background submanifold and denoted by $BSM=\{R_k\}_{k=1,...,m-l}$, where l is the number of signals to be excluded.

The clustering algorithm is as follows.

Step 1. Calculate the Frobenius norm of reference cells and store the set as $\{d_1,...,d_k\}$.

Step 2. Align the stored set $\{d_1,...,d_k\}$. Then $i=k/2$

Step 3. Suppose that more than half of the reference cells are elements of the background submanifold and calculate the mean and variance.

$$\gamma_{\max} = d_i$$

$$\bar{d} = mean\{d_i, \ldots, d_k\}$$

$$\sigma^2 = \frac{1}{k-i}\sum_{l=i}^{k}(d_l - \bar{d})^2$$

Step 4. $d_{k+1} < \bar{d} + 3\sigma, i = i+1$ returns to Step 3.

3. SIGNAL DETECTION BY GEODESIC PROJECTION

When $f(X)$ is a differentiable convex function defined on a positive Hermitian matrix manifold, the Bregman divergence between two positive Hermitian matrices X and Y is defined as follows.[6]

$$D(X,Y) = tr(f(\mathbf{X})) - tr(f(\mathbf{Y})) - tr(f'(\mathbf{Y})(\mathbf{X}-\mathbf{Y})) \tag{2}$$

When $f(X), X \in PH(n)$ is a differentiable convex function defined on a positive Hermitian matrix manifold, its gradient $X^* = \nabla f(X)$ is equal to the normal vector normal to the tangent space at the point X of a positive Hermitian matrix manifold.

Since different points of a positive Hermitian matrix manifold correspond to different normal vectors, this normal vector gives another coordinate system to a positive Hermitian matrix manifold. This coordinate system is called a dual coordinate system. Two coordinate systems X and X^* have the Legendre transformation relation.[3]

Defining

$$f^*(X^*) = X \cdot X^* - f(X) \tag{3}$$

as a dual convex function in the dual coordinates X^*, we have

$$\nabla f^*(X^*) = X + \frac{\partial X}{\partial X^*}X^* - \nabla f(X)\frac{\partial X}{\partial X^*} \tag{4}$$

Let $f(X) = tr(X^2)$ be a matrix convex function,and then $tr(f(\mathbf{X})) = tr(\mathbf{X}^2)$, $f'(\mathbf{Y}) = 2\mathbf{Y}$ is established. So the Bregman Matrix divergence and the dual Bregman Matrix divergence [4] denoted as following:

$$D(\boldsymbol{X},\boldsymbol{Y}) = tr(\boldsymbol{X}^2 - \boldsymbol{Y}^2 - 2\boldsymbol{Y}(\boldsymbol{X}-\boldsymbol{Y})) \tag{5}$$

$$D^*(\boldsymbol{X}^*,\boldsymbol{Y}^*) = tr(\boldsymbol{Y}^2 - \boldsymbol{X}^2 - 2\boldsymbol{X}(\boldsymbol{Y}-\boldsymbol{X})) \tag{6}$$

On the other hand, it can be seen from [2,3] that for a matrix Bregman divergence $D(\boldsymbol{X},\boldsymbol{Y})$ and a dual divergence $D^*(\boldsymbol{X},\boldsymbol{Y})$ by a matrix convex function $f(X)$, $D(\boldsymbol{X},\boldsymbol{Y}) = D^*(Y,X)$ holds. We also know that the divergence is similar to the square of the distance and that the generalized Pythagorean theorem and projection theorem on positive Hermitian matrix manifolds hold.[1,2] In particular, if the matrix convex function is $f(X) = tr(X^2)$,

then $g_{ij} = \delta_{ij} = \begin{cases} 1, i = j \\ 0, i \neq j \end{cases}$ is, and the Bregman divergence and the dual Bregman divergence due to this convex function are the same, and hence the self-dual flat manifold structure is introduced [3].

In the sense of divergence, the problem of finding the point of a submanifold BSM closest to a point R of a manifold has many applications. Hence we find the geodesic projection from point $R \in PH(n)$ to the background submanifold $BSM = \{R_k\}_{k=1,\dots,m-l}$.

Let $R \in HPD$ be a test cell and $BSM = \{R_k\}_{k=1,\dots,m-l}$ a background submanifold constructed in Step 2.

[Theorem] The geodesic projection point $\hat{R}_M \in BSM \subset PH(n)$ into the background submanifold BSM of the test cell $R_D \in PH(n)$ is

$$\hat{R}_M = \frac{R_D}{1 + \lambda\sqrt{1 + \| 2R_D \|_F^2}} \quad (7)$$

, where $\lambda = \dfrac{\dfrac{\| R_D \|_F}{\gamma_{\max}} - 1}{\sqrt{1 + \| 2R_D \|_F^2}}, \lambda > 0$.

Proof . Define a function

$$F(R) = D(R, R_D) + \lambda(\| R \|_F^2 - \gamma_{\max}^2), R \in BSM \quad (8)$$

by the Bregman divergence (5) ,where $\lambda > 0$ and find the point at which it gives its minimization, i.e.,

$$\hat{R}_M = \arg\min_{R \in BSM} F(R) \quad (9)$$

This is the solution of the following.

$$\begin{cases} \dfrac{\partial F(R,\lambda)}{\partial R} = 0 \\ \dfrac{\partial F(R,\lambda)}{\partial \lambda} = 0 \end{cases}, R \in M, \lambda \in K^{+} \tag{10}$$

Solving the first expression yields

$$R = \frac{R_D}{1 + \lambda\sqrt{1 + \| 2R_D \|_F^2}} \tag{11}$$

Also, solving the second equation yields

$$\| R \|_F^2 - \gamma_{\max}^2 = 0 \tag{12}$$

so

$$\left\| \frac{R_D}{1 + \lambda\sqrt{1 + \| 2R_D \|_F^2}} \right\|^2 = \lambda_{\max}^2 \tag{13}$$

Holds. hence

$$\lambda = \frac{\dfrac{\| R_D \|_F}{\gamma_{\max}} - 1}{\sqrt{1 + \| 2R_D \|_F^2}} \tag{14}$$

Since $\| R_D \|_F > \gamma_{\max}$ is, we have $\lambda > 0$. Substituting Eq. (15) into Eq. (12), we obtain the following result.

$$\hat{R}_M = \frac{R_D}{1 + \lambda\sqrt{1 + \| 2R_D \|_F^2}} \tag{15}$$

This is the projection point to be found. □

Using this projection point and the geometric distance from the test cell, a new detector is designed.

4. Simulation Experiment

Numerical experiments are carried out to analyze the performance of the proposed detector.

We analyze the performance of the proposed detector through computational simulations of the TSL detector proposed by [4], EMean detector proposed by [7], and BSMP detector proposed by us. In the simulation, we simulate the single-station signal by Matlab Phased Array System Toolbox with the parameters listed in Table 1 and the target parameters listed in Table 2.

Table1. *Radar Parameters.*

operating frequency	10^{10}Hz
Sampling rate	5.9958×10^{6}Hz
Pulse width	3.3356×10^{-7}s
Pulse repetition frequency	2.9979×10^{4}Hz
Noise Figure	3db
Receiver gain	20db
Transmitter gain	20db
Loss factor	10db
Pulse number	5
Peak transmitter power	3305watt

Table2. *Target Parameters.*

parameters	**Target1**	**Target2**	**Target3**
speed	0	0	0
cross section	1	1	1
distance	2509	2710	4504

With the design parameters, the maximum range is 5000 m, the resolution is 25 m, and the return signal consists of 200 cells.

In Figures 2-4, the horizontal axis represents the distance and the vertical axis represents the statistics.

After calculating the statistics of the corresponding detector in each cell, we normalize the statistics by dividing the statistics of each cell by the maximum.

Figure 2 shows the simulated signal echo.

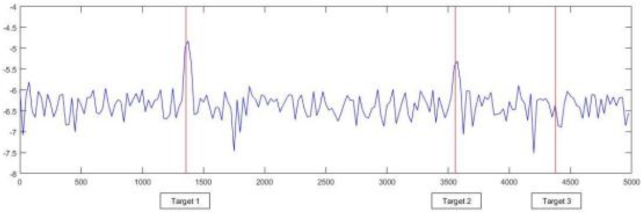

Fig2. *echo in the case of more than two targets*

As can be seen, if there are more than two targets, the target at greater distance is more difficult to distinguish between the scattered and the reflected target signals, with the signal energy weakening.

Figure 3 shows the normalized statistics by different detectors in the case where targets 1, 2, and 3 are one, respectively.

Figure 3 shows that the statistics for a cell with a target are much larger than those for a cell without a target.

This indicates that these three detectors can detect the target accurately.

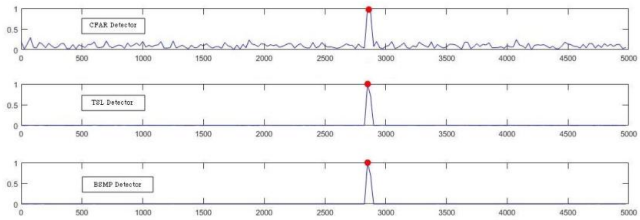

Fig3-1. T*he results of detection of different detectors in the case of one target*

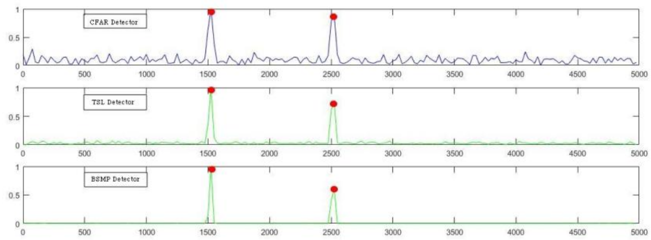

Fig3-2. *The results of the detection of different detectors in the case of two targets*

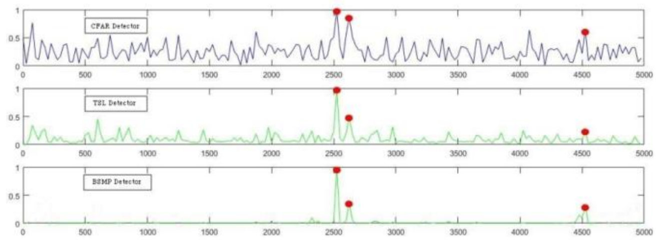

Fig4. *The results of the detection of different detectors in the case of three targets*

Figure 4 shows the normalized statistics by different detectors in the case of two and three targets.

The three detectors have detectability because of their large statistics on the nearby target compared to the target-free cell, but for the second target, they have no detectability because of their weak signal strength. However, the proposed detector has detectability even in the case of weak signal strength. This gives the possibility that the performance of the proposed detector can be higher than other detectors.

5. Conclusion

In this paper, we propose the idea that geodetic mapping of the detection signal into submanifold can improve the detection performance by constructing a background submanifold composed of noisy signals. Although there are several problems to construct the background submanifold, it can be seen that detection performance may be improved remarkably by well-constructed submanifold reflected whole noise signal.

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