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Some Results on Commutativity for Alternative Rings with 2-Torsion Free.

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Abstract: *Let R be a 2-torsion free Alternative ring with unity satisfy the following constrain:*

$$(p_1)[(xy)^2 - xy, x] = 0$$

$$(p_2)\ [(xy)^2 - (xy)^2, x] = 0. \quad \forall\, x, y \in R.$$

In this article we investigate and proved the commutativity of alternative ring with suitable constraints (p_1) and(p_2) .

Keywords: *Alternative ring, associative ring, commutator, prime rings, n-torsion free.*

1. Introduction

In this paper, we consider (Assing) R represents an alternative ring, $C(R)$ the commutator, $A(R)$ the assosymetric ring. $N(R)$ the set of nilpotent element. An alternative ring R is a ring in which$(xx)y = x(xy)$, $y(xx) = (yx)x$ for all x, y in R, these equations are known as left and right alternative laws respectively. An associator (x, y, z) we mean by $(x, y, z) = (xy)z - x(yz)$for all $x, y, z \in$ R. A ring R is called a prime if whenever A and B are ideals of R such that $AB = \{0\}$ then either $A = \{0\}$ or $B = \{0\}$. If in a ring R, the identity $(x, y, x) = 0$ i.e. $(xy)x = x\,(yx)$ for all x, y in R holds then R is called flexible. A ring R is said to be n-torsion tree if $nx = 0$ implies $x = 0$, n is any positive number for all $x \in R$.A non-associative rings R is an additive abelian group in which multiplication is defined, which is distributive over addition on left as well as on right $[(x + y)z = xz + yz,\ z(x + y) = zx + zy, \forall\, x, y, z \in R]$.

Abujabal and Khan [1] proved the commutativity of associative ring satisfies the identity $(xy)^2 = xy^2x$. Gupta [2] established that a division ring R is commutative if and only if $[xy, yx] = 0$. In addition, Madana and Reddy [3] have established the commutativity of non-associative ring satisfying the identities $(xy)^2 = x^2y^2$ and $(xy)^2 \in Z(R)\forall x, y \in R$.

Further, Madana Mohana Reddy and Shobha latha.[4] established the commutativity of non-associative primitive rings satisfying the identities: $x(x^2 + y^2)$+$(x^2 + y^2)x \in Z(R)$ and $x(xy)^2 - (xy)^2x \in Z(R)$. Recently Madana Mohana Reddy [5] show that some results on commutativity of some 2-torsion free non associative rings with unity satisfy:

$(\alpha\beta)^2 - \alpha\beta \in Z(R)$ $for\ all\ \alpha\beta\ in\ R$ Motivated by these observation it is natural to look commutativity of alternative rings satisfies:(p_1) & (p_2),

2. Main Results

Theorem 2.1

Let R be a 2-torsion free alternative rings with unity satisfy: $[(xy)^2 - xy, x] = 0 \quad \forall\, x, y \in R$. Then R $is\ commutative$.

Proof:

From the hypothesis : (p_1) we get

$x[(xy)^2 - xy] - [(xy)^2 - xy]x = 0$ (1)

$x(x^2y^2 - xy) = (x^2y^2 - xy)x$ (2)

Put $x = (x + 1)$ in 2 above

$(x + 1)[(x + 1)^2y^2 - (x + 1)y] = [(x + 1)^2y^2. -(x + 1)y](x + 1).(x + 1)[(x^2 + 2x + 1)y^2 - (xy + y)] = [(x^2 + 2x + 1)y^2 - (xy + y)](x + 1)(x + 1)(x^2y^2 + 2xy^2 + y^2 - xy - y) = (x^2y^2 + 2xy^2 + y^2 - xy - y)(x + 1)x(x^2y^2) + 2x^2y^2 + xy^2 - x^2y - xy + x^2y^2 + 2xy^2 + y^2 - xy - y = (x^2y^2)x + 2xy^2x + y^2x - xyx - yx + x^2y^2 + 2xy^2 + y^2 - xy - y$

Known by our hypothesis and using 2-torsion free we get

$xy^2 - xy - xy = y^2x - yx - xy$ this will be come

$xy^2 - xy = y^2x - yx$ (3)

(3) can be re-write as $xy^2 + yx = y^2x + xy$

Put $y = y + 1$ in above (3)

$x(y + 1)^2 + (y + 1)x = (y + 1)^2x + x(y + 1)$

$x(y^2 + 2y + 1) + yx + x = (y^2 + 2y + 1)x + xy + x$

$xy^2 + 2xy + xy + yx + x = y^2x + 2yx + x + xy + x$

Using (3) above and collecting like terms we get: $2xy = 2yx <=> 2(xy - yx) = 0$

$2(xy - yx) = 0$ (4)

$xy - yx = 0$ 2-torsion free is applied

Then $xy = yx$ or $[x, y]$ is commutative.

From the above R is commutative Ring and satisfy the Identities either

$(xx)y = x(xy)$ *or* $y(xx) = (yx)x$. So R is an Alternative rings, Hence an alternative rings with Identity together with commutativity yields $(x, x, y) = 0 = (y, x, x)$ *in complition.*

Theorem 2.2

Let R be a 2-torsion free alternative rings with unity satisfy: $[(xy)^2 - (xy)^2, x] = 0 \ \forall x, y \in R$. Then R *is commutative.*

Proof:

From our hypothesis (p_2) we get:

$x[(xy)^2 - (xy)^2] - [(xy)^2 - (xy)^2]x = 0$

$x[(xy)^2 - (xy)^2] = [(xy)^2 - (xy)^2]x$

$x(x^2y^2 - y^2x^2) = (x^2y^2 - y^2x^2)x$ (5)

Put $x = x + 1$ in (5) above

$(x + 1)[(x + 1)^2y^2 - y^2(x + 1)^2) = [(x + 1)^2y^2 - y^2(x + 1)^2](x + 1)$

$(x + 1)[(x^2 + 2x + 1)y^2 - y^2(x^2 + 2x + 1) = [(x^2 + 2x + 1)y^2 - y^2(x^2 + 2x + 1)](x + 1)$

$(x + 1)(x^2y^2 + 2xy^2 + y^2 - y^2x^2 - 2y^2x - y^2)$

$= (x^2y^2 + 2xy^2 + y^2 - y^2x^2 - 2y^2x - y^2)(x + 1)$

$x(x^2y^2) + 2x^2y^2 + xy^2 - x(y^2x^2) + 2xy^2x + xy^2 + x^2y^2 + 2xy^2 + y^2 - y^2x^2 - 2y^2x - y^2$

$= (x^2y^2)x + 2xy^2x + y^2x - (y^2x^2)x - 2y^2x^2 - y^2x) + x^2y^2 + 2xy^2 + y^2 - y^2x^2 - 2y^2x - y^2$

Using (5) and collecting like terms we get:

$2x^2y^2 + 2xy^2 = 2y^2x + 2y^2x^2$ <=> $2(x^2y^2 + xy^2) = 2(y^2x + y^2x^2)$

Known we had

$x^2y^2 = y^2x^2$ (6)

Put $x = x + 1$ in (6)

$(x+1)^2y^2 = y^2(x+1)^2$

$(x^2 + 2x + 1)y^2 = y^2(x^2 + 2x + 1)$

$x^2y^2 + 2xy^2 + y^2 = y^2x^2 - 2y^2x - y^2$

Apply 6 we get

$2xy^2 = 2y^2x$ <=> $2(xy^2 - y^2x) = 0$

Apply 2-torsion free we had: $xy^2 = y^2x$ (7)

Insert $y = y + 1$ in (7) $above$

$x(y+1)^2 = (y+1)^2x$

$x(y^2 + 2y + 1) = (y^2 + 2y + 1)x$

$xy^2 + 2xy + x = y^2x + 2yx + x$

Using 7 and apply 2-torsion free we get:

$xy = yx$ or $[x, y]$

Which is commutative.

As we seen From the above R is commutative Ring and satisfy the Identities either

$(xx)y = x(xy)$ or $y(xx) = (yx)x$. So R is an Alternative rings, Hence an alternative rings with Identity together with commutativity yields $(x, x, y) = 0 = (y, x, x) in\ complition.$

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