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Generalizing the Relationship of LCM and HCF for Numbers in Progressions

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Abstract: *This paper extends the known relationship between the Least Com- mon Multiple (LCM) and Highest Common Factor (HCF) of numbers in arithmetic progression (AP). Building on the work of Deepa Shetty, Paresh D. Nayak, and Sadhana S. in MATHEMATICAL FORMULA RELATIONSHIP OF LCM & HCF OF 3 NUMBERS IN AP (Int. Journal of Mathematical Archive-8(3), 2017, 99-103), we generalize their formula for three numbers to larger sets, specifically four num- bers in AP, and further extend it to other types of sequences like geometric and harmonic progressions. We also explore the behavior of these relationships when some of the numbers share common factors other than 1. New formulas and examples are provided, demonstrat- ing the broader applicability of these methods.*

1. Introduction

In their paper, Deepa Shetty, Paresh D. Nayak, and Sadhana S. derived a relationship between the LCM and HCF of three numbers in arithmetic progression. They provided a formula that, under specific conditions, sim- plifies the calculation of the LCM based on the HCF of the numbers. Their approach can be summarized in the following formula:

$$\mathrm{LCM}(a,b,c) = \frac{a \times b \times c}{\mathrm{HCF}^2}$$

when the numbers divided by their HCF are co-prime. If two out of three numbers share a common factor other than 1, the formula becomes:

$$\mathrm{LCM}(a,b,c) = \frac{a \times b \times c}{2 \times \mathrm{HCF}^2}$$

This method efficiently handles cases where the numbers form a simple arithmetic progression. However, there are opportunities to extend this for- mula to more complex situations, such as four or more numbers in arithmetic progression, geometric progressions, and harmonic progressions.

In this paper, we generalize their approach to address these more complex cases and provide detailed examples to illustrate the new formulas.

2. Literature Review

The work by Shetty, Nayak, and Sadhana was foundational in establishing a practical relationship between the LCM and HCF for three numbers in AP. Their formula simplified computations in number theory and demonstrated clear relationships between co-prime numbers and those with common fac- tors.

However, their method focused exclusively on three numbers in arithmetic progression. This left an open question as to whether their approach could be extended to larger sets of numbers or other types of progressions.

To fill this gap, our work proposes generalized formulas for sets of four or more numbers, as well as for geometric and harmonic progressions. Addi- tionally, we explore cases where two or more numbers share common factors other than 1, which were only briefly touched on in the original work.

3. Existing Method: Three Numbers in Arith- metic Progression

Shetty et al. provided a straightforward approach for calculating the LCM and HCF of three numbers in arithmetic progression. For example, consider the numbers a = 6, b = 12, and c = 18, which form an arithmetic progression with a common difference d = 6. The HCF of these numbers is 6.

Using the existing method:

$$\mathrm{LCM}(6,12,18) = \frac{6\times 12\times 18}{6^2} = \frac{1296}{36} = 36$$

For co-prime numbers, such as a = 4, b = 5, and c = 6, we use the simpler formula

$$\mathrm{LCM}(4,5,6) = \frac{4\times 5\times 6}{1^2} = 120$$

4. Proposed Method: Four Numbers in Arith- metic Progression

We now extend this formula to **four numbers** in arithmetic progression. Let a, b = a + d, c = a + 2d, and d = a + 3d represent four numbers in AP.

4.1 General Formula for Four Numbers in AP

For four numbers in arithmetic progression, we hypothesize the relationship:

$$\mathrm{LCM}(a,b,c,d) = \frac{a\times b\times c\times d}{\mathrm{HCF}^3}$$

4.2 Example 1: Four Numbers with HCF of 2

Consider the numbers $a = 2$, $b = 4$, $c = 6$, and $d = 8$, which form an arithmetic progression with a common difference $d = 2$ and HCF $= 2$. Applying the generalization:

$$\mathrm{LCM}(2,4,6,8) = \frac{2\times 4\times 6\times 8}{2^3} = \frac{384}{8} = 48$$

4.3 Example 2: Four Numbers with Common Factors

Now consider the numbers $a = 10$, $b = 20$, $c = 30$, and $d = 40$, where the HCF is 10. Since two of the numbers (20 and 30) share a common factor other than 1, we modify the formula:

$$\mathrm{LCM}(10,20,30,40) = \frac{10\times 20\times 30\times 40}{2\times 10^3} = \frac{240000}{2000} = 120$$

4.4 Example 3: Numbers with Mixed Common Factors

For $a = 12$, $b = 18$, $c = 24$, and $d = 30$, the HCF is 6. Since 18 and 24 share a common factor of 6, we apply the adjusted formula:

$$\mathrm{LCM}(12,18,24,30) = \frac{12\times 18\times 24\times 30}{2\times 6^3} = \frac{155520}{432} = 360$$

5. Extension to Other Types of Progressions

5.1 Geometric Progression (GP)

For numbers in a geometric progression, where each term is a constant multiple of the previous one, the relationship between LCM and HCF changes due to the exponential growth of the terms.

Example 4 (GP): Let $a = 3$, $ar = 9$, $ar^2 = 27$, and $ar^3 = 81$ with common ratio $r = 3$ and HCF $= 3$:

$$\mathrm{LCM}(3,9,27,81) = \frac{3\times 9\times 27\times 81}{3^3} = \frac{59049}{27} = 2187$$

5.2 Harmonic Progression (HP)

For harmonic progressions, where the reciprocals of the numbers form an arithmetic progression, we use the formula:

$$\text{LCM}(a, b, c, d) = \frac{a \times b \times c \times d}{\text{HCF}}$$

Example 5 (HP): For $a = 2$, $b = 3$, $c = 6$, and $d = 12$, which are in harmonic progression:

$$\text{LCM}(2, 3, 6, 12) = \frac{2 \times 3 \times 6 \times 12}{1} = 432$$

6. CONCLUSION

This article extends the formula for calculating the LCM of numbers in arith- metic progression, originally developed for three numbers, to four or more numbers and other types of progressions. By incorporating examples with common factors, we have demonstrated the flexibility and broader applicabil- ity of this method in number theory. The new formulas offer practical tools for solving problems involving LCM and HCF in more complex numerical sequences

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