Generalised Lattice Metrized Spaces (gl-metrized spaces)

Parimi Radha krishna Kishore*

Associate Professor, Department of Mathematics, Arba Minch University, Arba Minch, Ethiopia and also Guest Faculty, Department of Mathematics, SRM University-AP, Andhra Pradesh, Neerukonda(Village), Mangalagiri(Mandalam), Guntur (District), Andhra Pradesh (State), PIN Code: 522240, INDIA

*Corresponding Author: Parimi Radha krishna Kishore, Associate Professor, Department of Mathematics, Arba Minch University, Arba Minch, Ethiopia and also Guest Faculty, Department of Mathematics, SRM University-AP, Andhra Pradesh, Neerukonda(Village), Mangalagiri(Mandalam), Guntur (District), Andhra Pradesh (State), PIN Code: 522240, INDIA

Abstract: In this paper initially introduced the concept generalised lattice betweenness (gl-betweenness) relation in generalised lattice and observed transitivity properties. Later introduced the concept generalised lattice metrized space (gl-metrized space), imagined triangles, sides of the triangles and observed their properties.

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1. INTRODUCTION

Mellacheruvu Krishna Murty and U. MadanaSwamy (Professors of Andhra University) [6] introduced the concept of generalised lattice. The author P.R.Kishore [2,3,4] developed the theory of generalised lattices that can play an intermediate role between the theories of lattices and posets. The concept of Lattice metrized space (L-metrized space) is known from Leo Lapidus [5,7] and later in [8] the concept of Brouwerian generalised lattice introduced and developed by the author P.R.Kishore. In this paper section 2 contains some preliminary concepts that are from the references. In section 3 introduced two kinds of transitivity properties $t_1$, $t_2$ and generalised lattice betweenness (gl-betweenness) relation in a generalised lattice. Proved that the gl-betweenness relation satisfies the transitivity $t_1$. In section 4 introduced the concept generalised lattice metrized space (gl-metrized space), imagined triangles, sides of the triangles and observed their properties. Finally introduced a relation $P$-linear (Pl) in a gl-metrized space and proved that it satisfies the transitivity $t_2$.

2. PRELIMINARIES

This section contains some preliminaries from the references those are useful in the next sections.

[Murty [6]] For any finite subset $A$ of a poset $P$, define $L(A) = \{x \in P \mid x \leq a \text{ for all } a \in A\}$ and $U(A) = \{x \in P \mid a \leq x \text{ for all } a \in A\}$. Then the sets $L(P) = \{L(A) \mid A \text{ is a finite subset of } P\}$ and $U(P) = \{U(A) \mid A \text{ is a finite subset of } P\}$ are semi lattices under set inclusion.

Definition 2.1 [Murty [6]] Let $(P, \preceq)$ be a poset. $P$ is said to be a generalised meet semilattice if for every non empty finite subset $A$ of $P$, there exist a non-empty finite subset $B$ of $P$ such that, $x \in L(A)$ if and only if $x \preceq b$ for some $b \in B$. $P$ is said to be a generalised join semilattice if for every non empty finite subset $A$ of $P$, there exist a non-empty finite subset $B$ of $P$ such that, $x \in U(A)$ if and only if $b \preceq x$ for some $b \in B$. $P$ is said to be a generalised lattice if it is both generalised meet and join semilattice.

[Murty [6]] It is observed that if $P$ is a generalised meet (join) semilattice, then for any $L(A) \subseteq L(P)(U(A) \subseteq U(P))$ there exists a unique finite subset $B$ of $P$ such that $L(A) = \bigcup_{b \in B} L(b)$ ($U(A) = \bigcup_{b \in B} U(b)$) and the elements of $B$ are mutually incomparable and the set is denoted by $ML(A)$ ($MU(A)$). If a poset $P$ is a generalised lattice then $(L(P), \subseteq)$ and $(U(P), \subseteq)$ are lattices.

3. GENERALISED LATTICE BETWEENNESS (GL-BETWEENNESS)

Definition 3.1 Let $P$ be a generalised lattice and $\theta \subseteq PxP \times P$. Then $\theta$ is said to have the property of transitivity $t_1$ if for $a,b,c \in P$; $(a,b,c) \in \theta$ and $(a,x,b) \in \theta$ implies $(x,b,c) \in \theta$.
Definition 3.2 Let P be a generalised lattice and \( \theta \subseteq P^3 \). Then \( \theta \) is said to have the property of transitivity 1; if for \( a, b, c \in P \); \( (a, b, c) \in \theta \) and \( (a, x, c) \in \theta \) implies \( (a, x, c) \in \theta \).

Definition 3.3 Let \( P \) be a generalised lattice and \( a, b, c \in P \). Then \( b \) is said to be gl-between \( a \) and \( c \) in \( P \) if \( ML(mu(ML{a, b} \cup ML{b, c})) = \{ b \} = mu(ML(mu{a, b} \cup mu{b, c})) \), denoted by \( (a, b, c) \in P \).

Note: Let \( P \) be a generalised lattice. Then \( glb = \{(a, b, c) \in P^3 | (a, b, c) \in P \} \) ML(mu(ML{a, b} \cup ML{b, c})) = \{ b \} = mu(mu(ML{a, b} \cup mu{b, c})) \) is a 3-relation on \( P \).

Theorem 3.4 Let \( P \) be a generalised lattice. Then the 3-relation \( glb \) on \( P \) has transitivity 1.

Proof: Let \( a, b, c \in P \) and suppose \((a, b, c) \in P \). Then \( glb = \{(a, b, c) \in P^3 | (a, b, c) \in P \} \) ML(mu(ML{a, b} \cup ML{b, c})) = \{ b \} = mu(mu(ML{a, b} \cup mu{b, c}))

Defining 3.1 Let \( S \) be a gl-metrized space. Then \( d(a, b) \leq d(a, c) + d(c, b) \) if \( (a, b, c) \in S \).

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Definition 4.7 Let S be a gl-metrized space and a, b, c ∈ S. Then the triple of elements (a, b, c) is said to satisfy P-line segment property (Pls property) if (a, b, c) ∈ Pl and (a, b, c) ∈ Pl if and only if b=c.

Theorem 4.8 Let S be a gl-metrized space and a, b, c ∈ S. If a, b, c are vertices of an isosceles triangle then (a, b, c) ∈ Pl implies (a, c, b) ∈ Pl.

Proof: Suppose a, b, c are vertices of an isosceles triangle and say that a * b = a * c. Suppose (a, b, c) ∈ Pl. Then we have L(mu{a * b, b * c}) = L(a * c). Since L(P) is a lattice, we get L(a * b)/L(b * c) = L(a * c) = L(a * b). To show that (a, c, b) ∈ Pl: By definition 4.1 we have a * b ∈ L(mu{a * c, c * b}). Since L(P) is a lattice, we get L(a * b) ⊆ L(a * c) ∨ L(c * b) = L(a * c) ∨ L(b * c) = L(a * b) ∨ L(b * c) = L(a * b). Therefore L(mu{a * c, c * b}) = L(a * c) ∨ L(b * c) = L(a * b). That is (a, c, b) ∈ Pl. □

Theorem 4.9 Let S be a gl-metrized space and a, b, c ∈ S. Suppose (a, b, c) ∈ Pl. Then (a, b, c) not satisfies P-line segment property (Pls property) if and only if a, b, c are vertices of an isosceles triangle.

Proof: Suppose a, b, c are vertices of an isosceles triangle. That is a≠ b ≠ c. Given that (a, b, c) ∈ Pl. Then by theorem 4.8 we get (a, b, c) ∈ Pl. Since b≠ c, by definition 4.7, we can say that (a, b, c) not satisfies Pls property. Conversely suppose (a, b, c) not satisfies Pls property. To show that a, b, c are vertices of an isosceles triangle: If a = c then a * b = c * b = b * c and therefore ΔP(a,b,c) is an isosceles triangle. Suppose a≠ c. Then ΔP(a,b,c) is a P-triangle in S with a≠ c. Case(i): Suppose (a,c,b) ∈ Pl. Then we have (a, b, c) ∈ Pl and (a,b,c) ∈ Pl. Now by theorem 4.3 and definition 4.4 we have L(a * c) = L(mu{a * b, b * c}) = L(mu(a * c, c * b)) = L(a * b). Therefore a * c = a * b. Case(ii): Suppose (a,c,b) ∈ S^3 - Pl. Then since (a, b, c) not satisfies Pls property, by definition 4.7 we get b=c. Therefore a * c = a * b. Hence by both the cases we can say that ΔP(a,b,c) is an isosceles triangle. □

Theorem 4.10 Let S be a gl-metrized space. Then the 3-relation Pl on S satisfies the property of transitivity t2.

Proof: Let a,b,c,x ∈ S. Suppose (a,b,c) ∈ Pl and (a,x,b) ∈ Pl. To show that (a,x,c) ∈ Pl: By note after definition 4.5 we get L(a * b)/L(b * c) = L(mu{a * b, b * c}) = L(a * c) and L(a * x)/L(x * b) = L(mu{a * x, x * b}) = L(a * b). Then L(a * x)/L(x * b) ∨ L(b * c) = L(a * b)/L(b * c) = L(a * c). By definition 4.1 we have x * c ∈ L(mu{a * x, x * c}) and a * c ∈ L(mu{a * x, x * c}). Then L(a * c)⊆ L(a * x)/L(x * c) ∨ L(x * b)/L(b * c) = L(a * c). That is L(mu{a * x, x * c}) = L(a * x)/L(x * c) = L(a * c). Therefore (a,x,c) ∈ Pl. Therefore Pl satisfies the property of transitivity t2. □

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