The Implementation of Chirp Scaling Algorithm Via Fast Fourier Transform

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Abstract: Synthetic aperture radar uses the data collected by rather small antenna in the form of electromagnetic waves to image with high resolution imaging. This radar uses sending and receiving antennas or microwave pulses while moving forward as an advantage which resembles to the large antennas. In more simple terms, such forms of sending and receiving data by small antenna resemble to data which have been sent and received by an antenna during flight. To produce image from raw data four algorithms have been developed so far. Among these algorithms Chirp scaling with excellent power of the focusing on the target (higher resolution), optimized processing, and simple implementation has been cared for. The aim of this paper is to implement Chirp scaling algorithm using the Fast Fourier Transform in order to increase processing speed and to produce high-resolution images. The simulation result indicate that this proposed scheme is more accurate than other implementation.

Keywords: Synthetic aperture radar, Chirp Scaling Algorithm, Fast Fourier Transform

1. INTRODUCTION

It is necessary to use bulky aperture to produce high resolution images in optical systems. Since frequency in synthetic aperture radar is lower than that of optical systems then even obtaining mediocre resolution in synthetic aperture radar need a bulky antenna. For doing this, it needs antenna with hundreds meters’ length [1]. An aerial radar can collect data while making advance of distance to few hundred meters and process data in such way that it seems to be obtained from a physically bulky antenna. Synthetic aperture is defined as distance through which a flying object goes to make synthetic antenna. As a synthetic aperture radar moves it sends a pulse to each target. Reflecting echoes are taken by the receiver and kept in an echo-accumulating space. Since relating to the ground the radar is moving, reflecting echoes take Doppler displacement [2]. Comparing Doppler displacement frequency to a referent frequency makes it possible that large number of reflecting echo concentrate on one point; as a result, the length of antenna capturing a particular point will be increased effectively [3]. To produce image from raw data four algorithms have been developed so far. Among these algorithms Chirp scaling with excellent power of the focusing on the target (higher resolution), optimized processing [4], and simple implementation has been cared for. In spite of the Range Migration and Range Doppler Algorithms this algorithm can be implemented without the need of inter-acquire easily, it can also be used to point and band capturing. In point capturing Range Migration Algorithm, Range Doppler Algorithm, and Chirp Scaling Algorithm are used but in point capturing Polar Algorithm, Range Doppler Algorithm, and Chirp Scaling Algorithm are used. Fourier Transform is used to both implement and phase-keeping effectively. Another advantage of using Chirp Scaling Algorithm in terms of Secondary Range Compression is to provide range for bilateral processing of adapting filter. Secondary Range Compression is an effective method for joining Range-Bearing, it increasingly can be applied to pass large radar beam, large beam width, and high Squint inclination.

The structure of paper is as follow: section II present the chirp scaling algorithm. Section III demonstrate the results and conclusion is represented in section IV.
2. **Chirp Scaling Algorithm**

Chirp Scaling Algorithm which is called Differential Range Deramp Algorithm\(^1\) either, is a modified classic Range Doppler to produce high resolution image from synthetic aperture radar. In the Chirp Scaling Algorithm receiving extended echoes lead to effective implementation and Secondary Range Compression capability that it finally increases accurate angular focusing. This modification is due to the access of raw data in bilateral frequency's domain which gets Secondary Range Compression depended to bearing frequency while Range Doppler uses inter-acquire to implement range cell migration correction \([5]\).

Chirp Scaling Algorithm uses frequency shift to correct constant transfer vector and uses Chirp Scaling to correct linear transfer vector. The ultimate goal of shift operation and scaling is differential and Bulky Differential Range Deramp Algorithm\(^2\) which gets target energy concentrated to correct range position. Additionally, Chirp Scaling Algorithm is used against Range Doppler to both point and band capturing. Consider Chirp LFM signal as the below equation:

\[
S(t) = \exp(j2\pi(K_r/2)t^2)
\] (1)

Where, \(K_r\) equals to signal frequency modulation rate in Hertz which is sent by radar. Sent signal from referent target is reflected and transferred to different bearing position(\(\eta\)) which causes constant variation throughout the synthetic aperture. The time needed for signal to reach for the target and return back to receiver can be calculated by the following equation:

\[
\tau = \frac{2R}{c}
\] (2)

Where, \(R\) is the range between radar and target and \(C\) is the dispersion speed of electromagnetic waves \((C=3*10^8\, \text{m/s})\).

![Figure 1. Range transference from single target in bilateral memory. Each cell is compatible with heterogeneous video-signal sample(I&Q).](image)

Received base band signal can be obtained after range transferring from the following equation:

\[
S_r(t, \eta) = A_0w_r(t - \frac{2R}{c})w_a(\eta - \frac{\eta_c}{c})e^{-j4\pi f_0R_{\eta}/c}.
\] (3)

\[
S\left(t - \frac{2R\eta}{c}\right)
\]

Where within

\[
R_{\eta} = (R_0^2 + V_r^2\eta^2)^{1/2}
\] (4)

That \(\eta\) is the time-bearing (bearing in the time domain), \(A_0\) is desirable sundry constant, \(\eta_c\) is center beam (source) start time, \(w_r(t)\) is range push rectangular function, \(w_a(\eta)\) is bearing push squared function, \(f_0\) is central frequency of radar, \(R_{\eta}\) is traverse range in time, \(R_0\) is traverse range in the closest point of approach, \(V_r\) is the efficient speed of the radar.
Correction for the range migration and signal shift to proper condition, modulating frequency to Chirp coded signal to reach for a shift or scaling is exerted. The maximum of scale variations or shift could not be too much. It can be implemented by frequency modulation to avoid any problem with dependent variation to the central frequency of the signal and band width. This limitation by the use of the range cell migration is damped within two paces to the extent which the only difference in migrating range cell at various distances in the Chirp scaling operation should be corrected and bulky range cell migration will be completed in frequency bilateral domain with secondary range compression. In Figure 2 point \( \eta_0 \) hints to the wide surface of the range condition in which bearing frequency is equated to \( f_{\eta_c} \), or \( f_\eta = f_{\eta_c} \). (point here means the extent of visualization from radar real aperture as shown in Figure (2).

Chirp Scaling Algorithm includes multiplication three phases by four Fourier transforms. This problem is extracted by equation (3) for received base band signal and by equation (4) for range equation. Converting signal to bilateral frequency domain is occurred by unilateral Fourier transform. Major standing phase is used to Fourier transform integral approximation.

Following figure indicates block diagram of Chirp Scaling Algorithm. Various algorithm stages for the raw data in terms of time- range and time-bearing domains are as below:

First, to convert received data \( (S_r(t, \eta)) \) to Range Doppler, fast bearing Fourier transform gets calculated. To equate range migration of all targets, Chirp Scaling should be exerted to the first stage result. To convert received data to bilateral frequency domain, fast range Fourier transform is exerted. Phase multiplication is done by referent function. Secondary Range Compression and bulky range cell migration correction are exerted at this stage. To return data to Range Doppler domain, reverse Range Instant Fast Fourier transform is executed.

Phase multiplication is exerted for compressing bearing with congruous filter which is at variation by time. Additionally, as a result of Chirp Scaling in the second stage, phase correction is needed which can be added to phase multiplication.

Consequently, to convert compressed data to synthetic aperture radar image reversed bearing Fast Fourier transform is exerted to the sixth stage. Figure 3 indicates geometrical mono-plate turn up. Same Chirp Scaling Algorithm to flying object plate can be exerted. Real variable transverse range \( (R_\eta) \) fluctuates spontaneously in such a way that on-target trajectory beam moves with \( (V_r) \) speed [7].

Figure 2. Diagram block of Chirp Scaling Algorithm
As shown in Figure 3, target transverse range resembles hyperbole which causes constant variations in the range from antenna's wide side. Next Figure presents a definition to image data array and indicates how both the range and the bearing directions can be referent. As following Figure indicates range increases from the left to the right and bearing from the bottom to the up. The term fast-time is used to indicate range and range cell is used to indicate column vector at constant range. The term slow-time is used to indicate both bearing direction and the number of pulses of a horizontal vector (linear) from the constant bearing.

During the time of image-capturing or collecting, pulses are received with repeat frequency rate and with range sampling rate, in the form of bilateral data array (range and bearing matrix) appropriate to image processing, get sampled.

In Figure 4, variables $\rho_r$ and $\rho_a$ hint to range and bearing resolutions. The range for the radar signal is the linear measurement of the line of sight from radar to the target. To band capturing from the synthetic aperture radar with rather small squint angle, the range is at approximate vertical on flying way and range resolution is given through the following equation:
\[
\rho_r = \frac{c}{2BW}
\]

Where \(C = 3 \times 10^8\) m/s is dispersion speed of the electromagnetic wave (light), and \(BW\) is radar band width. To detect two targets in range direction their range value must be bigger than \(\rho_r\) value. Bearing is the linear range paralleled to radar flying way. It can also be defined as track way extension 1; as a result, a target position extension in radar field of view2 follows radar's line of sight.

To band capturing systems, synthetic aperture radar with rather small squint angle, bearing direction is at vertical on the range direction and image resolution bearing-wide is stated as follow:

\[
\rho_a = \frac{L_a}{2}
\]

Where \(L_a\) is an antenna's length toward bearing, to detect two targets in direction of bearing, their range must be bigger than \(\rho_a\) value.

- **Chirp Scaling**

In terms of bearing-time and range-time, target range curvature depends on both its range and baring positions. Bearing-time and range-time terms hint to Range Doppler, and as all target energy stand in constant range direction then range curvature will not depend upon target bearing position. \([7],[10]\) and \([12]\). Due to this, Chirp Scaling in bilateral domains is exerted by dependent bearing multiplied by range phase. First step in Chirp Scaling Algorithm is to calculate received signal Fourier Transform conversion in terms of Range Doppler. Since it is difficult to calculate such conversion, using an approximate obtained from exerted POSP is concerned. If from \(S_r(t, \eta)\) in equation 3 Fourier Transform gets taken (time-bearing/Doppler frequency \((f_r, \eta)\)).

\[
S_{rd}(t, f_r) = A \omega_r \left( t - \frac{2R_0}{cD(f_{r,ref}, V_r)} \right) w_d(f_r, \eta_r) * \exp \left\{ -j \frac{4\pi f_0 D(f_r, V_r)}{c} \right\} \ast \exp \left\{ \frac{j \pi K_m}{c^2} \left[ 1 - \frac{D(f_r, V_r)}{D(f_{r,ref}, V_{r,ref})} \right] \right\} \left[ \frac{R_0}{D(f_r, V_r)} - \frac{R_{ref}}{D(f_{r,ref}, V_{r,ref})} \right]^2
\]

Where \(A\) is constant sundry, \(w_d(f_r)\) is Doppler spectrum coverage or Fast Fourier Transform from \(D(f_r, V_r)\), and \(w_d(\eta)\) is a range parameter in Range Doppler for radar with speed \(V_r\) which can be computed through following formula:

\[
D(f_r, V_r) = \left( 1 - \frac{c^2 f_r^2}{4 V_r^2 f_0^2} \right)
\]

\(K_m\) Is a modified frequency modulation rate and can be computed through following formula:

\[
K_m = \frac{K_r}{1 - K_r \cdot \frac{R_0 f_0^2}{2 V_r^2 f_0^2 D^2(f_r, V_r)} D^3(f_r, V_r)}
\]

For Chirp LFM signal, scaling function dependent to the bearing can be computed through following formula:

\[
S_{sc}(t, f_r) = \exp \left\{ j \pi K_m \left[ \frac{D(f_{r,ref}, V_{r,ref})}{D(f_r, V_r)} - 1 \right] t^2 \right\}
\]

In the above equation it is assumed that target in \(R_{ref}\) is shiftless. By the use of \(S_{sc}(t, f_r)\) and \(S_{rd}(t, f_r)\) which obtained from equations (7) and (10) Chirp Scaling dependent to bearing can be obtained as follow:

\[
S(t, f_r) = S_{sc}(t, f_r) \cdot S_{rd}(t, f_r)
\]

- **Range Compression**

After multiplying Chirp Scaling phase, raw data via Fourier Transform, \(S(t, f_r)\), on t to \(f_r\) in terms of range-frequency and bearing-frequency get converted which result in \(S_2(t, f_r)\). Now
data are in the bilateral frequency domain and POSP to calculate integral and to determine $S_2(f_t, f_\eta)$ is used.

$$S_2(f_t, f_\eta) = A_1 \mathcal{W}_r(f_t). \mathcal{W}_a (f_\eta - f_{nc})$$

.. math::

   . \exp \left\{ -j \frac{4\pi R_0 D(f_\eta, V_r)}{c} \right\} \cdot \exp \left\{ -j \frac{\pi D(f_\eta, V_r)}{K_m D(f_{\eta ref}, V_r)} f_t^2 \right\}

.. math::

   . \exp \left\{ 1 - \frac{D(f_\eta, V_r)}{D(f_{\eta ref}, V_r)} \right\} \frac{R_0}{D(f_\eta, V_r)} \exp \left\{ D(f_\eta, V_r) \left( \frac{R_0}{D(f_{\eta ref}, V_r)} - \frac{R_{ref}}{D(f_\eta, V_r)} \right)^2 \right\}

Where $\mathcal{W}_r(f_t)$ is the range spectrum coverage or Fast Fourier Transform of $\mathcal{W}_r(t)$. The second phase multiplication by Chirp Scaling Algorithm is a bilateral range adapting filter which depends upon both range and bearing. It can be exerted to all range frequencies which are dependent to bearing frequencies. Adapted filter corrects bulky range cell migration and secondary range compression with removing the second and forth exponential expressions from the equation (12) which leads to $S_3(f_t, f_\eta)$ and can be obtained by below equation:

$$S_3(f_t, f_\eta) = A_1 \mathcal{W}_r(f_t). \mathcal{W}_a (f_\eta - f_{nc})$$

.. math::

   . \exp \left\{ -j \frac{4\pi R_0 D(f_\eta, V_r)}{c} \right\} \cdot \exp \left\{ -j \frac{4\pi R_0}{c. D(f_{\eta ref}, V_{ref})} f_t \right\}

$$\exp \left\{ 1 - \frac{D(f_\eta, V_r)}{D(f_{\eta ref}, V_r)} \right\} \frac{R_0}{D(f_\eta, V_r)} \exp \left\{ D(f_\eta, V_r) \left( \frac{R_0}{D(f_{\eta ref}, V_r)} - \frac{R_{ref}}{D(f_\eta, V_r)} \right)^2 \right\}

$$\exp \left\{ 1 - \frac{D(f_\eta, V_r)}{D(f_{\eta ref}, V_r)} \right\} \frac{R_0}{D(f_\eta, V_r)} \exp \left\{ D(f_\eta, V_r) \left( \frac{R_0}{D(f_{\eta ref}, V_r)} - \frac{R_{ref}}{D(f_\eta, V_r)} \right)^2 \right\}

Where, $\rho_r(t)$ is both the range coverage and the reversed Fast Fourier Transform of $\mathcal{W}_r(f_t)$ in above expression. Now $S_4$ signal is compressed with corrected range curvature for all radar line of sight targets range ward.

**Bearing Compression**

The third phase expression is adapting filter which can be the first and the fifth heterogeneous exponential expressions in equation (14). The first expression is bearing modulation the fifth expression is remained phase which is obtained from the multiplication of the first phase (scaling function). After exerting adapted- filter bearing dependent to range, range compression will be completed by exerting the
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unilateral reversed Fast Fourier Transform. As a result, received data have already been compressed in the forms of range and bearing and it produce a Single Look Complex (SLC) image.

\[ S_s(t, f_\eta) = A_4 \rho_s \left( t - \frac{2R_0}{cD(f_{\eta ref}, V_{ref})} \right) \rho_s(\eta - \eta_0) \exp \{ j\theta(t, \eta) \} \]  

(15)

Where \( \rho_s(\eta) \) is the reversed Fast Fourier Transform \( W_s(f_\eta) \) in equation (14).

- **Range Variable Scaling**

In the previous section, it was assumed that the transferred pulse was a LFM Chirp signal and \( V_r \) and \( K_m \) are invariable with range. Generally, it is not the way that \( V_r \) and \( K_m \) vary with range if upper degree expression in scaling function \( S_{sc}(t, f_\eta) \) occur. To calculate this rather small upper degree expression it is necessary to use more general equation [13]:

\[ S_{sc}(t, f_\eta) = \exp \left\{ 2\pi \int_0^1 K_m q_t(u, f_\eta) \, du \right\} \]  

(16)

That,

\[ q_t(u, f_\eta) = \left[ \frac{D(f_{\eta ref}, V_r)}{D(f_\eta, V_r)} - 1 \right] t + \frac{2R_{ref}}{c} \left[ \frac{D(f_{\eta ref}, V_r)}{D(f_{\eta ref}, V_{ref})} \frac{D(f_\eta, V_r)}{D(f_\eta, V_{ref})} \right] \]  

(17)

To calculate the value of the equation (16), the expression \( K_m q_t(u, f_\eta) \) must be extended around \( t \) with superscript series. Supposing scaling function frequency is locally linear it can be approximated as below:

\[ S_{sc}(t, f_\eta) = \exp\{ [g_0 t + g_1 t^2] \} \]  

(18)

Where therein coefficients \( g_0 \) and \( g_1 \) vary with \( t \) and \( f_\eta \) (range and bearing). With this approximation remained phase (the last expression in the equation (12)) can be obtained from the following equation:

\[ \phi_{res}(t, f_\eta) = \frac{4}{c^2} \pi K_m g_1 \left[ \frac{R_0}{D(f_\eta, V_r)} - \frac{R_{ref}}{D(f_{\eta ref}, V_{ref})} \right] + \frac{c g_0}{4 g_1} \left[ \frac{R_0}{D(f_\eta, V_r)} \right]^2 - \frac{R_0^2}{4 g_1} \]  

(19)

3. RESULTS

Against Range Migration and Range Doppler Algorithms, Chirp Scaling Algorithm can be easily implemented without inter-acquiring. It can be used to point and band capturing systems. Through this paper, Chirp Scaling Algorithm is implemented via fast Fourier Transform. The result of the Chirp Scaling Algorithm is a single sight compound image. Each pixel of the image has the I and Q vectors, real or unreal vectors, and often dynamic range from the amplitude is as:

\[ y(m, n) = [I^2(m, n) + Q^2(m, n)]^{1/2} \]  

(20)

There are many displayable algorithms and theories for remapping pixels. Large number of these theories utilizes logarithmic and digital images. Remapping a single sight compound image includes resolution extension in the form of non-linear which rewrites all pixel values between maximum and minimum amplitudes \( \sqrt{I^2 + Q^2} \) at eight bits. Each pixel value ranges from 0 to 255, Figure 5 with 1/8 resolution is captured in which bearing- time and range increase from bottom to up and from the left to the right respectively.

![Figure 5. Vancouver's image sent by the radar of the satellite 'RADARSAT1'](Image)

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Figure 6 with full resolution of the center of the image sent by satellite RADARSAT1 is used to evaluate this algorithm. First, perceiving proper understanding of 8 meters resolution should be concerned and to do this, marker is put on the point 08L – 26R of the image. According to the Vancouver airport brochure this part of runway is 60.96 meters wide.

Figure 6. Full-resolution image sent by satellite 'RADARSAT1'.

In this image, two white rectangular are displayed whose resolutions are indicated in the below table:

Table 1. Comparing the resolution of defined areas

<table>
<thead>
<tr>
<th>Location</th>
<th>Doppler Range Implementation</th>
<th>Chirp Scaling Implementation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loc 1</td>
<td>3.797</td>
<td>4.013</td>
</tr>
<tr>
<td>Loc 2</td>
<td>4.829</td>
<td>5.109</td>
</tr>
<tr>
<td>(2424,2415)</td>
<td>3.004</td>
<td>3.113</td>
</tr>
</tbody>
</table>

Figure 7 displays a target point in column 2415 and raw 2424. The left image is processed by the Range Doppler Algorithm and the right image is processed by the Chirp Scaling Algorithm. As it's clear the right image has the better resolution than the left one in range ward which is resulted by the correction of non-correspondent FM rate. In other words, passing range cells is at the same time with frequency or time.

Figure 7. Analysis of a target point in position(2424,2415) using MATLAB.

4. CONCLUSION

As previously stated, this paper implement Chirp scaling algorithm using the Fast Fourier Transform in order to increase processing speed and to produce high-resolution images. The simulation results were indicated that that proposed scheme was more accurate and faster than other implementation blueprints.

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