Analyzing and Implementation of Ant Colony Optimization in Relevance with Travelling Salesman Problem

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1. INTRODUCTION

ACO (Ant Colony Optimization) is an artificial system motivated and designed observing the behavior of ants and it finds its applications in designing discrete optimization problems. It was designed back in year 1992 and was the creation of man named Marco Dorigo. It found its first application in TSP (Traveling Salesman Problem). Researchers began to observe ant behavior and their modes of communication. Ants use pheromone while traveling from food to the nest and vice versa to communicate with each other and find the shortest path. Fig. 1 to Fig. 4 shows the approach adopted by ants in different circumstances [1, 2].
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Fig 3. The figure represents the random movement of ants between nest and food with obstacle placed in between

Fig 4. The figure represents the random movement of ants between nest and food with obstacle placed in between and ants preferring to cross the objective from above of the obstacle

Ants have to decide the course of their movement, whether they should adopt to go right or left and the choice made is purely a random decision. The shorter is the path, the faster is the rate of pheromone accumulation. The difference between the numbers of pheromone in the paths under study, makes the ants decide to choose the shorter path [1]. More the number of ants follow a particular path, the more eye-catching it becomes for other ants. Many problems relevant to optimization has been explored based on real ant behavior [2, 3].

An ant k at node r will choose the destination node s at a later stage with probability

$$p_k(r, s) = \begin{cases} \frac{\tau(r, s)^{\alpha} \eta(r, s)^{\beta}}{\sum_{u \in M_k} \tau(r, u)^{\alpha} \eta(r, u)^{\beta}}, & \text{for } s \in M_k \\ 0, & \text{otherwise} \end{cases}$$

α refers to the degree of importance of pheromone
β Refers to degree of visibility and
ueMk Refers to the choice that belongs to ant k when it was at node r.

The neighborhood of ant k at node r will contain the details of all the nodes that are possible to be addressed directly as they are connected to node r, excluding nodes that have been already visited [2, 4]. Visibility is replaced with 1/(distance between node r to node s).
Whenever an ant \( k \) passes through a segment, it is supposed to leave pheromone behind it. The number of pheromone contained in that particular segment \( ij \) after passing of the ant \( k \) is given by the formula

\[
\Delta \tau_{i,j}^{(k)} = \begin{cases} 
\frac{c f_{\text{best}}}{f_{\text{worst}}} & , \text{if } (i,j) \in \text{best global path} \\
0 & , \text{otherwise}
\end{cases}
\]

As the value of pheromone increases on the segment \( i-j \), the chances of the segment been chosen by ants during next iteration increases. After the node is passed, the pheromone evaporation takes place as per below mentioned rule [2, 5].

\[
\tau_{i,j} \leftarrow (1 - \rho)\tau_{i,j} ; \forall (i,j) \in A
\]

where \( \rho \in (0, 1] \) is the evaporation rate parameter and \( A \) represents segments that have been passed by ant \( k \) as part of the path from the nest to get to the food. As the number of pheromone decreases with time, the ants begin to explore different path during search process [1, 6]. This eliminates the chances of choosing the path that is not good enough. The number of pheromone added to the segment \( i-j \) by ant \( k \) is given as

\[
\Delta \tau_{i,j}^{(k)} = \frac{Q}{L_k}
\]

where \( Q \) is a constant and \( L_k \) represents the total distance traveled by ant \( k \) after returning to the nest (in the case of the TSP is the path traversed by the salesman). The value of \( Q \) is determined by the user [2, 7]. In many cases the value of \( Q= 1 \) gives good results. Or it could also be implemented in the following way

\[
\Delta \tau_{i,j}^{(k)} = \begin{cases} 
\frac{c f_{\text{best}}}{f_{\text{worst}}} & , \text{if } (i,j) \in \text{best global path} \\
0 & , \text{otherwise}
\end{cases}
\]

Where \( f_{\text{best}} \) represents the best value and \( f_{\text{worst}} \) refers to the worst value. Both the values are in relevance with the objective function. The constant \( c \) controls the global scale referring to pheromone updating. The greater is the value of \( c \), more are the number of pheromone added to the global path [1, 8].

2. WORKING OF ACO IN REFERENCE WITH TSP

ACO (Ant Colony Optimization) is the new technique that has been introduced in the field of swarm intelligence. The technique was developed by Marcodorigo and his companions. The technique was specialized for dealing with problems relevant to combinatorial optimization. The ACO algorithm imitates the behavior of real ant colonies in finding the shortest path between the food sources and the
nests. The ants release pheromone on the ground while moving from nest to food and vice versa. The movement of the ants is inspired by more amount of pheromones and it is followed by the other ants. The following ants tends to choose the shorter path having high concentration of pheromones. Artificial ants are capable of solving even more complicated problem than real ants [1, 9].

ACO has found its applications in handling combinatorial optimization problems such as TSP (Traveling Salesman Problem), VRP (Vehicle Routing Problem), JSP (Job-shop scheduling problem), and QAP (Quadratic Assignment Problem). The ACO suffers with the problems of premature convergence, stagnation, and the slow convergence speed and all these problems become more noticeable with the increase in the size of the problem [1, 2, 10].

The TSP is concerned with finding the shortest closed tour visiting all the cities under consideration within a problem. The TSP is further classified into two categories, symmetric TSP and asymmetric TSP. In symmetric TSP the traveling distance between two cities is same irrespective of direction of travel whereas in case of asymmetric TSP their exists variation in distance with the direction of the travel. In this research paper, the concentration is laid on symmetric TSP in which all the cities are on the same plane and there is existence of path between each pair of cities. The problem of TSP states that given N cities, if a salesman start his journey from his origin city and has to visit each city exactly once and get back to his home city, the need is to find the order of the tour in such a manner that total distance traveled should be least in terms of cost, money, time, and energy. A complete weighted graph G (N, E) represent a TSP. N refers to the set of n cities and E refers to the set of edges fully connecting all cities. Each edge (i,j) is a part of set E and assigned a cost of dij referring to the distance between cities i and j.

In the beginning, each ant is allocated a city randomly. A particular ant k having been present at city i prefers to move to city j via following below mentioned rule [2, 11].

\[
p^k_j(t) = \begin{cases} \frac{[\tau_{ij}(t)]^\alpha[\eta_{ij}]^\beta}{\sum_{k \in \text{allowed}_k} [\tau_{ik}(t)]^\alpha[\eta_{ik}]^\beta} & \text{if } j \in \text{ allowed}_k \\ 0 & \text{otherwise} \end{cases}
\]

Where nij represents the experimental visibility of edge (i,j) and can be calculated as nij = 1 / dij where d_{ij} is the distance between city i and city j.

City j refers to the set of the cities which are yet to be visited when the ant is present at city i.

\(\alpha\) and \(\beta\) refers to the adjustable positive parameters that are intended to manage the relative weights of the pheromone trail and heuristic visibility. If \(\alpha = 0\), the closed vertex i has greater chances of getting selected. If \(\beta = 0\), it means that pheromone amplification is at work. Ultimately this moves the system to a stagnate situation. Stagnate situation is the condition where all ants motivated to generate a sub-optimal tour. So there is a need of trade-off between pheromone intensity and edge length. After the completion of the tour by each ant, the amount carried by pheromone on each path gets adjusted as per equation (1 – \(\rho\)) which is pheromone decay parameter and value of \(\rho\) lies between 0 and 1 where it represents the trail evaporation when the city is chosen by the ant and it decides to move.

\(m\) represents the number of ants.

\(L_k\) is the length of the tour performed by ant k.

\(Q\) is an arbitrary constant.

On completion of the tour by all the ants, there is a need to update the pheromone as mentioned below.

\[
\tau_{ij}(t+1) = (1-\rho)\tau_{ij}(t) + \Delta \tau_{ij}(t)
\]

Where
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\[ \Delta \tau_y(t) = \sum_{k=1}^{m} \Delta \tau_y^k(t) \]

Tour Length: \[ \Delta \tau_{i,j}^k = \begin{cases} \frac{Q}{L_k} & \text{if } (i, j) \in \text{tour}_k \\ 0 & \text{otherwise} \end{cases} \]

Fig. 5 below shows the flowchart for ACO as under.

1. Initialize t=0, Randomly place ‘m’ ants on ‘n’ nodes
2. Select next node based on probability
   \[ p_{ij}^k(t) = \begin{cases} \frac{[\tau_{y}(t)]^m [\eta_y]^p}{\sum_{j \in \text{allowed}_k} [\tau_{y}(t)]^m [\eta_y]^p} & \text{if } j \in \text{allowed}_k \\ 0 & \text{otherwise} \end{cases} \]
3. Compute length of ‘\( L_k \)’, of every ant. Update the shorter tour found
4. For every edge \((i,j)\), compute
   \[ \tau_y(t+1) = (1-\rho)\tau_y(t) + \Delta \tau_y(t) \]
   \[ \Delta \tau_y(t) = \sum_{k=1}^{m} \Delta \tau_y^k(t) \]
   Tour Length: \[ \Delta \tau_{i,j}^k = \begin{cases} \frac{Q}{L_k} & \text{if } (i, j) \in \text{tour}_k \\ 0 & \text{otherwise} \end{cases} \]
5. Set t=t+1, NC=NC+1
6. Value of \( \Delta \tau_{i,j}^k \) is zero
7. NC=NC_{max} and stag
8. Yes END

Fig 5. The figure shows the flowchart detailing the working of ACO
3. CONTRIBUTION AND IMPLEMENTATION

This section elaborates the working of ACO using Matlab as a simulation tool. Consider an environment with 14 nodes. The first section of Fig. 6 shows the graph plotted considering average cost against number of cycles. The second section of Fig. 6 specifies the closed path covering all 14 nodes under consideration. The optimum course by the length is calculated as 45.562.

![Fig. 6. The figure shows the plotted graphs as per working concept of ACO with 14 nodes.](image1)

Fig. 7 again shows the graph plotted with the same scenario under consideration. The graph plotted in the first section considering average cost against number of cycles is different from the one plotted in Fig. 6. But despite of this, the optimum course length remains unchanged (45.562).

![Fig. 7. The figure shows the plotted graphs as per working concept of ACO with 14 nodes.](image2)
4. CONCLUSION

The research paper elaborated the working concept of Ant Colony optimization along with equations. The paper illustrated the functioning of ACO in relevance with TSP. The discussed idea has been implemented and simulated as discussed in section III of the paper. The advantages and disadvantages of the ACO are summarized as under.

Advantages

- ACO is capable of searching among a population in parallel.
- ACO provides rapid discovery of good solutions.
- ACO can easily adapt new changes like change in distance.
- ACO guarantees convergence.

Disadvantages

- Probability distribution can change for each iteration.
- ACO have a very difficult theoretical analysis.
- ACO have dependent sequences of random decisions.
- ACO have more experimental than theoretical research.
- ACO have uncertain time to convergence.

REFERENCES


AUTHOR’S BIOGRAPHY

Dr. Gagandeep Jagdev, is working in the capacity of a faculty member in Dept. of Computer Science, Punjabi University Guru Kashi College, Damdama Sahib (PB). His total teaching experience is more than 12 years and has 130 international and national publications in reputed journals and conferences to his credit. He is also a member of editorial board of several international peer-reviewed journals and has been active TPC member of several international and national conferences conducted by renowned universities and academic institutions. He has been awarded Best Research Paper award at NCEEITET – 2015 held at Govt. College of Engineering &Technology, Jammu. He has also been awarded with Best Research Paper Award by National Institute of Technical Teachers Training and Research (NITTTR), Chandigarh at International Conference ICCCN – 2017 conducted by Dept. of CSE. His field of expertise is Big Data, Data Mining, Image processing, ANN, Biometrics, RFID, Cloud Computing, Cloud security, Cryptography, MANETs, and VANETs.