Research on the Mathematical Model of a Thermostatic Medical Irrigation Pump

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Abstract: The medical irrigation pump is a common medical device for cleaning the wound in the surgery. A thermostatic irrigation pump with a PID (proportional-integral-derivative) controller was designed to provide warm solution nearing the body core temperature for surgical irrigation. Firstly, an experiment was designed to obtain the step response curve, so as to derive the mathematical model of the system at different flow rates. Then, a simulation model was established to verify our design. Finally, discussions were given to the experiment. The result showed that the temperature of the solution could maintain at 36±1.5°C with slight fluctuation.

Keywords: Medical irrigation pump, PID controller, Step response curve, Thermostatic control.

1. INTRODUCTION

During and after abdominal surgery, it is imperative to clean the abdominal cavity and wound with warm stroke-physiological irrigation fluid, so as to remove deeper debris, and to assist with the visual examination. Typically, it has been thought that such irrigation serves to dilute out both bacteria and those adjuvant substances (necrotic debris, barium, bile, and hemoglobin) which foster bacterial growth and it can also prevent surgery-related infection and shorten the process of rehabilitation [1]. Moreover, during the surgical operation, especially for the surgery lasting for several hours, the body tissue and other body parts of the patient are exposed to exterior environment for a long time, which leads to heat loss of the body or hypothermia. Under this circumstance, if the cold irrigation fluid is used, the core temperature of the patient will be decreased drastically soon, which will cause a series of adverse reactions, such as shivering and mental confusion, with the worst is hypothermia. Hypothermia may cause a higher incidence of cardiovascular complications, inhibit blood coagulation, impact on the immune system and slow the metabolism of anesthetic agents; all of these impairments may extend hospitalization [2]-[6]. Generally, in order to avoid the problems mentioned above, the irrigation fluid should be heated before the surgery. Thus, the thermostatic medical irrigation pump, which can provide warmed saline solution fluid for irrigation, would be helpful in clinical.

Practically, there are many kinds of medical irrigation pumps at present. But most of the medical irrigation pumps heat the irrigation fluid in advance before the surgery. When needed, take a bag of the irrigation fluid, and connect it serially to the irrigation pump. But if preheated fluid is exposed to the ambient room temperature, the amniowrization fluid will cool down within the fluid containing bag as it is hung on the pole above the pump in the operating room. Much more important is the heat loss of the fluid when it runs through the irrigation lines, which have a larger surface compared to their volume. This may cause a drop of temperature in the amniotic cavity as low as 34.06°C [7]. In order to control the temperature of the medical irrigation fluid, we designed a medical thermostatic irrigation pump that can warm the irrigation fluid in real time during operation.

In order to provide warm 36°C irrigation fluid in the medical irrigation pump, a PID controller was used. In order to tune the parameters of the PID controller in more detail, we tried to derive the mathematical model of the irrigation pump and verify the model we achieved. Correspondingly, the paper will mainly introduce how to establish the mathematical model of the thermostatic medical irrigation pump, as well as present the experiment we have done.
2. METHODS

2.1. System Description

The thermostatic irrigation pump is designed to control the temperature of the medical irrigation fluid for the surgical purpose. The outline of the irrigation pump is shown in Fig.1 (a). As we can see in Fig.1 (a), the irrigation fluid is pumped from a supply bag, which has scale lines showing the volume of the bag, into the instrument by the movement of the peristaltic pump through the soft tube. And there is a panel in front of the instrument. The panel is composed of five buttons, two LEDs (Light Emitting Diode) and a LCD (Liquid Crystal Display). The five buttons can be used to adjust the flow rates and the maximum irrigation pressure. The two LEDs indicate the status of the power supply and the irrigation status of the instrument respectively. Furthermore, the LCD shows the essential parameters of the irrigation pump, such as the actual flow rates, the irrigation pressure, the temperature of the irrigation fluid and other related information. When the instrument works in the normal model, the thermal fluid flows out of the instrument through the soft tube and then pours into the patient’s body during the surgery.

Fig.1(b) shows the central core of the irrigation pump, which is a square aluminum heating plate. Over its surface, a thin supply water-bag is mounted through which the fluid runs. Subsequently, a second parallel heating plate is locked over the coiled water-bag, sandwiching the fluid film passing in between the heating elements. The thermal flow is connected serially to the irrigation pump. In our study, the temperature is ideally set to 36°C.

Fig1. The schematic diagram of the medical irrigation pump. (a) Outline of the device; (b) System composition.

2.1.1. The Fluid System

A peristaltic pump, a coiled water-bag, and a flexible tube are the main parts of the fluid system [8]. The peristaltic pump is a type of positive displacement pump used for pumping the fluids, i.e., saline solution or distilled water. The fluid is contained within the flexible tube fitted inside the pump casing. Like peristalsis, the pumping principle is based upon alternating compression and relaxation of the tube to draw the saline solution in and propel it away from the pump. Because the mechanical workings of the peristaltic pump never come into contact with the fluid directly, this kind of pump can avoid cross contamination in hospital, where sterile fluids are required. Actually, the peristaltic pump regulates pressurized liquid by repeatedly squeezing a flexible tube in the same location with circular rollers. Each turn of the roller moves a small volume of the liquid forward. Its speed is controlled by the peristaltic pump motor driver, and thus the flow speed is controlled.

The coiled water-bag is a container to temporarily deposit the saline solution. The saline solution after being pumped into the coiled water-bag is heated gradually along this s-type tube until its temperature reaches about 36°C, and flows out of the heater.

2.1.2. The Temperature Sensors

Three temperature sensors in the system are used for the MCU to decide how to provide the power energy for the heater to heat the saline solution in the water-bag.

As shown in Fig.1(b), three temperature sensors are employed in the system. The first one (Tsin) is placed at the inlet port of the soft tube, which is connected to the saline water-bag, to indicate
the ambient temperature, which equals the temperature of the incoming saline solution. The second one (Tsout) is placed at the outlet port of the soft tube to indicate the temperature of the outgoing saline solution, which will be very close to 36°C with some error. The third one (Thin) is laid on one of the center of one of the aluminum plates to monitor the heating temperature of the two paralleled plates. These three temperature signals will be collected by the MCU after they are converted to the corresponding electric signals by their temperature transmitters. The decisions on how to control the PID algorithm, correspondingly control the heater, are made based upon these three temperature sensors.

2.1.3. The Heating System

Mechanically, from the coiled water-bag outward of the heating system are two sheets of aluminum to get a more uniform temperature, which keep the saline solution inside the coiled water-bag to be warmed gradually until it reaches 36°C, the two sheets of resistance wires using the electrical Ohm’s law to getting heat, and two sheets of thermal insulation to reduce heat loss. From the electricity, the heating process is controlled by a custom-built controller using a solid state relay (SSR), and a PID controller with ramp/soak programming features. The SSR can switch many times a second if necessary allowing pretty finely control over how much heat is delivered via the two sheets of aluminum [9].

Considering the dangers inherent within electricity as a medical device, which can do damage to the human body directly, such as stoppage of breathing or irregular heartbeats, or burns, 50V safe voltage is adopted in the system from the point of view of safety voltage.

2.1.4. The MCU and its Peripheral Devices

The MCU is the core of the system. It performs the thermostatic control algorithm, controls the flow rate of the saline solution by regulating the speed of the peristaltic pump, as well as interfaces with other peripheral devices. The signals from three temperature sensors are transported to the MCU after they are transmitted or conditioned from their non electric-quantities.

Other peripheral devices are composed of a liquid crystal display (LCD), an alarm component, and four setting buttons. The LCD displays the temperatures at the incoming inlet and the outgoing outlet port, the flow rate of the saline solution, and other necessary prompt information. The alarm component gives sound and light alarm if one of the followings happens, (1) the temperature at the outgoing outlet port is higher than 40°C, which is very hot to human tissue, and gives a message to the MCU to stop heating; (2) the pressure inside the tube is higher than a threshold, which may be caused by the system error, the speed of the peristaltic pump increase for example; (3) the temperature of the heater is higher than the value that is set by the MCU. Four setting buttons include a function setting button to switch between the flow parameter setting and the pressure threshold setting, an increment setting button, pressed when an increment setting is needed; a decrement setting button, pressed when a decrement setting is needed; an enter button, pressed when a confirmation is needed.

2.2. The PID Controller

Despite the recent development of sophisticated electrical controllers, the simple proportional-integral-derivative (PID) algorithm is still the majority of the controllers found in industrial applications [10-13]. Among the many control methods for heat pumps and other heating systems, the PID algorithm is still a common one [14]. In the paper we firstly adopt the PID algorithm to maintain the heating plates at a relatively constant temperature. After the design of mechanical structure and the electrical control system, we try to obtain the step response curve of the irrigation pump system by experiments; and then calculate the characteristic parameters of the transfer function according to the response curves; finally we use the Simulink Toolbox of the MATLAB software to verify the accuracy of the transfer function.

The PID controllers are the largest number of controllers found in industries sufficient for solving many control problems [15-17]. The PID control law is the sum of three types of control actions: a proportional, an integral and a derivative control action. Mathematically, the PID controller in the time-domain is given by the following equation [18, 19]:
\( u(t) = K_p \left[ e(t) + \frac{1}{T_i} \int_0^t e(\tau) d\tau + T_d \frac{d}{dt} e(t) \right] \)  
\((1)\)

where \( u(t) \) is the controller output (input signal to the model), the error signal \( e(t) \) is defined as \( e(t) = r(t) - y(t) \), \( r(t) \) is the reference input signal while \( y(t) \) is the output. The controller parameters are proportional gain \( K_p \), integral time \( T_i \), and derivative time \( T_d \) [19,20].

The functionalities of each part are highlighted by the following [21,22]:

- The proportional part - providing an overall control action proportional to the error signal through the all-pass gain factor.
- The integral part - reducing steady-state errors through low-frequency compensation by an integrator.
- The derivative part - improving transient response through high-frequency compensation by a differentiator.

If a mathematical model of the PID-controlled system can be derived, the various design techniques for determining the controller parameters can be applied. However, if the system is so complex that its mathematical model cannot easily be obtained, then analytical approach to design PID controller is not possible [23]. In that case, one must resort to experimental approaches for tuning the PID controllers.

### 2.3. The Step Response Curve

A large number of industrial systems can approximately be modeled by the first order plus dead time (FOPDT) model with transfer function as follows [24-26]:

\[ G(s) = \frac{K}{Ts + 1} e^{-\tau s} \]
\((2)\)

The model is characterized by three parameters: the static gain \( K \), the time constant \( T \), and the dead time \( \tau \) [20,25,26]. There are many methods to obtain the mathematical model of the system, such as analysis and experiments. The step response is the method used in our work.

The step response is a convenient way to characterize process dynamics because of its simple physical interpretation. Many tuning methods are based on it. A formal mathematical model can also be obtained from the step response. Moreover, models obtained from a transient experiment are often sufficient for PID controller tuning. The methods are also very simple to use. And then general methods for the design of control systems can then be used [20].

Fig 2. The step response curve of a first-order inertia system with dead time. (a) The step excitation; (b) The step response curve.

Here we use the step response curve (shown in Fig.2) combined with the Cohen-Coon formula to derive the mathematical model of the irrigation pump. The Cohen-Coon formula is as follows [27, 28]:

\[ K = \frac{\Delta out}{\Delta in} = \frac{y(\infty) - y(0)}{u(0)} \]
\((3)\)
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\[ T = 1.5(t_{0.632} - t_{0.28}) \]  \quad (4)

\[ \tau = 1.5(t_{0.28} - \frac{t_{0.632}}{3}) \] \quad (5)

where, \( y(0) \) is the initial value of the output; \( y(\infty) \) is the final value of the output; \( u(0) \) is the input step excitation; \( \Delta u \) equals \( u(0); \Delta out \) is the difference between the final value of the output and the initial value of the output; \( t_{0.28} \) is the time for heating plates to reach \( 0.28\Delta out \) in the step response curve; \( t_{0.632} \) is the time for heating plates to reach \( 0.632\Delta out \) in the step response curve.

The step response experiment can be determined as follows:

- Provide an excitation to the system as follows: leave the temperature controllers aside and wait for the system to stabilize; give the solid state relay a square wave whose cycle time is 4s, and then the equivalent signal of the square wave is a step signal for the plates.
- Record the temperature of the heating plates every second.
- Plot the step response curve according to the recorded data.
- Calculate the transfer function parameters – \( K, T, \tau \), according to the Cohen-Coon formula from (3)-(5).
- Calculate the PID controlling parameters – \( K_p, K_i, K_d \) according to Ziegler-Nichols formula [29, 30].
- Verify the transfer function and the controlling parameters by simulating the PID controlling system by Simulink Toolbox.
- Apply the PID controlling parameters to the real irrigation pump.

3. DATA PROCESSING

![Fig3. The step response of the irrigation pump. (a) The flow rate is 200ml/min; (b) The flow rate is 400ml/min.](image)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>200</th>
<th>400</th>
</tr>
</thead>
<tbody>
<tr>
<td>Duty cycle of square wave(%)</td>
<td>15</td>
<td>20</td>
</tr>
<tr>
<td>( y(\infty) / ) (°C)</td>
<td>48.2</td>
<td>48.4</td>
</tr>
<tr>
<td>( y(0) / ) (°C)</td>
<td>28.6</td>
<td>28.2</td>
</tr>
<tr>
<td>( u(0) / ) (volt)</td>
<td>7.5</td>
<td>10.0</td>
</tr>
<tr>
<td>( t_{0.28} / ) (s)</td>
<td>44.1</td>
<td>31.8</td>
</tr>
<tr>
<td>( 0.28y / ) (°C)</td>
<td>34.0</td>
<td>34.0</td>
</tr>
<tr>
<td>( t_{0.632} / ) (s)</td>
<td>99.5</td>
<td>73.2</td>
</tr>
<tr>
<td>( 0.632y / ) (°C)</td>
<td>41.0</td>
<td>41.0</td>
</tr>
<tr>
<td>( K )</td>
<td>2.61</td>
<td>2.02</td>
</tr>
<tr>
<td>( T )</td>
<td>83.1</td>
<td>62.1</td>
</tr>
<tr>
<td>( \tau )</td>
<td>16.4</td>
<td>11.1</td>
</tr>
</tbody>
</table>
The irrigation pump that we designed can work at two different flow rates, namely 200ml/min for the minor surgery with small incisions and 400ml/min for the major surgery with large incisions. Therefore, we analyze the experimental results for these two rates respectively (shown in Fig.3). Firstly, we analyze the experimental data to obtain the initial value of the output \( y(0) \), the final value of the output \( y(\infty) \), the time \( t_{0.28} \) and \( t_{0.632} \). At last we get the \( K, T, \tau \) of the transfer function from formula (3) to formula (5). The parameters are listed in the Tab.1.

The “Parameter” column is the list of different parameters used and derived in the process of the system.

200 refers to that the flow rate of the infusion fluid is 200 ml/min.

400 refers to that the flow rate of the infusion fluid is 400 ml/min.

We can conclude from the table that the transfer function of the irrigation pump system at the flow rate of 200ml/min is:

\[
G(s) = \frac{2.61}{1+83.1s} e^{-16.4s}
\]  

Also, the transfer function at the flow rate of 400ml/min is:

\[
G(s) = \frac{2.02}{1+62.1s} e^{-11.1s}
\]  

4. SIMULATION

Now that we have obtained the transfer functions of the system, we want to find out whether these two transfer functions can describe the characteristics of the system. Thus, we use the Simulink Toolbox of the MATLAB software to establish the simulation model and then simulate the mathematical model and controlling parameters [31], so as to find out whether the step response can stabilize. If the output of the step excitation could maintain at a prospective value, then we can conclude that the transfer function is correct; otherwise, the transfer function need to be corrected.

Fig.4 shows the simulation model, where ‘Step’ is the step pulse, ‘Integrator’ the integral part, ‘Derivative’ the derivative part, ‘Tran Fcn’ the transfer function at different flow rate, ‘Transport Delay’ the dead time. And the PID controlling parameters are ‘Gain’, ‘Gain1’, ‘Gain2’, namely the proportional gain, the integral gain and the derivative gain.

![Simulink simulation model of the thermostatic irrigation pump](image)

According to the Ziegler-Nichols formula, we can derive the PID controlling parameters. The formula is as follows [30]:

\[
K_p = \frac{1.2* T}{K* \tau} , \quad K_i = \frac{K_p}{2\tau} , \quad K_d = \frac{K_p * \tau}{2}
\]  

The calculated controlling parameters from (7) at different flow rate are listed in Tab.2.
Table 2. The parameters of the PID controller of thermostatic irrigation pump.

<table>
<thead>
<tr>
<th>PID parameter</th>
<th>200</th>
<th>400</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_p$</td>
<td>0.23</td>
<td>0.33</td>
</tr>
<tr>
<td>$K_i$</td>
<td>0.007</td>
<td>0.015</td>
</tr>
<tr>
<td>$K_d$</td>
<td>1.89</td>
<td>1.83</td>
</tr>
</tbody>
</table>

The “PID parameters” column is the list of the PID controlling parameters derived from the transfer function.

200 refers to that the flow rate of the infusion fluid is 200 ml/min.

400 refers to that the flow rate of the infusion fluid is 400 ml/min.

Fig. 5 displays the results of the model simulation at different flow rate. According to Fig. 5(a), we can see that the response of the step excitation maintains at 48.2°C after a period of time of fluctuation when the flow rate is 200ml/min. Thus we can conclude that the transfer function can describe the dynamic characteristics of the irrigation pump correctly and the transfer function is accurate and applicable. It is the same with the simulation result at the flow rate 400ml/min.

5. RESULT

Our ultimate goal was to control the temperature of the irrigation fluid at the outlet of the irrigation pump to reach about 36°C, a little lower than the body core temperature. Therefore, we applied the PID algorithm to our system and recorded the supply voltage presented on the heater as well as the temperature of the heating plates and the temperature of the irrigation fluid at the outlet of the irrigation pump at different flow rate, and then process these data as shown in Fig. 6.

![Fig6. Temperature at the outlet of the thermostatic irrigation pump.](image)

Fig. 6 showed that the irrigation fluid reached a relatively proper temperature after 2 minutes’ warm (segment A), and then the temperature maintained at 36±1.5°C with slight fluctuation (segment B), as about 36.0°C with flow rate 200ml/min and about 36.5°C with flow rate 400ml/min.

![Fig7. The overall heating voltage and temperature recordings.](image)

(a)

(b)
Fig.7 (a) and Fig.7 (b) showed the overall heating voltage and temperature recordings of the irrigation pump. The right vertical axis was the reference of the input voltage, while the left vertical axis was the reference of the temperatures. The black line in Fig.7 was the voltage presented on the heater, when the output was 50, the aluminum plates were heating; and when the output was 0, the aluminum plates were out of work. The green line was the variation of the heating plate’s temperature and the red line was the variation of the temperature of the irrigation fluid at the outlet.

From Fig.7, we could see that voltage switching on the heater was very frequent and the temperatures of the heating plates and the irrigation fluid at the outlet rose rapidly during the first 150s in Fig.7 (a) and the first 180s in Fig.7 (b), and then each maintained at a certain value with slight fluctuation. According to the PID algorithm, we knew that the variations of the voltage and the temperature could be attributed to the PID controller. At the beginning, the irrigation fluid was cold (about 20°C), which made the real temperature of the heating plates far from the prospective one. In order to let the temperature of the heating plates reach the respective one, the supply voltage presented on the heater had a bigger duty cycle, which made the temperature of the heating plates and the temperature of the irrigation fluid rise rapidly. When the temperature of the heating plates approaching to the prospective temperature, the difference between them became smaller, resulting in a smaller duty cycle of the supply voltage presented on the heater. And then the temperature of the irrigation fluid at the outlet maintained at a certain value gradually. With the slight fluctuation of the heating plates’ temperature, the duty cycle of the supply voltage presented on the heater changing slightly, attributing to the slight variation of the temperature of the irrigation fluid at the outlet. Finally, the temperature of the irrigation fluid maintained at a certain temperature with slight fluctuation.

The result illustrated that the obtained transfer function parameters could accurately reflect the characteristics of the irrigation pump system. And the controlling parameters obtained from the transfer functions could provide good control of the irrigation pump. This could further prove that the transfer function was accurate and reliable.

6. CONCLUSION

In order to control the temperature of the irrigation fluid at the outlet of a medical irrigation pump, we tried to design a PID controller. But if a mathematical model of the PID-controlled plant can be derived, the various design techniques for determining the controller parameters can be applied. Therefore, we designed an experiment to obtain the mathematical model of the irrigation pump system to prepare for detail controlling parameters tuning. We designed two experiments to get the step response curves of the pump at two different flow rates, and calculated their parameters of the transfer functions to get the PID controlling parameters. And then we simulated the obtained transfer functions to verify their accuracies. After the verification, we applied the controlling parameters to the real irrigation pump, and we found that the transfer functions we obtained could really describe the characteristics of the irrigation pump system correctly and the temperature of the irrigation fluid at the outlet could maintain at the prospective temperature with slight fluctuation (36±1.5°C).

These results illustrated that our design of experiment was correct and reliable, and the mathematic model we obtained was accurate and applicable.

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AUTHOR’S BIOGRAPHY

Rongguo Yan achieved his bachelor degree in 1997 from Huaihai Institute of Technology, master degree in 2001 from Nanjing University of Aeronautics and Astronautics and Ph.D. degree in 2006 from Shanghai Jiaotong University; he is an associate professor in University of Shanghai for Science and Technology at present and his research direction is biomedical engineering.