Estimation of Mean Time to Recruitment for a Two Grade Manpower System with Inter - Exit Times as Geometric Process When Threshold has Two Components

A.Srinivasan¹
Professor Emeritus, PG & Research Department of Mathematics,
Bishop Heber College, Trichy-17.
mathsrinivas@yahoo.com

P. Arokkia Saibe²
M.Phil Scholar,
PG & Research Department of Mathematics,
Bishop Heber College, Trichy-17
shaibeglitz@gmail.com

Abstract: In this paper, for a two grade manpower system, three mathematical models are constructed and using univariate policy of recruitment based on the shock model approach, the expected time to recruitment is obtained when (i) the loss of manpower due to attrition form an ordinary renewal process (ii) inter – exit times form a geometric process and (iii) the threshold for each grade has two components. A different probabilistic analysis is used to derive the analytical result.

Keywords: Two grade manpower system, attrition, ordinary renewal process, exit times, geometric process, recruitment.

AMS Mathematics Subject Classification (2010): 990B70, 91B40, 91D35, 60H30

1. INTRODUCTION

Exit of personnel, voluntary and involuntary, is a common phenomenon in any marketing organization. This leads to reduction in the total strength of marketing personnel and will adversely affect the sales turnover of the organization, if recruitment is not planned. In fact, frequent recruitments may also be expensive due to the cost of recruitments and training. As the loss of manpower is unpredictable, a suitable recruitment policy has to be designed to overcome this loss. A univariate recruitment policy, usually known as CUM policy of recruitment in the literature, is based on the replacement policy associated with the shock model approach in reliability theory and is stated as follows: Recruitment is made whenever the cumulative loss of man hours exceeds a breakdown threshold. Several researchers have studied the problem of time to recruitment for a two grade manpower system using shock model approach. In this context, in [18], [11], [12], [3], [16], [14], [15], [19],[13] and [17] the authors have obtained the variance of the time to recruitment for a two grade manpower system using several recruitment policies under different conditions on the loss of man power, breakdown thresholds when the inter – policy decision times form an ordinary renewal process. In [20], [2] and [4] the authors have estimated the mean time to recruitment using geometric process for inter – decision times. In [6], [5], [7] and [8] the authors have studied the problem of time to recruitment with two sources of depletion under different conditions on the inter-policy decisions, inter-transfer decisions when the breakdown threshold for each grade has only one component. In [1] the authors have determined the mean time to recruitment for a single manpower system with policy decisions forming the only one source of depletion when the threshold for the cumulative loss of manpower has three components. Recently, in [9] and [10] the authors have extended this work for a two grade man power system according as the inter-policy decision times and inter-transfer decision times form the same or different ordinary renewal processes respectively. The objective of this paper is to derive the mean time to recruitment for a two grade manpower system using univariate CUM policy of recruitment when (i) the inter-exit times (exit times can be either voluntary or involuntary) form a geometric process and (ii) the breakdown threshold for each grade has two components.

2. MODEL DESCRIPTION

Consider an organization consisting of two grades (grade A and grade B) in which exit of personnel takes place voluntarily (due to policy decisions, VRS etc.) or involuntarily (death, retirement etc.). Let
Xₙ be the loss of man power in the organization at the nᵗʰ exit point, n=1,2,3,… . It is assumed that (Xₙ)ₙ≥₁ is a sequence of independent and identically distributed exponential random variables with tail distribution \( \bar{G}(.) \) and mean \( \frac{1}{\alpha} \) (\( \alpha > 0 \)). Let \( \chi_A(.) \) indicator function of the event A. Let \( S_n \) be the cumulative loss of man power in the organization corresponding to the first n exits. Let \( U_n \) be the time between (n-1)ᵗʰ and the nᵗʰ exit times, n=1,2,3,… and \( R_i+1 \) be the waiting time up to (i+1)ᵗʰ exit. It is assumed that \( \bar{U}_n \) is a geometric process with rate ‘a’ and \( E(U_1) \) be the mean of \( U_1 \). Let T be the breakdown threshold for the cumulative loss of man power in the organization with probability density function \( h(.) \). For grade A, let \( T_{A1} \) be the normal exponential threshold of depletion of manpower with mean \( \frac{1}{\theta_{A1}}(\theta_{A1} > 0) \) and \( T_{A2} \) be the exponential threshold of frequent breaks of existing workers with means \( \frac{1}{\theta_{A2}} \). For grade B, let \( T_{B1} \) and \( T_{B2} \) be the normal exponential threshold of depletion of manpower, exponential threshold of frequent breaks of existing workers with means \( \frac{1}{\theta_{B1}}, \frac{1}{\theta_{B2}} \) respectively. Let \( S_i, T \) be the joint density function of \( S_i \) and T. The following recruitment policy, known as CUM policy of recruitment is employed in the present work. Recruitment is done whenever the cumulative loss of man power in the organization exceeds the threshold level T. Let W be the time to recruitment with mean \( E(W) \).

3. **Main Results**

By the recruitment policy, recruitment is done whenever the cumulative loss of man power exceeds the threshold T. When the first exit takes place, recruitment would not have been done for \( U_1 \) units of time. If the loss of man power \( X_1 (= S_1) \) at the first exit point is greater than T, then recruitment is done and in this case \( W = U_1 = R_1 \). However, if \( S_1 \leq T \) the non – recruitment period will continue till the next exit point. If the cumulative sum \( S_2 \) of the loss of man power up to the second exit point exceeds T, then recruitment is done and \( W = U_1 + U_2 = R_2 \). If \( S_2 \leq T \), then the non – recruitment period will continue till the next exit point and depending on \( S_3 > T \) or \( S_3 \leq T \), recruitment is done or the non – recruitment period continues and so on. This observation leads to the following expression for the time to recruitment.

\[
W = \sum_{i=0}^{\infty} R_{i+1} \mathcal{X}(S_i \leq T < S_{i+1})
\]  
\[
(1)
\]

Since \( E(R_{i+1}) = \sum_{j=0}^{\infty} E(U_{j+1}) \), from (1) we get

\[
E(W) = \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} E(U_{j+1}) P(S_i \leq T < S_{i+1})
\]

(2)

Using the law of total probability and the result \( E(U_k) = \frac{E(U_k)}{a_k-1}, k=1,2,… \) in (2), we get

\[
E(W) = \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} E(U_{j+1}) \int_{0}^{\infty} \int_{0}^{t} G(t-x) \left( e_{S_i,T}(x,t) \right) dx dt
\]

(3)

We now consider different forms for T and obtain the mean for time to recruitment.

**CASE (I):** \( T = \max (T_{A1}+T_{A2}, T_{B1}+T_{B2}) \)

The present case for the breakdown threshold T is the best choice when we permit mobility of personnel from one grade to another in order to compensate loss of man power which is larger among the two grades. In this case it is found that

\[
h(t) = \frac{\theta_{A1}\theta_{B2}e^{-(\theta_{B2})t}}{(\theta_{B1}+\theta_{B2})} - \frac{\theta_{B1}\theta_{B2}e^{-(\theta_{B1})t}}{(\theta_{B1}+\theta_{B2})} - \frac{\theta_{A1}\theta_{B1}\theta_{B2}e^{-(\theta_{B1})t}}{(\theta_{B1}+\theta_{B2})} + \frac{\theta_{A1}\theta_{B1}\theta_{B2}e^{-(\theta_{A1}+\theta_{B2})t}}{(\theta_{B1}+\theta_{B2})} + \frac{\theta_{A1}\theta_{B2}e^{-(\theta_{A1}+\theta_{B2})t}}{\theta_{B1}+\theta_{B2}} + \frac{\theta_{A1}\theta_{B2}e^{-(\theta_{A1}+\theta_{B2})t}}{\theta_{B1}+\theta_{B2}} - \frac{\theta_{A1}\theta_{B2}e^{-(\theta_{A1}+\theta_{B2})t}}{(\theta_{A1}+\theta_{B2})}.
\]

(4)

Since \( S_i \) and T are independent, using (4) in (3) and on simplification, we find the following expression for mean time to recruitment for the present case.
Estimation of Mean Time to Recruitment for a Two Grade Manpower System with Inter-Exit Times as Geometric Process When Threshold has Two Components

\[
E(W) = 
\begin{pmatrix}
\frac{\theta B_1 (a + \theta B_2)}{(a + \theta B_2)^2} - \frac{\theta B_2 (a + \theta B_1)}{(a + \theta B_1)^2} - \frac{\theta A_1 \theta B_1 (a + \theta B_2 + \theta B_1)}{(a + \theta B_2 + \theta B_1)^2} + \\
\frac{\theta A_1 \theta B_1 (a + \theta B_2 + \theta B_2)}{(a + \theta B_2 + \theta B_2)^2}
\end{pmatrix}
\]

\[
E(U_1) = 
\begin{pmatrix}
\frac{\theta A_1 \theta B_1 (a + \theta B_2)}{(a + \theta B_2)^2} - \frac{\theta A_1 \theta B_1 (a + \theta B_1)}{(a + \theta B_1)^2} - \frac{\theta A_1 \theta B_1 (a + \theta B_2 + \theta B_1)}{(a + \theta B_2 + \theta B_1)^2} + \\
\frac{\theta A_1 \theta B_1 (a + \theta B_2 + \theta B_2)}{(a + \theta B_2 + \theta B_2)^2}
\end{pmatrix}
\]

(5) gives the mean time to recruitment to the present case.

**CASE (II):** T = \(T_{A_1} + T_{A_2} + T_{B_1} + T_{B_2}\)

The present case for the breakdown threshold T is the best choice when we assume transfer of personnel between grades is not permitted. In this case it is found that

\[
h(t) = \frac{\theta A_1 \theta B_1 \theta B_2 e^{-(\theta A_1 + \theta B_1) t}}{(\theta A_1 - \theta B_1)(\theta B_1 - \theta B_2)} - \frac{\theta A_1 \theta B_1 \theta B_2 e^{-(\theta A_1 + \theta B_2) t}}{(\theta A_1 - \theta B_2)(\theta B_1 - \theta B_2)} + \frac{\theta A_1 \theta B_1 \theta B_2 e^{-(\theta A_1 + \theta B_1) t}}{(\theta A_1 - \theta B_1)(\theta B_1 - \theta B_2)}
\]

\[
E(W) = E(U_1) a
\]

(7) gives the mean time to recruitment to the present case.

**CASE (III):** T = \((T_{A_1} + T_{A_2}) + (T_{B_1} + T_{B_2})\)

The choice for T cited in case(iii) provides a better maximum allowable loss of man power in the entire organization compared to the choices mentioned in cases (i) and (ii). In this case it can be shown that

\[
h(t) = \frac{\theta A_1 \theta B_1 \theta B_2 e^{-(\theta A_1 + \theta B_1) t}}{\theta A_1 - \theta B_1}(\theta B_1 - \theta B_2) - \frac{\theta A_1 \theta B_1 \theta B_2 e^{-(\theta A_1 + \theta B_1) t}}{\theta A_1 - \theta B_2}(\theta B_1 - \theta B_2) + \frac{\theta A_1 \theta B_1 \theta B_2 e^{-(\theta A_1 + \theta B_1) t}}{\theta A_1 - \theta B_1}(\theta B_1 - \theta B_2)
\]

\[
E(W) = E(U_1) a
\]

(8) gives the mean time to recruitment to the present case.
The analytical results for the mean time to recruitment when the inter – exit times form an ordinary renewal process can be deduced from our results by taking \( a=1 \).

1. If the threshold for grade A (grade B) has a third component corresponding to backup or reservation sources, the analytical result for all the three cases can be obtained by convoluting the distribution of this component with that of the two components considered in this paper.

From the organization’s point of view, case (iii) is more suitable than cases (i) and (ii) as the time to recruitment is elongated compared to cases (i) and (ii).

**References**


Estimation of Mean Time to Recruitment for a Two Grade Manpower System with Inter-Exit Times as Geometric Process When Threshold has Two Components


AUTHORS’ BIOGRAPHY

Dr. A. Srinivasan, Professor Emeritus PG & Research Department of Mathematics Bishop Heber College (Autonomous), Tiruchirappalli-17, has obtained M.Sc. Mathematics Degree in 1976 and Ph.D Degree in 1985 from Annamalai University. He joined Bishop Heber College in 1988 as Assistant Professor in Mathematics. He has 27 years of teaching and research experience and produced 10 Ph.D’s and 28 M.Phil Degree students. At present he is guiding 8 students for Ph.D program. He has published 98 research papers in International Journals, 72 in National Journals, 23 papers in the Proceedings of the International conferences and 28 in that of National conferences. He has completed one UGC Minor Research Project. He has refereed research papers for International Journals. The thrust areas of his research are Stochastic manpower models, Queues, Retrial Queues, Fuzzy inventory models and First passage times. He is a life member in the Indian Mathematical Society, the Indian Society for Probability and Statistics and the forum for Inter disciplinary Research in Mathematics. He visited Manila in Philippines to present a research paper in an International conference in 2000. He presented a research paper at the International Congress for Mathematicians held in the University of Hyderabad during 19-27 August 2010. He is a resource person and Chairperson for several International and National level conferences / Seminars and Refresher courses for University and College teachers sponsored by UGC and DST. He is a member in several academic bodies and recruitment boards.

Ms. P. Arokkia Saibe, Mphil Scholar, PG & Research Department of Mathematics, Bishop Heber College (Autonomous), Trichy-17, was born on 27/02/1992 at Trichy district. she obtained her B.Sc degree in 2012, M.Sc degree in 2014 and pursuing her M.phil degree in Bishop Heber College (Autonomous), Trichy-17. She has published a Research paper in Stochastic Processes in International Journal

International Journal of Managerial Studies and Research (IJMSR)