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Flow of a Maxwell Dusty Fluid Over a Stretching Surface

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Abstract: *The flow of Maxwell dusty fluid over a stretching surface with variable thickness is considered. The governing partial differential equations are transformed to a system of ordinary differential equations. These equations are solved numerically by RKBS-45 method. The effects of various physical parameters such as fluid particle interaction parameter, and wall thickness parameter are presented and discussed. The comparison with previous results shows an excellent agreement.*

Keywords: *Maxwell Fluid, Dusty fluid, Stretching surface, Variable thickness*

1. INTRODUCTION

The investigation of flow over a stretching surface takes place in a lot of industrial applications such as tap manufacturing, plastic and glass forming…etc., a lot of researches considered the Newtonian fluids and others considers the non-Newtonian fluids. A lot of mathematical models simulate the non-Newtonian fluids. From the technology point of view, the study of non-Newtonian fluid flow is more reliable than the Newtonian one. The non-Newtonian fluid takes different models such as third grade model, viscoelastic, micropolar, dusty…, etc. Studies on the third grade non-Newtonian fluid can be seen in refs. [1-3] to show the shear thickening and shear thinning characteristics. Hayat et al. [1] considered the effect of MHD flow over an exponentially stretching sheet in the presence of first order chemical reaction, while Salahuddin et al. [2] examined the effects of temperature dependent viscosity and thermal conductivity on MHD stagnation point flow over a stretching cylinder. Rashidi et al. [3] studied the convective flow due to a linearity stretching sheet subjected to magnetic field.

The viscoelastic non-Newtonian fluid introduces the viscous and elastic approaches. These approaches are investigated in refs [4-6]. Baag et al. [4] analyzed the entropy generation of an electrically conducting viscoelastic liquid over a stretching flat surface, Babu and Sandeep [5] presented the cross-diffusion effects on the MHD Williamson fluid flow across a variable thickness stretching sheet by viewing velocity slip, and Elbashbeshy et al. [6] considered the effects of unsteadiness and micropolar on the Maxwell fluid.

The study of dusty fluid introduces the effect of the fluid-particle interaction as in refs. [7-16]. Vajravelu and Nayfeh [7] analyzed the hydro magnetic flow of a dusty fluid over a stretching sheet showing the effects of fluid-particle interaction, particle loading, and suction on the flow characteristics, while Gireesha et al. [8] considered the flow of dusty fluid over a vertical stretching surface, Gireesha et al. [9]extended the study for the unsteady flow of a dusty fluid over stretching surface in the presence of non-uniform heat source/sink, and Gireesha et al. [10]considered the viscous dissipation while studying the dusty fluid flow over a flat stretching surface.

Ramesh et al. [11] studied the MHD flow of dusty fluid over an inclined flat stretching sheet with non-uniform heat source/sink. Pavithra and Gireesha [12] considered the dusty fluid flow over exponentially stretching flat surface in the presence of viscous dissipation and internal heat generation/absorption. Ramesh and Gireesha [13] analyzed the radiation effect on the flow of a dusty fluid over a stretching flat surface. Gireesha and Chamkha [14] studied the unsteady convective flow of a dusty fluid over a stretching flat surface in the presence of thermal radiation and space-dependent heat source/sink. Pavithra and Gireesha [15] investigated the unsteady flow of a dusty fluid over an exponentially stretching flat surface subjected to suction. Rauta and Mishra [16] investigated the two phase flow in a porous medium over a stretching flat surface with internal heat generation.

Nomenclature			
a, b	Constants	β	The elasticity parameter
C_f	Local skin-friction coefficient	η	Dimensionless coordinate
f	Dimensionless functions	λ	The relaxation time of the fluid
F	Dimensionless functions	μ	Dynamical viscosity, $Kg/m.s$
G, H	Dimensionless functions	υ	Kinematical viscosity= $\mu/\rho, m/s^2$
K	Stokes resistance	ρ	Fluid density, Kg/m^3
n	Velocity power index	σ	Electrically conductivity
Re_x	Local Reynold's number	ψ	Physical stream function
U_w	Velocity of solid surface	Subscripts	
u, v	Velocity components along x and y, m/s	w	Condition on the wall
x, y	Distance along and normal to the surface	∞	Free stream (ambient) condition
Greek symbols		O	Reference value
α	Wall thickness parameter	P	Particle

A lot of researches included the effect of nanoparticles on the dusty fluid such in refs. [17-23]. Sandeep et al. [17] analyzed the unsteady MHD radiative dusty nanofluid over an exponentially permeable stretching flat surface in the presence of volume fraction of dust and nanoparticles. Also, Manjunatha and Gireesha [18] studied the flow of MHD dusty fluid over an unsteady stretching flat surface. Sandeep and Sulochana [19] analyzed the flow of MHD nanofluid embedded with conducting dust particles past a stretching surface in the presence of volume fraction of dust particles. Daniel et al. [20] considered the MHD flow of nanofluid over nonlinear stretched surface with variable thickness in the presence of electric field. Bhatti et al. [21] studied the influence of nonlinear thermal radiation on the laminar incompressible, dissipative electro-magneto-hydrodynamic peristaltic propulsive flow of non-Newtonian dusty fluid through a porous planar channel. Reddy et al. [22] studied the flow with heat and mass transfer of nanofluid over a stretching surface with variable thickness and variable thermal conductivity under the radiation effect. Hayat et al. [23] studied the MHD flow of Powell-Eyring nanofluid past a non-linear stretching sheet of variable thickness.

The present study introduces the influence of the elasticity parameter and the variable thickness on the flow of a Maxwell dusty fluid over a stretching surface. The schematic of the industrial process concerning this study is shown in Fig. 1(a) where the melted material is extruded from a die to be stretched by wind-up roll. The thickness of the stretching surface is assumed to be decreased gradually until getting a uniform thickness as shown in Fig. 1(b). The mathematical formulation and the mathematical solution will be introduced next.

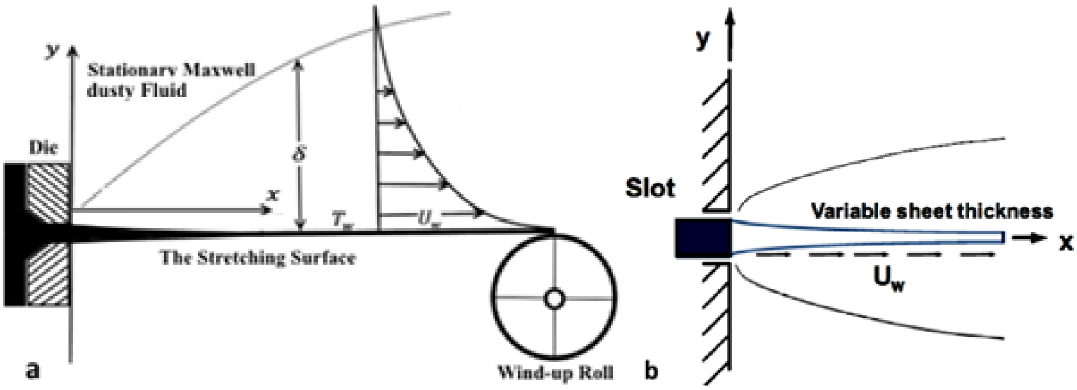

Fig1. *Schematic for flow above stretching surface*

2. MATHEMATICAL FORMULATION

Consider a two-dimensional boundary layer flow of steady laminar Maxwell dusty fluid over a stretching surface. The surface profile is described by$y = a(x + b)^{(1-n)/2}$ and is stretched by the velocity $U_w = a(x + b)^n$ where a and b are constants, n is the power index of the stretching velocity. The surface is assumed to be impermeable ($v_w = 0$). The x-axis is chosen along the stretching surface in the direction of the motion and y-axis is taken normal to it.The mathematical model governing this type of flow can be formulated as follows [6, 13]:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + \lambda\left[u^2\left(\frac{\partial^2 u}{\partial x^2}\right) + v^2\left(\frac{\partial^2 u}{\partial y^2}\right) + 2uv\left(\frac{\partial^2 u}{\partial x \partial y}\right)\right] = \nu\frac{\partial^2 u}{\partial y^2} + \frac{KN}{\rho}(u_p - u) \tag{2}$$

$$\frac{\partial u_p}{\partial x} + \frac{\partial v_p}{\partial y} = 0 \tag{3}$$

$$u_p\frac{\partial u_p}{\partial x} + v_p\frac{\partial u_p}{\partial y} = \frac{KN}{\rho}(u - u_p) \tag{4}$$

subject to boundary conditions

$$\begin{aligned} &at\ y = \mathrm{a}(x+b)^{\frac{1-n}{2}}:\ u = U_w = \mathrm{a}(x+b)^n, \qquad v = 0 \\ &\qquad as\ y \to \infty:\ u \to 0,\ v_p \to v,\ u_p \to 0 \end{aligned} \tag{5}$$

where u and v are the velocity components along x and y, respectively,u_p and v_p are the velocity components of the dusty particles along x and y, respectively, λ is the relaxation time parameter,σ is the electrical conductivity. The density isρ, and the dynamic viscosity isν of the fluid. K is the Stokes resistance, and N is the number density of dust particles. The Maxwell dusty fluid is viscous for $\lambda > 0$ while it is inviscid when $\lambda = 0$.

As suggested in [6, 24], the previous mathematical model is simplified by introducing the following dimensionless similarity variables

$$\begin{aligned} &u = a(x+b)^n G'(\xi);\ v = -\sqrt{\frac{n+1}{2}\nu a(x+b)^{n-1}}\left(G + \frac{n-1}{n+1}\xi G'\right) \\ &;\ \xi = \sqrt{\left(\frac{n+1}{2}\right)\frac{\mathrm{a}(x+b)^{n-1}}{\nu}}y;\ u_p = a(x+b)^n H'(\xi); \\ &v_p = -\sqrt{\frac{n+1}{2}\nu a(x+b)^{n-1}}\left(H + \frac{n-1}{n+1}\xi H'\right) \end{aligned} \tag{6}$$

to convert the partial differential equations into a nonlinear ordinary differential equations where $G(\xi)$and $H(\xi)$are dimensionless function and $\alpha = a\sqrt{a(n+1)/2\nu}$ is the wall thickness parameter describes the wall profile of the stretched surface. The equations (2-5) are transformed into ordinary differential equations using the similarity transformation technique. The equation of continuity is satisfied if a stream function $\psi$is chosen such that

$$u = \frac{\partial \psi}{\partial y}, \qquad v = -\frac{\partial \psi}{\partial x} \tag{7}$$

to guarantee that u and v in Eq.(6) satisfying Eq.(1).Substituting equations (6) in equations (2-5) obtains the following ordinary differential equations.

$$\begin{aligned} &G''' + GG'' - \frac{2n}{n+1}G'^2 + \beta\left((3n-1)GG'G'' - \frac{2n(n-1)}{n+1}G'^3 + \frac{n-1}{2}\eta G'^2 G'' - \frac{n+1}{2}G^2 G'''\right) \\ &- \frac{n+1}{2}\beta_V(H' - G') \\ &= 0 \end{aligned} \tag{8}$$

$$n\,H'^2 - \frac{n+1}{2}H\,G'' + \beta_V(H' - G') = 0 \tag{9}$$

subject to the boundary conditions

$$G(\alpha)=\alpha\left(\frac{1-n}{1+n}\right),\qquad G^{'}(\alpha)=1,$$
$$\lim_{\xi\to\infty}G'(\xi)=0,\lim_{\xi\to\infty}H'(\xi)=0,\lim_{\xi\to\infty}G(\xi)=\lim_{\xi\to\infty}H(\xi), \tag{10}$$

where $\beta=\lambda a(x+b)^{n-1}$ is the elasticity number [24] and $\beta_V=KN/\rho_o a$ is fluid particle interaction. Let $G(\xi)=f(\xi-\alpha)=f(\eta)$ and $H(\xi)=F(\xi-\alpha)=F(\eta)$. Therefore, the similarity equations (8), (9) in combined with the boundary conditions (10) becomes

$$f^{'''}+ff^{''}-\frac{2n}{n+1}f^{'2}+\beta\left((3n-1)ff^{'}f^{''}-\frac{2n(n-1)}{n+1}f^{'3}+\frac{n-1}{2}\eta f^{'2}f^{''}-\frac{n+1}{2}f^{2}f^{'''}\right)-\frac{n+1}{2}\beta_V\left(F^{'}-f^{'}\right)=0 \tag{11}$$

$$n\,F^{'2}-\frac{n+1}{2}F\,F^{''}+\beta_V(F^{'}-f^{'})=0 \tag{12}$$

subject to the boundary conditions

$$f(0)=\alpha\left(\frac{1-n}{1+n}\right),\qquad f^{'}(0)=1,$$
$$\lim_{\eta\to\infty}f'(\eta)=0,\lim_{\eta\to\infty}F'(\eta)=0,\lim_{\eta\to\infty}f(\eta)=\lim_{\eta\to\infty}F(\eta), \tag{13}$$

The local skin friction is defined as follows

$$\mathrm{C_f}=-2\sqrt{\frac{\mathrm{n}+1}{2}}\mathrm{Re_x}^{-\frac{1}{2}}\mathrm{f}^{''}(0), \tag{14}$$

Where $C_f\sqrt{Re_x}=-2\sqrt{(n+1)/2}f^{''}(0)$ is the modified skin-friction, C_f is the local skin-friction coefficient, and $Re_x=U_w(x+b)/v$ is the local Reynold number [25].

3. Results And Discussion

The equations (11) and (12) in combine with the boundary conditions (13) are solved numerically by the fourth/fifth Runge-Kutta method. The method is detailed by Elabshbeshy et al. [26] and applied using the software Mathmatica. The accuracy of this numerical method was validated by comparison with numerical results reported by [27-28]. It can seen from Tab. 1 that there is a very good agreement achieved between results. Table 2 presents the influences of elasticity β, wall thickness α parameters on the modified skin friction. Finally, the influences of elasticity β, wall thickness α parameters, and the velocity power index n on the velocity profiles are shown in the graphs from Fig. 2 to Fig. 6. Table 1 discloses that the present numerical results for $-f^{''}(0)$ at different α (neglecting the effect of the porous medium) show an excellent agreement with the results in [27- 28] for four digits at different values of n.

3.1. Modified Skin-Friction

The ratio of the inertial force to the viscous force in a fluid is defined as dimensionless number called Reynold number, which defines the flow type to be laminar, transient, or turbulent. The friction force between the stretching surface and the adjacent fluid particles called skin-friction, which creates a drag force depending on the value of Reynold number. When it takes different values depending on the value of the Reynold number, it is called the local skin-friction, while considering the effect of the wall profile, it is called modified-skin friction.

Table 2 introduces the influences of the wall thickness, and the elasticity parameters on the modified skin friction. The first part in Tab. 2 shows that the modified skin friction increases with increasing the velocity power index.

Physically, increasing the power index n increases the effect of the friction between the surface and the adjacent fluid. Part two and three introduce the effect of the wall thickness parameter where the modified-skin friction increase or $n>1$, while for $n>1$ it has a vice versa effect.

Table1. *A comparison with others for the value of* $-f''(0)$ *at different* α

n	$\alpha = 0.25$			$\alpha = 0.5$		
	Fang[27]	Khader[28]	Present Work	Fang[27]	Khader[28]	Present Work
10	1.1433	1.1433	1.14332	1.0603	1.0603	1.06032
9	1.1404	1.1404	1.14039	1.0589	1.0588	1.05891
7	1.1323	1.1322	1.13228	1.0550	1.0551	1.05504
5	1.1186	1.1186	1.11859	1.0486	1.0486	1.04861
3	1.0905	1.0904	1.09049	1.0359	1.0358	1.03586
1	1.0000	1.0000	1.00000	1.0000	1.0000	1.00000
0.5	0.9338	0.9337	0.93382	0.9799	0.9798	0.97994
0	0.78439	0.7843	0.78427	0.9576	0.9577	0.95764
-1/3	0.5000	0.5000	0.50000	1.0000	1.0000	1.00000
-0.5	0.0833	0.0832	0.08333	1.1667	1.1666	1.16666

The second and third parts show the effect of the wall thickness parameter α on the modified-skin friction, the modified-skin friction increases with α for $n < 1$ while it decreases for $n > 1$.

For high β, the viscosity and resistivity of the fluid are increased inducing more friction within the fluid and reduces the velocity of the flow. These effects appear in the increasing of the modified-skin friction as tabulated in the fourth and fifth parts of Tab. 2. The modified-skin friction increases for $n > 1$ more than for $n < 1$ with increasing β. Finally, part 6 introduces the effect of fluid-particle interaction parameter β_V in increasing the modified-skin friction.

Table2. *The modified skin-friction and* $-f''(0)$ *at differentn,* α, β, *and* β_v

	n	α	β	βv	$-f''(0)$	$C_f\sqrt{Re_x}$
	0	0.25	0.2	0.2	0.80934	1.14458
n	0.5	0.25	0.2	0.2	0.97331	1.68582
	2	0.25	0.2	0.2	1.18994	2.91474
	0.5	0.1	0.2	0.2	0.94038	1.62878
α	0.5	0.8	0.2	0.2	1.10655	1.91660
$n < 1$	0.5	1.2	0.2	0.2	1.21765	2.10903
	1.2	0.1	0.2	0.2	1.49736	3.14088
α	1.2	0.8	0.2	0.2	1.07059	2.24568
$n > 1$	1.2	1.2	0.2	0.2	1.04526	2.19255
	0.5	0.25	0	0.4	0.97212	1.68376
β	0.5	0.25	0.3	0.4	0.99651	1.72600
$n < 1$	0.5	0.25	0.9	0.4	1.04997	1.81860
	1.2	0.25	0	0.4	1.04276	2.18731
β	1.2	0.25	0.3	0.4	1.13752	2.38608
$n > 1$	1.2	0.25	0.9	0.4	1.30540	2.73823
	0.5	0.25	0.2	0.2	0.97331	1.68522
βv	0.5	0.25	0.2	0.5	0.99374	1.72120
	0.5	0.25	0.2	1	1.00578	1.74206

3.2. The Index Power n

The surface profile of the stretching surface (see Fig. 1b) and the stretching wall velocity are described according to the index power n. Increasing the value for n leads to a greater decreases in the surface thickness. Also, the index power n describes the stretching wall velocity U_w. The effect of this parameter on the dimensionless velocity, is illustrated in Figure 2 for $n = 0, 0.5, 2$ at $\alpha = 0.25$, $\beta = 0; 0.2$, and $\beta_V = 0.4$.

It is important to denote that the dimensionless velocity f' represents the ratio of the fluid velocity component u to the stretching velocity U_w. For that, increasing the velocity index power n means an increase in the stretching surface velocity and the curvature of the profile surface. Consequentially, it causes a decrease in the dimensionless velocity as shown in Fig. 2b, and so a decrease in the momentum boundary layer. it seen that, this effect is inversed after $\eta > 1.3$.In the same figure, the velocity of the dusty particles decreases. Due to this fact, the increase in this parameter increases the resistive force to inhibit the Maxwell dusty fluid. Figure 2a presented the behavior of the flow for Newtonian dusty fluid where the flow velocity increases with increasing the index power.

3.3. Wall Thickness Parameter α

The effect of wall thickness parameter $\alpha = a\sqrt{a(n+1)/2v}$ on the velocity for the fluid flow is investigated and the results are shown in Figure 3 for $\alpha = 0.1, 0.8, 1.2$at $n = 0.5, 1.2$, $\beta = 0.2$,and $\beta_V = 0.4$. From Fig 3a, it is clear that for $n < 1$, as α increases the velocity decreases, consequently, the boundary layer becomes thinner and thinner, while Fig. 3b shows that for $n > 1$ the velocity slightly increases and the boundary layer becomes thicker. The same behavior for the velocity of the dusty particles is shown.

3.4. Elasticity Parameter β

The elasticity parameter β of Deborah number is a dimensionless number represents the ratio of the time scale flow to the relaxation time flow. Where, the time scale is used to distinguish the type of the unsteady flow. For instant, when the time scale approaches infinity, the flow becomes steady and the relaxation time is the time required to the viscous fluid to retain after the flow stopped due to the effect of shear stress. The effect of the elasticity parameter on the dimensionless velocity is shown in the Figure 4 for $\beta = 0, 0.3, 0.9$ at $n = 0.5{:}\,1.2$, $\alpha = 0.25$, and $\beta_V = 0.4$.

Figure 4 shows a decrease in the velocity while an increasing in the elasticity number. This is due to the fact that increasingly β makes the fluid more viscous and so the resistance within the fluid is increased. The effect of the elasticity parameter for $n > 1$ (Figure 4b) is a little significant than for $n < 1$ (Figure 4a). The velocity of the dusty particles does not change with the elasticity parameter.

3.5. Fluid Particle Interaction β_V

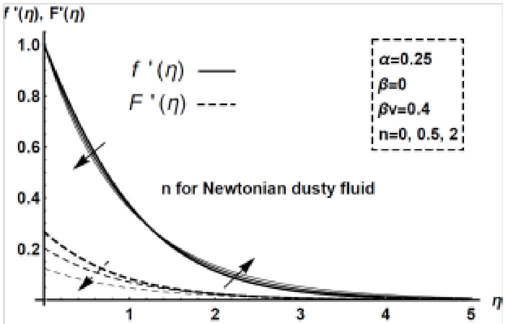

Fig2a. *Effect of n for Newtonian dusty fluid*

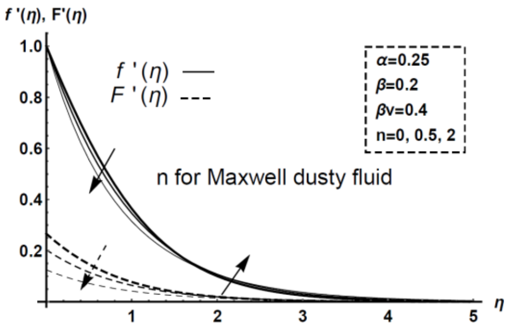

Fig2b. *Effect of n for Maxwell dusty fluid*

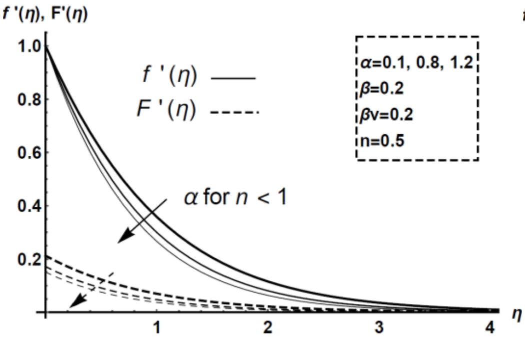

Fig3a. *Effect of* α *for* $n < 1$

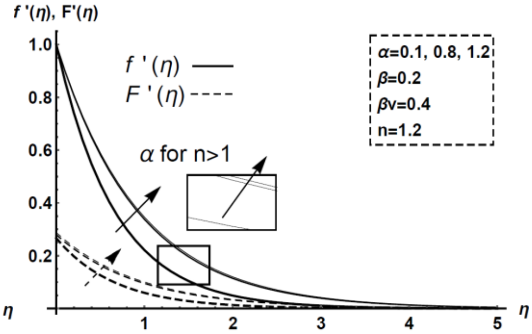

Fig3b. *Effect of* α *for* $n > 1$

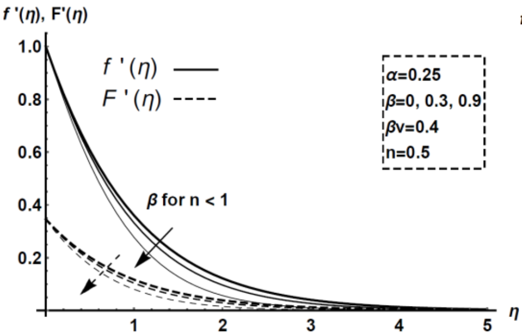

Fig4a. *Effect of* β *for* $n < 1$

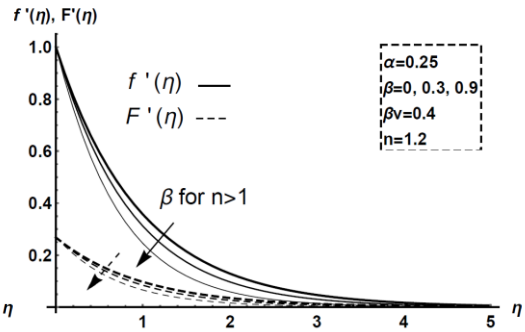

Fig4b. *Effect of* β *for* $n > 1$

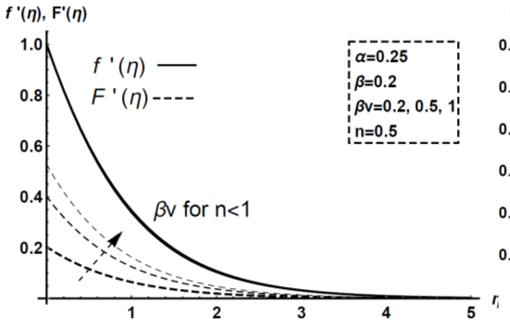

Fig5a. *Effect of* β_V *for* $n < 1$

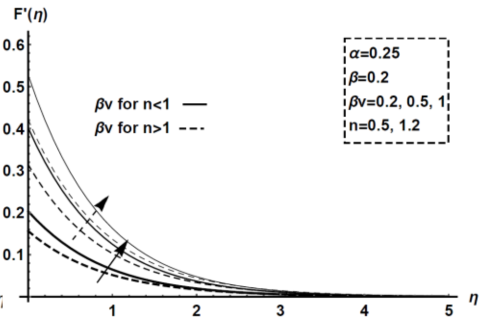

Fig5b. *Effect of* β_V *for different* n

4. CONCLUSION

Mass transfer of a Maxwell dusty fluid over a stretching surface with variable thickness have been investigated numerically. A similarity transformation for the governing boundary layer equations has been done based on wall thickness, elasticity, and fluid-particle interaction parameters. The influence of these parameters on the flow velocity have been presented graphically. The results for the modified-skin friction have been tabulated. The observations of the present study can be summarized in the following points.

- The velocity power index n and the wall thickness parameter α control the mechanical properties of the stretching surface. For $n < 1$, the wall thickness parameter α slightly decreases the velocity while, for$n > 1$, it strongly increases them.
- The elasticity parameter βdecreases the velocity profile.
- The modified skin-friction $C_f\sqrt{Re_x}$ with increasing α (for $n < 1$), and β. While it is decreased with increasing α (for $n > 1$).

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