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Geometrical Conditions Verification of Over Constrained Mechanism with Elastic Deformation Based on Bennett Mechanism

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Abstract: *The paper develops a new method to energetically solve the verification problem in redundant systems. The geometrical conditions of over constrained mechanism can be verified based on C++. Bennett linkage is used as the basic model to explain the way in verifying geometrical conditions. The main ideal is to optimize the potential energy of the system during its moving cycle. The kinetic and potential energy analysis is based on simulation of the elastic model. And the flexible body modeling method is introduced during the simulation. Quasi-Perfect Bennett mechanism is modeled with an alternative modeling environment on higher flexibility due to the kinematic constraints by elastic forces. The understanding of simulation and optimization method based on C++ in parameters verfication of redundant systems, therefore openning the new way to search for more redundant mechanisms which is movable.*

Keywords: *Over constrained mechanism, Modeling of Bennett Linkage, Geometric parameters verification, Potential energy analysis, Elastic links, Optimization*

1. INTRODUCTION

The theory of the constrained dynamics is well developed in the mechanics and widely used in the motion simulation of multibody system [1-4]. Due to the spatial kinematic characteristics using fewer links and joints, over constrained mechanisms are often good candidates in modern linkage design. Usage of various types of expensive and heavy gears can be avoided, when the change of axis orientation of revolute motions is required. Since the important usage of the over constrained mechanisms, many researchers study on the development of structure and machine design promotes the concept of over constrained mechanism [5]. The Bennett linkage [6-7] is a 4R four-bar spatial closed kinematic chain which is used as the basic construct unit to form different over-constrained 5R and 6R linkages, such as Myard linkages, extended Myard linkage[8], Goldberg's 5R and 6R linkages[9], Waldron's hybrid 6R linkages [10], Yu and and Baker's syncopated 6R linkage [11] and so on. The development in mechanics based design of reconfigurable structures have emerged into three major categories [12]. The first is based on the re-assembly of identical or similar robotics modules [13-14].The second is the metamorphic mechanism [15-16], which can generate different topologies for reconfigurations. The third is to incorporate certain bifurcation behaviors to the existing linkage's kinematic paths [17-18]. The class of over constrained linkages which are usually obtained by combination of other over constrained linkages. In some cases analytical methods were used to reveal over-constrained mechanisms [19]. The different nature and behavior of the over-constrained mechanism is the reason for the development of specific approaches for the kinematic and dynamic analysis and simulation for almost every single case. Even for a single mechanism it could happen that different approaches are to be applied during its motion. That significantly slows down the effectiveness of the computations [20]. and only for few cases such geometrical considerations could be regarded. Thus, the unified method to consider the geometrical conditions for each case is required. Furthermore the over constrained mechanisms under the consideration of inaccurate geometrical parameters are caused by manufacturing defect, the inaccurate geometrical parameters caused by the manufacturing defect [21] is also hard to verified.

In this paper, the redundant mechanism is modeled and analyzed in virtual system and Bennett linkage is used as the basic element to reveal the general solution to verify the certain conditions of over-constrained linkage parameters. Further more, the method was developed by taking into account of the potential energy of elastic links and the geometric conditions of Bennett mechanism can be automatically verified with the optimization of system elastic energy. The optimization method combined with virtue modeling and programming language which can make the verification process automatically achieved. The method can be applied to multi-loop linkages and the kinematic and dynamic analysis can be solved in this virtual system. The conclusion and future work are display in last section.

2. PRELIMINARY

The original setup of the Bennett linkage [6-7] is shown in Fig. 1(a). Its geometry conditions and closure equations are

$$a_{12}=a_{34},\alpha_{12}=\alpha_{34},a_{23}=a_{41},\alpha_{23}=\alpha_{41},R_i=0(i=1,2,3,4), \tag{1a}$$

$$\frac{\sin\alpha_{12}}{a_{12}}=\frac{\sin\alpha_{23}}{a_{23}}; \tag{1b}$$

And

$$\theta_1+\theta_3=0,$$

$$\theta_2+\theta_4=0, \tag{2a}$$

$$\tan\frac{\theta_1}{2}\tan\frac{\theta_2}{2}=\frac{\sin\frac{\alpha_{23}+\alpha_{12}}{2}}{\sin\frac{\alpha_{23}-\alpha_{12}}{2}},$$

Respectively.

The proportional relationship of the sine of twist over link length is called the Bennett ratio, as shown in Eq. (1b). The geometry conditions in Eqs. (1a) (1b) are typically in a line-symmetric form. But the closure equations in Eqs. (2a) do not follow the line-symmetric condition. This is because the revolute axes of the Bennett linkage in Fig. 1(a) are not set up in a line-symmetric manner. As shown in Fig. 2(b), by reversing the axes of joints 3 and 4, the resultant linkage becomes a Bennett linkage in a line-symmetric setup[22]. The corresponding geometry conditions and closure equations are

$$a_{12}=a_{34},\alpha_{12}=\alpha_{34},a_{23}=a_{41},\alpha_{23}=\alpha_{41},R_i=0(i=1,2,3,4), \tag{3a}$$

$$\frac{\sin\alpha_{12}}{a_{12}}=-\frac{\sin\alpha_{23}}{a_{23}}; \tag{3b}$$

And

$$\theta_1=\theta_3,$$

$$\theta_2=\theta_4,$$

$$\tan\frac{\theta_1}{2}\tan\frac{\theta_2}{2}=\frac{\cos\frac{\alpha_{23}+\alpha_{12}}{2}}{\cos\frac{\alpha_{23}-\alpha_{12}}{2}}, \tag{4a}$$

Respectively.

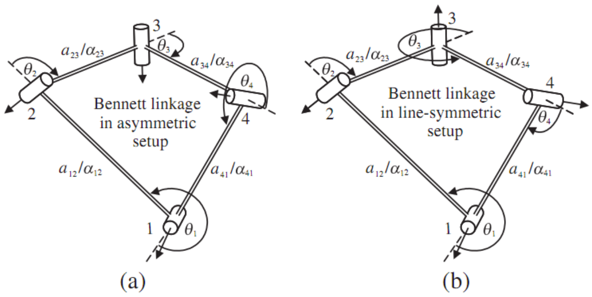

Fig1. *The two different joint-axis setups of Bennett linkage: (a) in the asymmetric setup; (b) in the line-symmetric setup.*

3. THE KINETIC AND POTENTIAL ENERGY ANALYSIS OF ELASTIC OVER-CONSTRAINED MECHANISM

The potential energy changing between the rigid link and elastic links of the over constrained mechanism are investigated. The method of kinetic and potential energy of over-constrained mechanisms are discussed.

3.1. Method of Modeling Elastic Links

Many closed chain mechanisms are movable, while the topology analysis shows zero or negative dof. This is because of the kinematic parameters for which the constraints equations are dependent. For numerical simulation of flexible system discretization of continuum. Mass and stiffness properties of the flexible bodies are to be reduced to a finite number of points called nodes. The node of a flexible element is a free object that, in three dimensional space, has six degrees of freedom. The node motion is restricted by elastic forces acting between the neighbor nodes.

Based on the ideal, the principle proposed in the paper consists in transformation of a closed chain into open branch cutting from the flexible links instead of joints. Using this approach allows the kinematic constraints to be substituted by force constraints, i.e.by elastic forces in the nodes.

The first step is the transformation of the closed chain into open branches representing the connectivity between the coordinate systems of the rigid bodies and the coordinate systems of the nodes of the flexible elements. For example, the connectivity of the coordinate system is shown in Fig. 2. The coordinate systems with indices 2’ and 2’’ correspond to the nodes of the flexible beam-link.

According to the results derived from [23], It could be seen that longitudinal elastic forces are exaggerated in the initial stage for going out from the singular configuration and become extremely high at the interruption of the integration process. So the mainly potential energy is stored in the longitude direction and it is assumed that the potential energy stored in other direction can be neglected. Based on this principle, a prismatic joint and a revolute joint were inserted between two links where the cutting from the elastic link. The translational and rotational axis are oriented along the link. The translational and torsion spring are attached into the prismatic and revolute joint respectively, thus the elastic energy can be detected by this springs during the its moving cycle. Then, the potential energy V_s of over-constrained mechanism can be calculated.

$$V_s = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{m} k_{ij}\varphi_{ij}^2 \quad \text{where n=1 m=1} \tag{7}$$

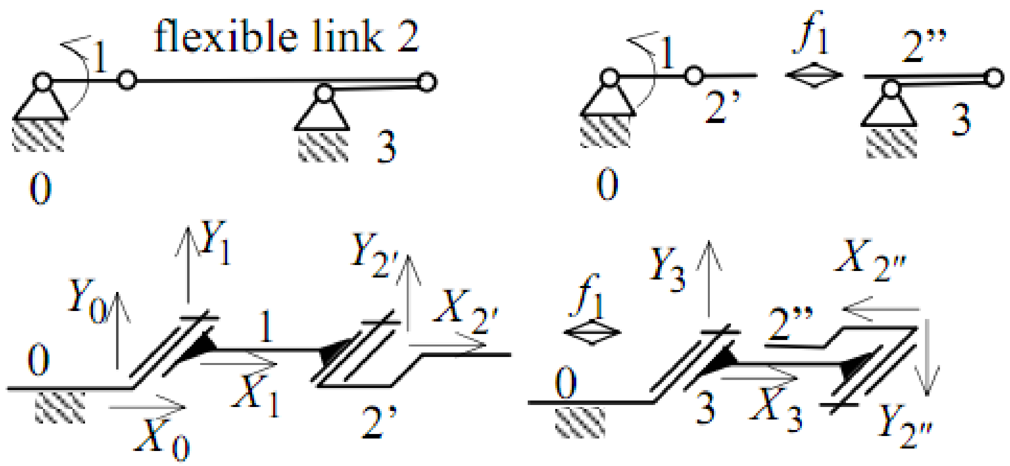

Fig2. *Connectivity of the coordinate systems of rigid links and nodes*

3.2. Configuration of Quasi-Perfect Bennett-Linkage

According to the geometric setup of the Bennett linkage [6,7], the Modeling of the Bennett linkage is showed in Fig. 3.

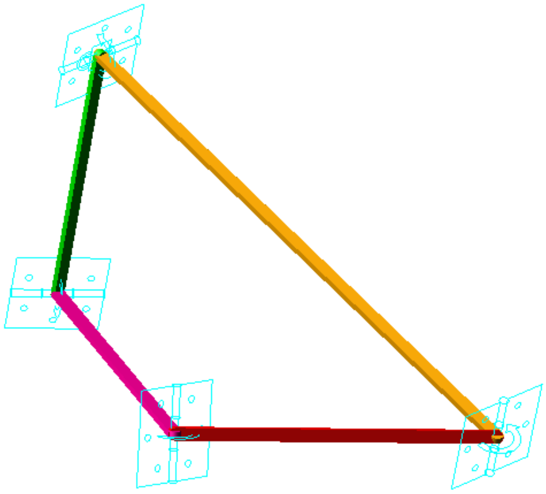

Fig3. *Model of Bennett linkage*

Based on the principle described in chapter 3.1, the rigid links are replaced by elastic links. The prismatic and revolute joint are inserted into each link, where the translational and rotational axis are oriented along the link. The translational and torsion spring are attached on the prismatic and revolute joint respectively which is shown in Fig.4.

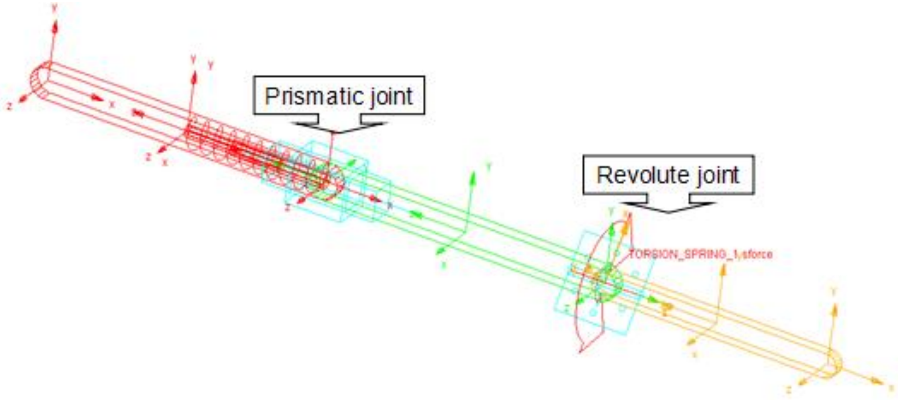

Fig4. *Set up of elastic links*

The stiffness coefficient of the spring is set to a very high level so that the model of the elastic link can be considered as the rigid one while the elastic energy variation still can be detected. When assembling the elastic links with high stiffness according to the Bennett conditions, the system can be called quasi-perfect Bennett mechanism which is shown in Fig.5.

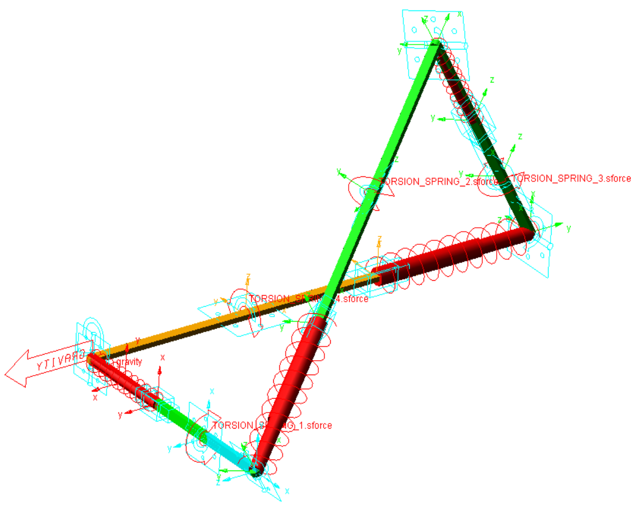

Fig5. *Model of quasi-perfect Bennett linkage*

3.3. Potential Energy Analysis of Quasi-Perfect Bennett-Linkage

The potential energy of the translational spring and torsion spring are taken into consideration.

$$V_s = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{m} k_{ij}\varphi_{ij}^2 \tag{8}$$

Potential Energy of a translational spring $V_{trans} = \frac{1}{2}k_{ij}x_{ij}$ (9)

Potential Energy of a torsion spring $V_{torsion} = \frac{1}{2}k_{ij}\theta_{ij}$ (10)

k_{ij} : stiffness coefficient of spring

x_{ij} : deformation of the translational spring

θ_{ij} : deformation of the torsion spring

Total potential energy $V_{total} = V_{trans} + V_{torsion}$ (11)

One revolute joint is defined by the constant speed, the total potential energy can be detected by torsion and translational spring while the mechanism is running the cycle. Theoretically, the bennet mechanism has one degree of fredom which indicated that there should be no deformation of elastic beam during the moving cycle. Thus, no potential energy can be detected. The ideal is supported by the simulation shown in Fig.6. The potential energy is detected to be zero. The result from the simulation experiment shows that the rigid body system can be replaced with elastic links and the potential energy can also be detected.

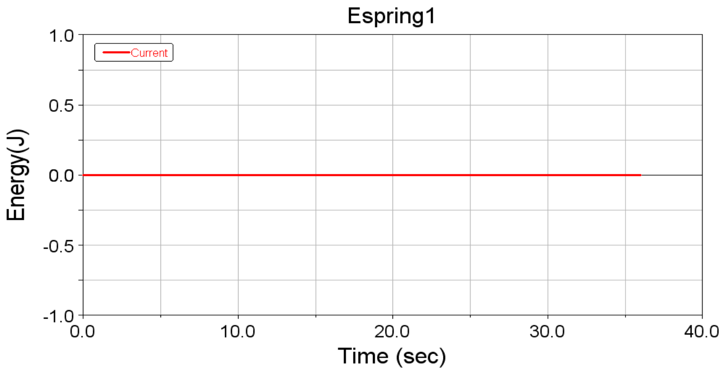

Fig6. *Total potential energy of quasi-Bennett linkage over a movement cycle*

After increasing the length of one link, the geometrical conditions of the Bennett linkage will be no more satisfied. The simulation experiment shows that the potential energy is increased when geometrical conditions of Bennet mechanism are not satisfied (Fig. 7.)

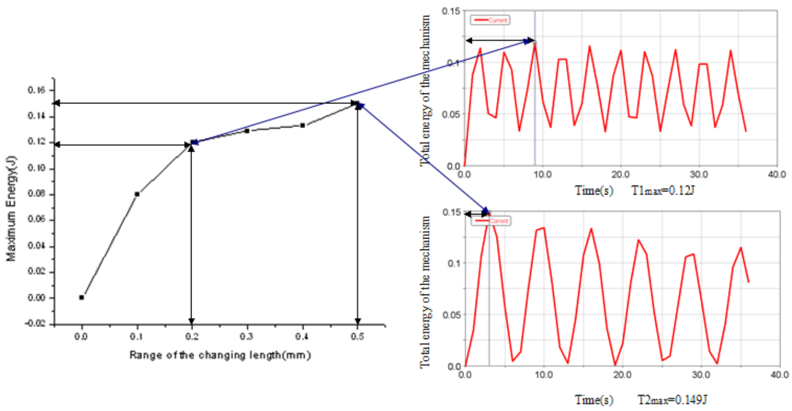

Fig7. *Total potential energy changing of quasi-Bennett linkage after geometrical conditions changes*

In summury, the experiment shows that when the geometrical conditions of the over constrained mechanism is satisfied, the total elastic energy of each links are detected to be zero. While the geometrical condition of the Bennett mechanism is no more satisfied, the potential energy of links can be dectected. In order to verify the exact geometrical properties of the Bennett mechanism, the minimum energy in a defined scope should be maintained minimum (zero) and geometric parameters is in favor of being detected automatically.

The purpose can be achieved with an alternative modeling environment with higher flexibility based on C++ will be applied. Although the simulation experiment from the ADAMS shows the method of modeling quasi-perfect over constrained mechanism perfectly, however compared with the flexibility environment, the optimization can be automatically realized based on C++.

4. Modeling and optimization of quasi-perfect Bennett mechanism with higher flexibility based on C++ [24]

4.1. Principle of The Modeling in MOBILE Based on C++

Representation of mechanical entities as objects is capable of transmitting motion and force across the system. The Portable and efficient implementation based on the object-oriented programming language C++ makes the modeling environment much more flexible. The Built-in interfaces for three-dimensional graphic libraries for animation with direct user feed-back allows the user to imbed the resulting modules in existing libraries.

One of the main feature of MOBILE is that it allows the user to model mechanical systems as executable programs that can be used as building blocks for other environments.

The transmission function for the rigid link are defined as (quantities of frame k' are marked with a prime):

Position: (forwards)

$$R' = R \cdot \Delta R \tag{12}$$

Velocity: (forwards)

$$\begin{bmatrix} w' \\ v' \end{bmatrix} = \begin{bmatrix} \Delta R^T & 0 \\ -\Delta r \cdot \Delta R^T & \Delta R^T \end{bmatrix} \cdot \begin{bmatrix} w \\ v \end{bmatrix} \tag{13}$$

$$r' = \Delta R^T r + \Delta r \tag{14}$$

Acceleration: (forwards)

$$\begin{bmatrix} \dot{w}' \\ a' \end{bmatrix} = \begin{bmatrix} \Delta R^T & 0 \\ -\Delta r \cdot \Delta R^T & \Delta R^T \end{bmatrix} \cdot \begin{bmatrix} \dot{w} \\ a \end{bmatrix} + \begin{bmatrix} 0 \\ \varepsilon_a \end{bmatrix} \tag{15}$$

$$\varepsilon_a = \omega' \times (\omega' \times \Delta r)$$

For the definition of a joint in MOBILE, one specifies the reference frames at the input and output, the joint variable and optionally an argument indicating which coordinate axis to use for the transformation. The type of joint, whether it is prismatic or revolute, is recognized by the type of the variable.

4.2. Method of Modeling the Quasi-Perfect Bennett Mechanism

The objective is to model the Bennett mechanism and compute the motion of the system at a prescribed configuration of the input variables. The concept of modeling the quasi-perfect Bennett mechanism is based on C++ which is showed in Fig. 8.

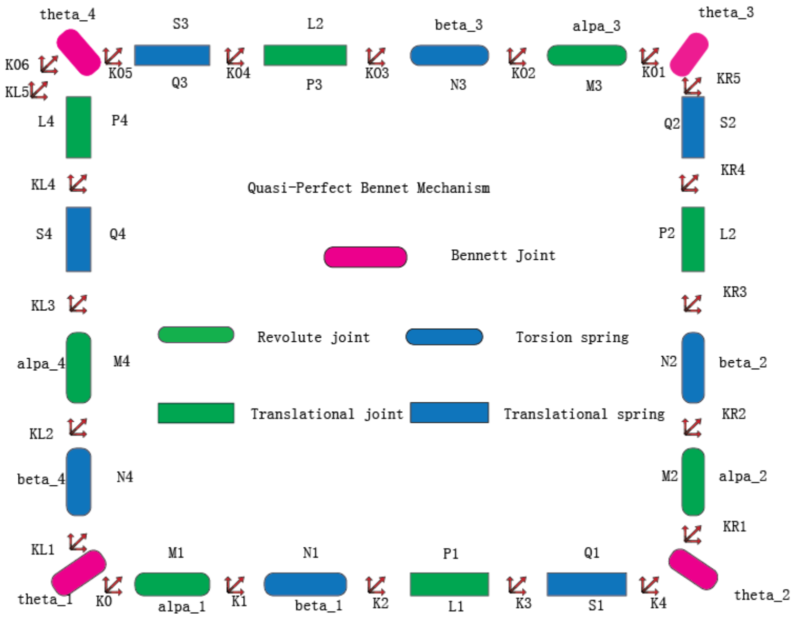

Fig8. *Configuration of quasi-perfect Bennett mechanism in MOBILE*

The regarded Bennett mechanism involves twelve joint variables. The revolute joints theta_1, theta_2,theta_3, theta_4 which are the Bennett joints and M1, M2, M3, M4 which is modeled as parameters to control the angel between the two links. The translation joints P1, P2, P3, P4 which is modeled as parameters to control the length variation between the Bennett joints. The elastic energy can be detected as the torsion and translational spring are attached to the link. Q1, Q2, Q3, Q4 are modeled as translational spring , and the translational deformation can be detected. N1, N2, N3, N4 are modeled as torsion spring so that torsion deformation can be detected. Only one of these parameters can be chosen as kinematic input according to the six general spatial loop closure conditions

In this experiment, the rotation joint B1 is defined as the input. The kinematic input is given by the rotation joint R1 for the convenience of analyze.

All the modeling is based on C++ language in the MOBILE, and the Bennett Mechanism is shown in Fig.9.

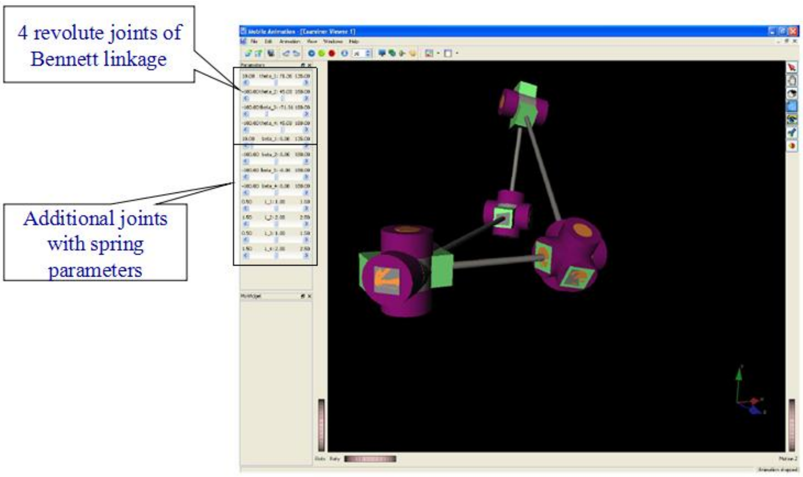

Fig9. *Model of quasi-perfect Bennett mechanism in MOBILE*

4.3. Simulation and Analysis of Quasi-Perfect Bennett Mechanism

The modeling of quasi-perfect Bennett mechanism is based on the principle introduced in chapter 4.1. The alternation and relationship of geometrical parameters can be detected and controlled based on C++. The deformation of translation spring can be derived from the parameters S1, S2, S3, S4. The deformation of torsion spring can be derived from $Beta_1, Beta_2, Beta_3, Beta_4$.

Thus, the total elastic energy of the system can be obtained. The Bennett joint derived from modeling is shown in Fig.10., which is perfectly satisfied the geometrical conditions of Bennett link. [6-7]

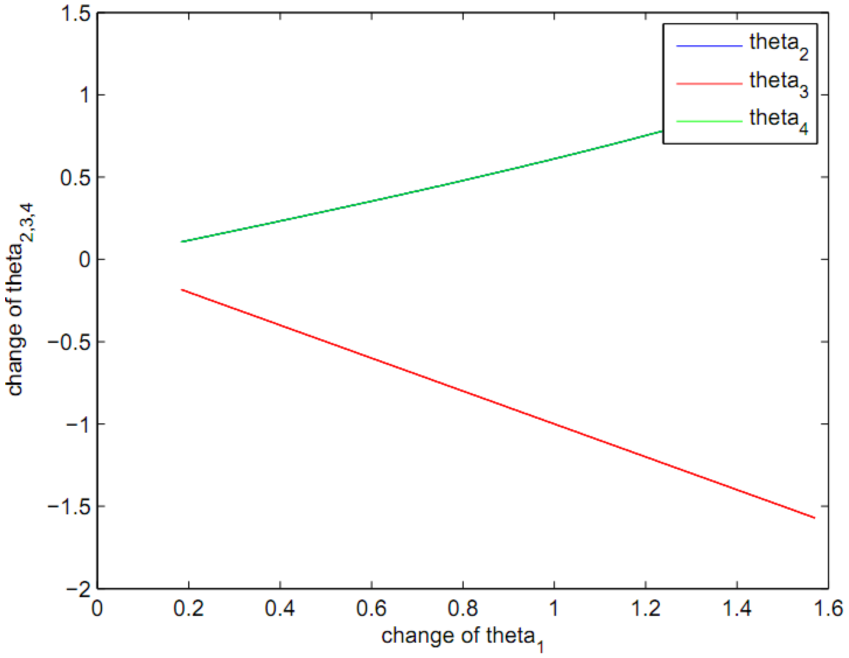

Fig10. *Geometrical conditions of Bennett joint angel*

4.4. Design the Optimizer

The experimental analysis of elastic Bennett linkage shows that the total potential energy of the system over a movement cycle is almost equals to zero. And when the geometrical conditions of the Bennett linkage is not satisfied, the total potential energy is increased. Therefore, if the minimum potential energy (equals to zero) of the mechanism can be found when searching the geometrical conditions iteratively, the over-constrained mechanism is movable and the geometrical conditions are verified.

The basic concept of the optimization is shown in Fig. 11. The length of each link and twisted angel are both variable parameters inflencing the system behaviour. The spatial four bar mechanism can be treated as Bennet mechanism when the combination of these parameters are satisfied certain conditions which is explained in section 2. The method is to optimized the potential energy to be zero while the geometrical parameters of system can be determined.

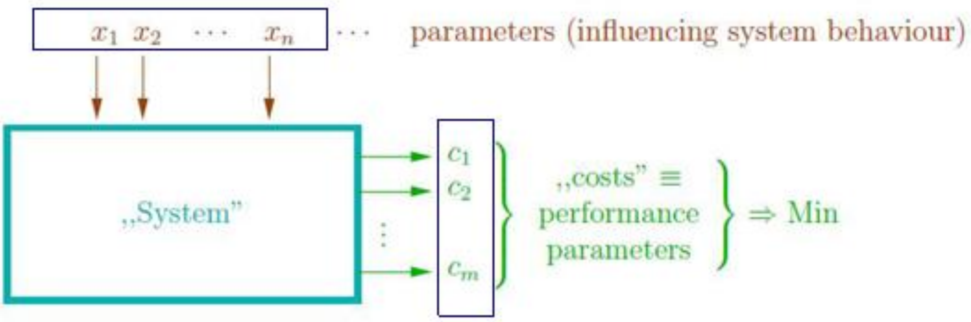

Fig11. *The basic concept of optimization*

Where parameters x_i stands for L1, L2, L3, L4 (length of the links)

Alpa_1,Alpa_2,Alpa_3,Alpa_4 (Twisted angel)

Costs c_i stands for the total potential energy.

Now the method of unconstrained minimization will be introduced. Allow the cost function $f(x)$ to be nonlinear. The minimization problem can be find an optimal point x^* such that

$$f(x^*) = Min\ f(x)$$

Let vector d denote the search direction and α denote the step size along the search direction. Assume d is a feasible direction, such that if one is located at the minimum x^* then

$$x = x^* + \alpha d \in \Omega$$

Then, starting from x^* one obtains a new set of design variables x. Then, it must hold $f(x) - f(x^*) \geq 0$ and after inserting equation which has already made, one can obtain $f(x^* + \partial d) - f(x^*) \geq 0$

Considering the scalar function $g(\alpha) = g(0) + \dfrac{dg}{d\alpha} d\alpha$

Thus the condition of local minimum becomes

$$\frac{dg}{d\alpha} \cdot d\alpha + g(0) - g(0) \geq 0$$

$$\frac{dg}{d\alpha} \cdot d\alpha \geq 0$$

and the necessary condition for a local minimum is $\dfrac{dg}{d\alpha} = 0$

Vector function $\dfrac{dg}{d\alpha} = \dfrac{\partial f}{\partial x} \dfrac{\partial (x^* + \alpha d)}{\partial \alpha} = \nabla_x fd$

Thus the necessary condition for vector function becomes $\nabla_x fd = 0$ at a minimum, any feasible direction must be orthogonal to the gradient. If the problem is unbounded, then all possible directions $d \in \Re^n$ are feasible and thus the necessary condition becomes $\nabla_x f = 0$. That meas necessary condition for unconstrained and unbounded problem is at the minimum, the gradient vanishes. The extend library in MOBILE is designed based on C++ according to the optimization principle.

4.5. Experimental Verification of the Model

In chapter 4, there are twelve geometrical parameters in the Bennett model. According to the geometry conditions described in equation 1a,1b,2a, the parameters define to be optimized are two links length L1, L2 and two twisted angle Alpa_1, Alpa_2. The two Bennett joint angle theta_1 ,theta_2 are considered to be the initial conditions of the system.

By applying the extend library in the direction of optimization in MOBILE, any geometrical variables of the mechanism can be optimized according to optimization principle. The initial conditions of Bennett joint angle (theta_1 and theta_2) are set to be 120° and 90° respectively.

Running the multibody system for cycles by initial conditions while the geometric condition changes and define the certain range of L1, L2, Alpa_1 and Alpa_2. The elastic energy of the system can be detected and optimized as the minimum. When the minimum elastic energy of the system was automatically found. The geometrical conditions can be verified during this optimization process.

As it is shown in Fig.13., each potential energy is responsible for one combination of link length during the iteration. For instance, there are two combination of link length shown in Fig.12. During the cycle running of the system, the maximum elastic energy can be detected. The first combination is L1=1.97 mm and L2=4.03 mm, and the second is L1=2.03 mm and L2=4.01 mm. The maximum elastic energy is represented by point $E1_{max}$ and $E2_{max}$ in Fig.13

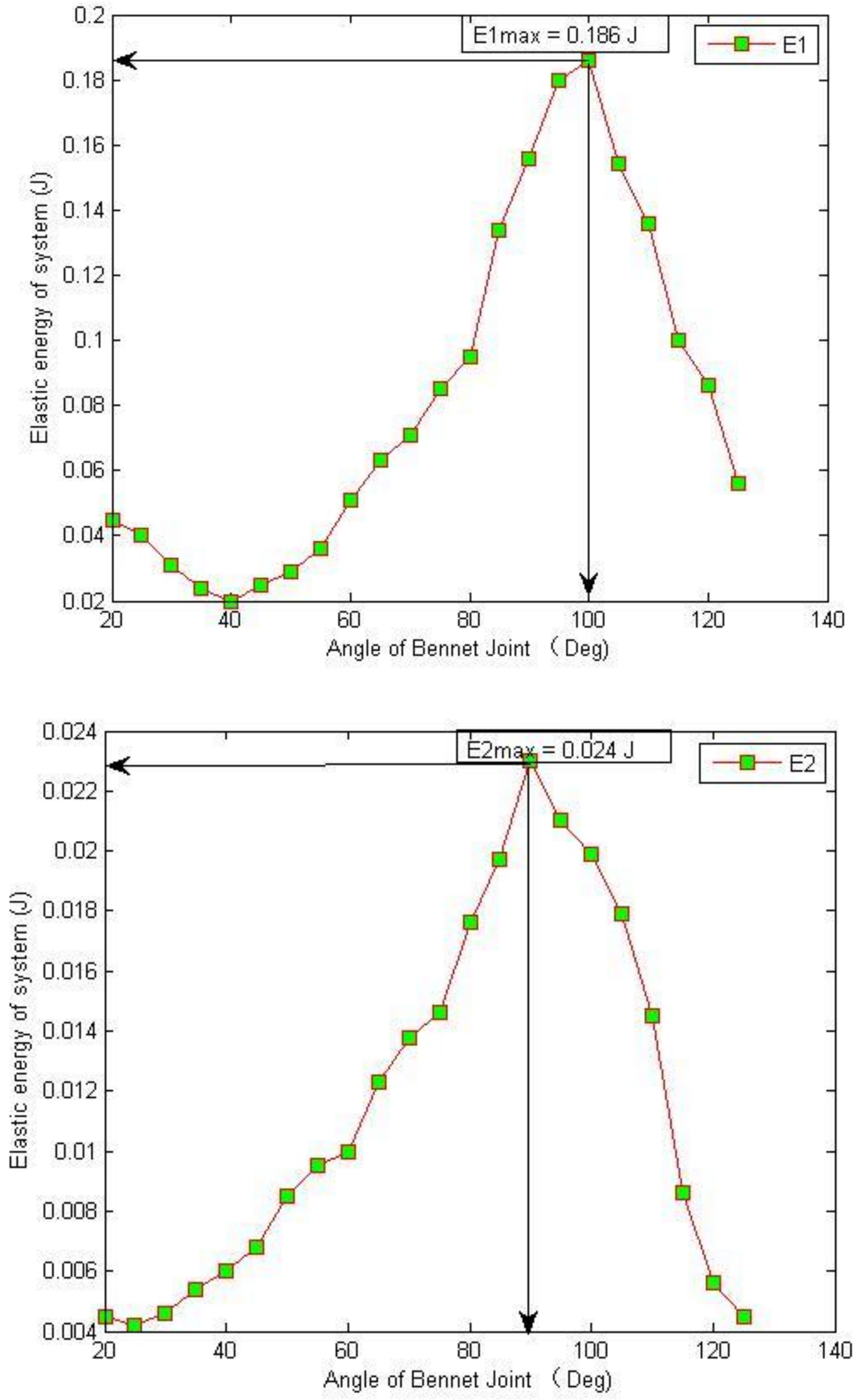

Fig12. *The elastic energy of system in one moving cycle during the link length iteration*

By searching the minimum elastic energy according to the optimization algorithm, the combination of link length which is fully satisfy the geometrical conditions of over constrained mechanism can be automatically detected.

The given scope of L1 is from 1.96 mm to 2.05 mm and L2 is from 3.96 mm to 4.05 mm. The twisted angle of Alpa_1 and Alpa_2 are given from 0° to 180° in the optimizer. The elastic energy over a period movement cycle can be obtained during the iteration and the optimization algorithm is designed based on C++.

As it is shown in Fig.13, the optimized geometrical conditions of link length is equal to 4 mm and 2 mm respectively. And the twisted angle are optimized to be 90° and 30° respectively in Fig.14.

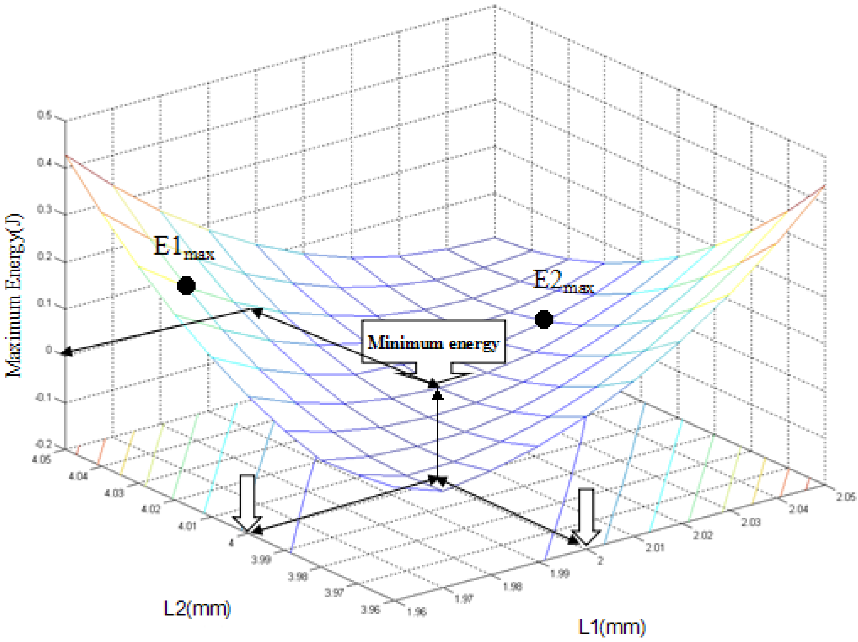

Fig13. *Optimization of system elastic energy during the link length iteration*

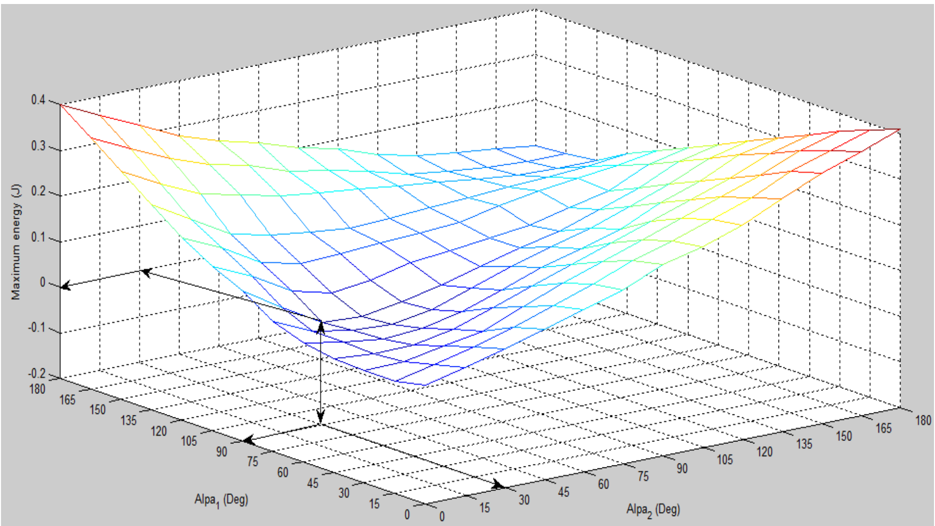

Fig14. *Optimization of system elastic energy during the twisted angle iteration*

The verified the parameters are listed in table 1. Reset the geometrical conditions to the geometrical conditions of the Bennett linkage. The results shows that the verified the geometrical parameters are perfectly satisfied the geometrical conditions of Bennett linkage.

Table1. *Verified geometrical conditions of the over constrained mechanism*

Over constrained Mechanism	Link Length	Twisted Angle	Bennett Joint Angle
L1 (mm)	2		
L2 (mm)	4		
Alpa_1 (Deg)		90	
Alpa_2 (Deg)		30	
Theta_1 (Deg)			120
Theta_2 (Deg)			90

5. Conclusion and Future Work

In this paper, a systematic approach to simulate rigid and flexible multibody system is proposed. The potential energy of the flexible links are detected based on the Bennett mechanism. By applying with an alternative modeling environment with higher flexibility, the minimum potential energy of the mechanism can be optimized. And the precise geometrical conditions of linkage structural parameters of the mechanism are determined by designing the optimizer based on C++. The modeling and optimization method provides a general solution for verification problem in redundant systems. Furthermore, the method provides a new way to find the possibility of the other new over-constrained mechanisms which is movable. The process is showed in Fig. 15.

In the future, it will be interesting to run an optimization loop with a large number of initial conditions taken from a random number generator and see where the optimizer lands and also let this random initial condition cases for days and weeks and list all cases with their corresponding minimum, then sort minimum from zero upwards and take all zero minimum cases. Then we can run the optimization for several million cases and thus to obtain all existing cases of other over constrained mechanism in the literature.

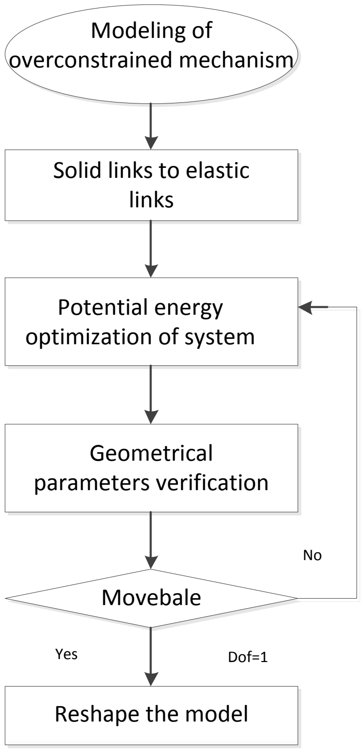

Fig15. *Geometrical verification of over constrained mechanisms*

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