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Abstract: Compare to the traditional limit equilibrium methods, finite element method has some obvious advantages for analyzing slope stability. As a result, strength reduction factor method based on the finite element theory is widespread used. The paper presents the fundamental principle of the shear strength reduction approach for finite element method calculations of slope factor of safety. Also, the influence factors for accuracy of the slope safety factor were discussed. It is found that slope model should be built by the six-node triangle type, and the element size ratio is 0.03, and it is found that distance from the crest of the slope to the left boundary should be more than 2.5 times of the slope width; the distance from the toe of the slope to the bottom boundary should be at least 2 times of the slope height. Finally, based on the optimal initial slope model, the safety factor of the initial end-wall slope in the studied coal mine is 1.52. The outcome shows that the simulation results by optimal slope model have the good accuracy.

Keywords: Accuracy analysis, Slope stability, Shear strength reduction, finite element method, factor of safety.

1. INTRODUCTION

In recent years, shear strength reduction (SSR) method based on the the finite element method (FEM) is widespread used to analyze the slope stability¹⁻⁵. Compare to the traditional limit equilibrium methods, the obvious advantages of the finite element method are as follows⁶⁻¹²:

(1) No assumption needs to be made in advance about the shape or location of the failure surface. Failure occurs "naturally" through the zones within the soil or rock mass in which the shear strength is unable to resist the applied shear stresses.

(2) Since there is no concept of slices in the finite element method, there is no need for assumptions about slice side forces. The finite element method preserves global equilibrium until "failure" is reached.

(3) The finite method is able to monitor progressive failure up to and including overall shear failure.

However, many factors will affect the accuracy of the slope safety factor in the finite element slope stability model¹³⁻¹⁷, such as the element type, size and model boundary range. In this paper, we try to get the suitable element type, size and boundary range for the studied slope model. The results can be as guidelines for solving the homogenous issue.

2. THE SHEAR STRENGTH REDUCTION (SSR) APPROACH

The shear strength reduction technique of finite element slope stability analysis is a simple approach that involves a systematic search for a strength reduction factor (SRF) or factor of safety value that brings a slope to the very limits of failure¹⁸⁻²⁰.

The SSR technique described in literature assumes Mohr-Coulomb strength for slope materials. The Mohr-Coulomb strength envelope is the most widely applied failure criterion in geotechnical

engineering. A unique feature of this linear failure model is the fact that it can be simply and explicitly expressed in both principal stress and shear-normal stress. The description of strength behavior is for a wide range of materials, and easy-to-obtain parameters of the Mohr-Coulomb criterion account for its popularity.

For Mohr-Coulomb material the factored or reduced shear strength can be determined from the Equation (1):

$$\frac{\tau}{F} = \frac{c}{F} + \sigma \frac{\tan \varphi}{F} \tag{1}$$

Where τ is shear stress. *c* is cohesion. σ is normal stress. φ is internal friction angle. *F* is strength reduction factor or slope safety factor.

This equation can be re-written as

$$\frac{\tau}{F} = c' + \sigma \tan \varphi' \tag{2}$$

Where $c' = \frac{c}{F}$ and $\varphi' = \arctan(\frac{\tan \varphi}{F})$ are factored Mohr-Coulomb shear strength parameters.

The steps for systematically searching for the critical factor of safety value *F* that brings a previously stable slope ($F \ge 1$) to the verge of failure are as follows:

Step 1: Develop a FE model of a slope, using the appropriate material deformation and strength properties. Compute the model and record the maximum total deformation.

Step 2: Increase the value of F (or SRF) and calculate factored Mohr-Coulomb material parameters as described above. Enter the new strength properties into the slope model and re-compute. Record the maximum total deformation.

Step 3: Repeat Step 2, using systematic increments of F, until the FE model does not converge to a solution, i.e. continue to reduce material strength until the slope fails. The critical F value just beyond which failure occurs will be the slope factor of safety.

For a slope with a factor of safety less than 1, the procedure is the same except fractional F values will be systematically decremented (translating into increments in the factored strength parameters) until the slope becomes stable.

The principal advantage of the SSR technique is its use of factored strength parameters as input into models, which enable the technique to be used with any existing FE analysis software. All the approach requires of a slope analyst is computation of factored Mohr-Coulomb strength parameters.

3. INFLUENCE FACTORS ANALYSIS FOR ACCURACY OF THE SLOPE SAFETY FACTOR

In order to study the influence factors for the accuracy of the slope safety factor, numerical simulation was carried out with two-dimension finite element method software Phase2. As it is know, the element type, size and boundary range can affect the simulation results greatly for FEM software. Therefore, some significant researches about the suitable element type, size and boundary range for slope stability analysis were implemented firstly.

4. ELEMENT TYPE AND SIZE ANALYSIS

In order to find the suitable element type and size for the slope stability analysis model, based on the geological conditions of end-wall slope in an open pit coal mine (Table 1), an end-wall slope model was built as shown in Fig. 1. In the model, according to the actual geological conditions, thirteen different rock materials are included in total. The average height of bench is 15 m, and slope angle of loess bench and rock bench is 55° and 70° , respectively. There are four haul benches for inpit dumping system in total, which are called loess haul bench, No.1 rock haul bench, No.2 rock bench and No.3 haul bench in top-to-bottom order. Meanwhile, the width of loess haul bench is 40 m and the other three rock haul benches are all 35 m. Besides, the overall end-wall slope height, width and angle are 198 m, 312 m and 32° , respectively. Also, the ground pressure that the haul trucks imposed on the

haul benches was considered, according to the field data, ground pressure on the loess haul bench is 0.08 MPa, and the rock haul benches is 0.58 MPa.

In order to decrease the effect of the boundary condition on the simulation result, the boundary was set far from the slope. As a result, in the model, the distance from the crest of the slope to the left boundary and that of the toe of the slope to the right was both three times of the slope width. Moreover, the distance from the toe of the slope to the bottom boundary was also three times of the slope height. The analysis boundary condition was set as follows: the ground surface was free plane; the lateral boundaries were horizontally fixed; the lower boundary was vertically fixed; and both corner points in the lower boundary were fully fixed.

For the slope model, the slope safety factor depends on the mechanical properties of slope strata layers. Hence, the slope safety factor also can be analyzed by the limit equilibrium method (LEM), and which result can be used as the basis for optimizing the FEM model. However, LEM model cannot simulate the slope excavate, support and complexity of the boundary condition. Fig. 2 shows the lowest slope safety factor and the critical slip surface position determined by the limit equilibrium method. Therefore, it can be thought that the safety factor of the initial model of end-wall slope is 1.52.

Lithology	Depth	Average	Unit	Young's	Poisson's	Cohesion	Friction	Tensile
	(m)	thickness	weight	modulus	ratio	(MPa)	angle	strength
		(m)	(kN/m^3)	(MPa)			(°)	(MPa)
Loess	30	30	19.6	15	0.42	0.085	12	0.0125
Weathered	44	14	23.0	2000	0.36	2.5	38	0.75
sandstone								
Sandstone	74	30	23.8	4200	0.32	3.0	39	0.9
Mudstone	98	24	24.9	2800	0.34	2.0	38	0.6
Siltite	110	12	23.2	4600	0.32	3.5	36	1.0
Sandstone	122	12	23.8	4000	0.30	4.0	40	1.2
No. 4	130	8	14.4	1000	0.38	1.62	36	0.7
coal seam								
Shale	145	15	24.5	2400	0.33	3.0	42	0.9
Siltite	160	15	26.0	4800	0.32	5.0	38	1.2
Shale	170	10	25.8	3500	0.35	5.0	38	1.0
No. 9	183	13	13.3	1200	0.36	1.62	36	0.7
coal seam								
Sandstone	195	12	23.8	2900	0.28	5.0	41	1.2
No. 11	198	3	14.0	1300	0.35	1.62	36	0.7
coal seam								

 Table1. Physical and mechanical properties of coal and rock of end-wall slopein studied open pit coal mine



Fig1. Overall view of the slope model



Fig2. The slope safety factor and critical slip surface position of the slope model by limit equilibrium method

In this research, the effect of the element size on slope safety factor is described with element length to slope height ratio, as shown in Equation (3). For Phase2 software, there are four kinds of element type, including three-node triangle, six-node triangle, four-node quadrilateral and eight-node quadrilateral. The influence result of the element size and element type to the slope safety factor is shown in Fig. 3.

$$i = \frac{h}{H} \tag{3}$$

Where i is element size ratio. h is element length, m. H is lope height, m.



Fig3. Slope safety factor with different element size ratio and element type

From the results of Fig. 3, it is found that the slope safety factor results have obvious errors when the element type adopts three-node triangle or four-node quadrilateral. Even if the element size ratio is smaller, the accuracy is still unacceptable. On the other hand, the simulation results by six-node triangle and eight-node quadrilateral are stable. Meanwhile, six-node triangle type has higher accuracy than the eight-node quadrilateral type under the same element size ratio. Therefore, during the mentioned element types, the six-node triangle type is the best choice for simulating the slope stability. By the six-node triangle type, the slope safety factor is 1.52 when the element size ratio is 0.03, and it is same with the result calculated by limit equilibrium analysis method. In this research, it is recommended that the slope model should be built by the six-node triangle type, and the element size ratio is 0.03.

5. MODEL BOUNDARY RANGE

As is well known, the boundary range of model will also affect the simulation result for the FEM model. In general, in order to eliminate the boundary effect, the boundary range is set largely, for instance, three or four times of the slope width and height. As a result, the model size is very large and the calculation time will be longer. Therefore, the reasonable boundary range should be studied.

When the element type uses the eight-node quadrilateral and the element size ratio is about 0.03, the

slope safety factors for different boundary ranges are shown in Fig. 4. In the figure, L is the distance from the crest of the slope to the left boundary, m; R is the distance from the toe of the slope to the right boundary, m; a is the slope width, m; B is the distance from the toe of the slope to the bottom boundary, m; b is the slope height, m.

Based on the simulation results, it is recommended that distance from the crest of the slope to the left boundary should be more than 2.5 times of the slope width; the distance from the toe of the slope to the right boundary should be at least 2 times of the slope width; and the distance from the toe of the slope to the bottom boundary should be at least 2 times of the slope height. Under these boundary ranges, the twenty six corner points on end-wall slope were chosen as measuring points between the primary boundary and optimized boundary models in order to compare the simulation results including the major and minor principal stresses, the maximum shear strain, horizontal and vertical downward displacements. The comparison results are shown in Fig. 5. From this figure, it is found that all the simulation results by the optimized boundary model can be in accord with the primary boundary model. Therefore, the recommended boundary range is acceptable for this study model.



Fig4. The slope factors for different boundary ranges



(b) Minor principal stress



(e) Vertical downward displacement

Fig5. Comparison results between the primary boundary and optimized boundary model

6. OPTIMAL SLOPE MODEL

Based on the mentioned results, the optimal initial model of the research can be built as shown in Fig. 6. Fig. 7 shows the slope safety factor and slip surface position by Phase2 software. Also, we import the slip surface from the limit equilibrium software to Phase2 interpret. As a result, we can see the very good agreement between the limit equilibrium analysis and the Phase2 finite element analysis.



Fig6. The initial slope model based on the reasonable mesh type, size and boundary range



Fig7. The slip surface position by FEM and limit equilibrium method

7. CONCLUSION

The paper presents the fundamental principle of the shear strength reduction approach for finite element method calculations of slope factor of safety. Also, the influence factors for accuracy of the slope safety factor were discussed. It is recommended that the end-wall slope model should be built by using the eight-node quadrilateral type, and the element size ratio is 0.03. Moreover, it is found that distance from the crest of the slope to the left boundary should be more than 2.5 times of the slope width; the distance from the toe of the slope to the right boundary should be at least 2 times of the slope width; and the distance from the toe of the slope to the bottom boundary should be at least 2 times of the slope height. Finally, based on the optimal initial slope model, the safety factor of the initial end-wall slope in the studied coal mine is 1.52.

REFERENCES

- [1] Chakraborti S., Konietzky H. and Walter K., A Comparative Study of Different Approaches for Factor of Safety Calculations by Shear Strength Reduction Technique for Non-linear Hoek–Brown Failure Criterion, *Geotechnical and Geological Engineering*, **30**, 1-10 (**2012**)
- [2] Wan S.S., Nian Y.K., Jiang J.C. and Luan M.T., Discussion on several issues in slope stability analysis based on shear strength reduction finite element methods, *Rock and soil mechanics*, **31**, 2283-2316 (**2010**)
- [3] Che L.H., Yu S. and Zhang H.T., Some issues on shear strength reduction finite element method, *Chinese Journal of Rock Mechanics and Engineering*, **32**, 433-437 (**2011**)
- [4] Zhao S.Y., Zheng Y.R. and Zhang Y.F., Study on slope failure criterion in strength reduction finite element method, *Rock and Soil Mechanics*, **26**, 332-336 (**2005**)
- [5] Gillie J. L., Rodriguez-Marek A. and McDaniel C., Strength reduction factors for near-fault forwarddirectivity ground motions, *Engineering Structures*, **32**, 273-285 (**2010**)
- [6] Kim H. M., Hong S. Y., Kwon H. W., Song J. H., Jeon, J. J., and Jung W. J., Numerical Simulation of Submarines with Anechoic Coatings for Acoustic Target Strength Reduction, *Naval Engineers Journal*, 124, 49-58 (2012)
- [7] Song E.X., Finite elements analysis of safety factor for soil structures, *Chinese Journal of Geotechnical Engineering*, **19**, 1-7 (**1997**)
- [8] Wang D., Nian S.K. and Chen Y.M., Three problems in slope stability analyses with finite element method, *Rock and Soil Mechanics*, **28**, 2309-23 13 (**2007**)
- [9] Nian Y.K., Zhang K.L., Liu H.S. and Xu H.Y., Stability and failure mechanism of three-dimensional slope using strength reduction method, *Journal of Jilin University (Earth Science Edition)*, **25**, 178-185 (**2013**)

- [10] Reid M. E., Keith T. E., Kayen R. E., Iverson N. R., Iverson R. M. and Brien D. L., Volcano collapse promoted by progressive strength reduction: new data from Mount St. Helens, *Bulletin of volcanology*, 72, 761-766 (2010)
- [11] Sarkar R. and Chellapandi P., A design approach for establishing creep strength reduction factor for repaired welds for fast reactor fuel pin end plugs, *Nuclear Engineering and Design*, **52**, 192-200 (**2012**)
- [12] Sevostianov I., Zagrai A., Kruse W. A. and Hardee H. C., Connection between strength reduction, electric resistance and electro-mechanical impedance in materials with fatigue damage, *International Journal of Fracture*, **14**, 159-166 (**2010**)
- [13] Song K., Yan E. C., Mao W. and Zhang T.T., Determination of shear strength reduction factor for generalized hoek-brown criterion, *Chinese Journal of Rock Mechanics and Engineering*, 21, 106-112 (2012)
- [14] Tang, Z.Z., Han, Y.S., Dong, Y.L. and Zhao, Y.M., The influence on calculation precision of strength reduction with slope main factors, Jiangsu Construction, 16, 127-130 (2012)
- [15] Wang P.T., Yang T.H., Zhu L.K. and Liu H.L., Strength reduction method for rock slope stability analysis based on PFC2D, *Journal of Northeastern University (Natural Science)*, 35, pp. 127-130 (2013)
- [16] Tang Z.Z., Han Y.S., Dong Y.L. and Zhao Y.M., The influence on calculation precision of strength reduction with slope main factors, *Jiangsu Construction*, **21**, 127-130 (**2012**)
- [17] Guo Y.C., Chen T. and Qian H., The determination method of dynamic safety factor for slope based on strength reduction, *China Civil Engineering Journal*, **12**, 117-120 (**2012**)
- [18] Liang Q.G. and Li D.W., Discussion on strength reduction FEM in geotechnical engineering, *Rock and Soil Mechanics*, **33**, 3054-3058 (**2008**)
- [19] Liao Y., Zeng X.G., Fu W.X. and Liu W.G., Linearization method of non-linear strength of Hoek-Brown rock mass, *Journal of Central South University (Science and Technology)*, **20**, 4902-4911 (**2012**)
- [20] Zhao S.Y., Zheng Y.R., and Zhang Y.F., Study on slope failure criterion in strength reduction finite element method, *Rock and Soil Mechanics*, **25**, 332-336 (**2005**)

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