Yukawa Potential in Klein-Gordon Equation in Cosmological Inertial Frame

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Abstract: We study Yukawa potential dependent about time in cosmological inertial frame. If we solve Klein-Gordon equation, we obtain Yukawa potential dependent about time in cosmological inertial frame.

Keywords: Yukawa potential; Klein-Gordon equation; Cosmological inertial frame

PACS Number: 03.30.+p, 03.65

1. INTRODUCTION

Our article’s aim is that we make Yukawa potential theory in cosmological inertial frame.

At first, Robertson-Walker metric is

$$d\tau^2 = dt^2 - \frac{1}{c^2} \Omega^2 (\ell_0) \left[ \frac{dx^2}{1-kr^2} + x^2 d\Omega^2 \right]$$

(1)

According to $\Lambda CDM$ model, our universe’s k is zero. In this time, if $t_0$ is cosmological time[3],

$$k = 0, t = t_0 >> \Delta t, \Delta t \ is \ period \ of \ matter’s \ motion$$

(2)

Hence, the proper time is in cosmological time,

$$d\tau^2 = dt^2 - \frac{1}{c^2} \Omega^2 (\ell_0) \left[ dx^2 + dy^2 + dz^2 \right]$$

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$$\gamma^2 = \frac{dx^2 + dy^2 + dz^2}{dt^2}$$

(3)

In this time,

$$d\tau = dt, d\vec{x} = \Omega (\ell_0) dx, d\vec{y} = \Omega (\ell_0) dy, d\vec{z} = \Omega (\ell_0) dz$$

(4)

Cosmological special theory of relativity’s coordinate transformations are

$$ct = \gamma (ct + \frac{v}{c} \Omega (\ell_0) \vec{x} \cdot \vec{e}) = \gamma (ct + \frac{v}{c} \Omega (\ell_0) \vec{x} \cdot \Omega (\ell_0))$$

$$\vec{x} = x\Omega (\ell_0) = \gamma (x + v_\vec{x} \Omega (\ell_0) \vec{e} \cdot \vec{e}) = \gamma (x + v_\vec{x} \Omega (\ell_0) \vec{e} \cdot \Omega (\ell_0))$$

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\[
\begin{align*}
\bar{y} &= \Omega(t_0)y = \bar{y}' = \Omega(t_0)y', \\
\bar{z} &= \Omega(t_0)z = \bar{z}' = \Omega(t_0)z'
\end{align*}
\]
\[\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}, \quad \Omega(t_0) = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}
\]  

(5)

2. YUKAWA POTENTIAL IN KLEIN-GORDON EQUATION IN COSMOLOGICAL INERTIAL FRAME

If we focus Klein-Gordon equation about Yukawa potential \( \phi \) dependent about time,

\[
\frac{m^2 c^2}{h^2} \phi + \partial_{\mu} \partial^{\mu} \phi = \frac{m^2 c^2}{h^2} \phi + \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \phi - \nabla^2 \phi = 0
\]

(6)

In this time, Yukawa potential \( \phi \) dependent about time is.

\[\phi = -\frac{g^2}{r} \exp\left(-\frac{m \pi r}{\hbar}\right) + A_y \sin \omega t
\]

Frequency \( \omega = \frac{m \pi c^2}{\hbar} \), \( m \pi \) is meson’s mass

(7)

Eq(6)-Klein-Gordon equation is satisfied by Eq(7)-Yukawa potential dependent about time

In cosmological inertial frame, Klein-Gordon equation is[2]

\[
-\Omega(t_0) \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \phi' + \frac{1}{\Omega(t_0)} \nabla^2 \phi' = \frac{m^2 c^2}{h^2} \phi'
\]

(8)

In this point, in cosmological inertial frame, space-time transformations in the type A of wave function and the other type B of the expanded distance are

Type A: \( r \rightarrow r \sqrt{\Omega(t_0)}, t \rightarrow \frac{t}{\sqrt{\Omega(t_0)}} \), Type B: \( r \rightarrow r \sqrt{\Omega(t_0)}, t \rightarrow t \)

(9)

Space-time transformation of Yukawa potential \( \phi' \) is depend on Type A

Hence, Yukawa potential \( \phi' \) dependent about time is

\[\phi' = -\frac{g^2}{r \sqrt{\Omega(t_0)}} \exp\left[-\frac{m \pi r \sqrt{\Omega(t_0)}}{\hbar}\right] + A_y \sin \left(\frac{\omega t}{\sqrt{\Omega(t_0)}}\right)
\]

Frequency \( \omega = \frac{m \pi c^2}{\hbar} \), \( m \pi \) is meson’s mass

(10)

Eq(8)-Klein-Gordon equation is satisfied by Eq(10)-the solution.

3. CONCLUSION

We solve Klein-Gordon equation in cosmological inertial frame. Hence, we found Yukawa potential dependent time in cosmological inertial frame.

REFERENCES

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Science, 711 (2020), pp. 4-9


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