Electromagnetic Wave Functions of CMB and Schwarzschild Space-Time

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Abstract: In the general relativity theory, we find electro-magnetic wave functions of Cosmic Microwave Background and Schwarzschild space-time. Specially, this article is to find electromagnetic wave equations treated by gauge fixing equations in Robertson-Walker space-time and Schwarzschild space-time.

Keywords: General relativity theory, Electro-magnetic wave equations; Electromagnetic wave functions; Cosmic Microwave Background; Schwarzschild space-time

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INTRODUCTION

In the general relativity theory, our article’s aim is that we find electro-magnetic wave equations and functions by gauge fixing equations in Robertson-Walker space-time and Schwarzschild space-time.

At first, Electro-magnetic field equations are in general relativity theory

$$F_{\mu\nu} = \frac{4\pi}{c} j^{\mu}$$

(1)

$$F_{\mu\nu,\lambda} + F_{\nu\lambda,\mu} + F_{\lambda\mu,\nu} = 0$$

(2)

The electro-magnetic field is

$$F_{\mu\nu} = A_{\nu;\mu} - A_{\mu;\nu} = \frac{\partial A_{\mu}}{\partial x^{\nu}} - \frac{\partial A_{\nu}}{\partial x^{\mu}}$$

(3)

The gauge fixing equation in general relativity theory

$$A^\mu_{;\mu} = \frac{\partial A^\mu}{\partial x^\mu} + \Gamma^\mu_{\mu\rho} A^\rho$$

$$\rightarrow \partial_\mu (A^\mu + g^{\mu\nu}\partial_\nu \Lambda) + \Gamma^\mu_{\mu\rho}(A^\rho + \partial^\rho \Lambda)$$

$$= \partial_\mu (A^\mu + g^{\mu\nu}\partial_\nu \Lambda) + \Gamma^\mu_{\mu\rho}(A^\rho + g^{\mu\rho} \partial_\rho \Lambda)$$

(4)

1. ELECTRO-MAGNETIC WAVE EQUATION IN ROBERTSON-WALKER SPACE-TIME

Because the gauge fixing equation is the electro-magnetic wave equation, the electro-magnetic wave equation is in Robertson-Walker space-time.
The Robertson-Walker solution is
\[ d\tau^2 = dt^2 - \frac{1}{c^2} \Omega^2(t) \left[ \frac{dr^2}{1-kr^2} + r^2 d\Omega^2 \right] \] (5)

In this time, 2-dimensional solution is
\[ d\Omega = 0 \]
\[ d\tau^2 = dt^2 - \frac{1}{c^2} \Omega^2(t) \frac{dr^2}{1-kr^2} \] (6)

The gauge fixing equation is in 2-dimensional solution
\[ \partial_{\mu}(A^\mu + g^{\mu\nu}\partial_{\nu}\Lambda) + \Gamma_{\nu\rho}^{\mu}(A^\rho + g^{\mu\nu}\partial_{\nu}\Lambda) \]
\[ = \partial_{\rho}A^\rho + \Gamma_{10}^1 A^0 + \Gamma_{11}^1 A^1 + \partial_{\mu}g^{\mu\nu}\partial_{\nu}\Lambda + g^{\mu\nu}\partial_{\mu}\partial_{\nu}\Lambda + \Gamma_{10}^1 g^{00} \frac{1}{c} \frac{\partial \Lambda}{\partial t} + \Gamma_{11}^1 g_{11} \frac{\partial \Lambda}{\partial r} \] (7)

Hence, we can find electro-magnetic wave equation in 2-dimensional Robertson-Walker space-time.
\[ \partial_{\mu}g^{\mu\nu}\partial_{\nu}(\sin\Phi) + g^{\mu\nu}\partial_{\mu}\partial_{\nu}(\sin\Phi) + \Gamma_{10}^1 g^{00} \frac{1}{c} \frac{\partial \sin\Phi}{\partial t} + \Gamma_{11}^1 g_{11} \frac{\partial \sin\Phi}{\partial r} \]
\[ = \left[ -\frac{2kr}{\Omega^2(t)} \frac{\partial}{\partial r} - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} + \frac{1-kr^2}{\Omega^2(t)} \frac{\partial^2}{\partial r^2} - \frac{\dot{\Omega}}{c\Omega} \frac{1}{c} \frac{\partial}{\partial t} + \frac{kr}{\Omega^2(t)} \frac{\partial}{\partial r} \right] \sin\Phi = 0 \] (8)

In this time, we can think the shape of electro-magnetic wave function from 2-dimensional Robertson-Walker space-time. In this case, light is
\[ d\tau^2 = dt^2 - \frac{1}{c^2} \Omega^2(t) \frac{dr^2}{1-kr^2} = 0 \]
\[ \int \frac{dt}{\Omega(t)} = \frac{1}{c} \int \frac{dr}{\sqrt{1-kr^2}} \] (9)

Hence, electro-magnetic wave function is in 2-dimensional Robertson-Walker space-time-
\[ \vec{E} = E_0 \sin\Phi, \vec{B} = B_0 \sin\Phi \]
\[ \Phi = \omega_b \left[ \int \frac{dt}{\Omega(t)} - \frac{1}{c} \int \frac{dr}{\sqrt{1-kr^2}} \right] \]
i) \[ k = 1, \Phi = \omega_b \left[ \int \frac{dt}{\Omega(t)} - \frac{1}{c} \sin^{-1} r \right] \]
ii) \[ k = 0, \Phi = \omega_b \left[ \int \frac{dt}{\Omega(t)} - \frac{1}{c} r \right] \]
iii) \[ k = -1, \Phi = \omega_b \left[ \int \frac{dt}{\Omega(t)} - \frac{1}{c} \sinh^{-1} r \right] \] (10)

The electro-magnetic wave equation-Eq(8) is satisfied by the electro-magnetic wave function-Eq(10).

### 2. ELECTROMAGNETIC WAVE EQUATION IN SCHWARZSCHILD SPACE-TIME

Because the gauge fixing equation is the electro-magnetic wave equation, the electro-magnetic wave equation is in Schwarzschild space-time.
Electromagnetic Wave Functions of CMB and Schwarzschild Space-Time

The Schwarzschild solution is

\[ d\tau^2 = (1 - \frac{2GM}{rc^2}) dt^2 - \frac{1}{c^2} \left[ \frac{dr^2}{1-\frac{2GM}{rc^2}} + r^2 d\Omega^2 \right] \]  \hspace{1cm} (11) \]

In this time, 2-dimensional solution is

\[ d\tau^2 = (1 - \frac{2GM}{rc^2}) dt^2 - \frac{1}{c^2} \frac{dr^2}{1-\frac{2GM}{rc^2}} \]  \hspace{1cm} (12) \]

The gauge fixing equation is in 2-dimensional solution

\[ \partial_\mu (A^\mu + g^{\mu\nu}\partial_\nu \Lambda) + \Gamma^\mu_{\mu\nu}(A^\nu + g^{\alpha\nu}\partial_\alpha \Lambda) \]

\[ = \partial_\mu A^\mu + \Gamma^0_{01}A^1 + \Gamma^1_{11}A^1 + \partial_\mu g^{\mu\nu}\partial_\nu \Lambda + g^{\mu\nu}\partial_\nu \partial_\mu \Lambda + \Gamma^0_{01}g^{11} \frac{\partial \Lambda}{\partial r} + \Gamma^1_{11}g^{11} \frac{\partial \Lambda}{\partial r} \]

\[ = \partial_\mu A^\mu + \partial_\mu g^{\mu\nu}\partial_\nu \Lambda + g^{\mu\nu}\partial_\nu \partial_\mu \Lambda \]

\[ \Gamma^0_{01} = \frac{GM}{r^2c^2} - \frac{1}{1-\frac{2GM}{rc^2}} \,, \quad \Gamma^1_{11} = -\frac{GM}{r^2c^2} + \frac{1}{1-\frac{2GM}{rc^2}} \]  \hspace{1cm} (13) \]

Hence, we can find electro-magnetic wave equation in 2-dimentional Schwarzschild space-time.

\[ \partial_\mu g^{\mu\nu}\partial_\nu (\sin \Phi) + g^{\mu\nu}\partial_\mu \partial_\nu (\sin \Phi) \]

\[ = \left[ \frac{2GM}{r^2c^2} \frac{\partial}{\partial r} - \frac{1}{1-\frac{2GM}{rc^2}} \frac{1}{c^2} \frac{\partial^2}{\partial t^2} + \left( 1 - \frac{2GM}{rc^2} \right) \frac{\partial^2}{\partial r^2} \right] \sin \Phi = 0 \]  \hspace{1cm} (14) \]

In this time, we can think the shape of electro-magnetic wave function from 2-dimentional Schwarzschild space-time. In this case, light is

\[ d\tau^2 = (1 - \frac{2GM}{rc^2}) dt^2 - \frac{1}{c^2} \frac{dr^2}{1-\frac{2GM}{rc^2}} = 0 \]

\[ \frac{t}{c} = \frac{1}{c} \int \frac{dr}{1-\frac{2GM}{rc^2}} = \frac{r}{c} + \frac{2GM}{c^3} \ln \left| \frac{r}{c} - \frac{2GM}{c^2} \right| \]  \hspace{1cm} (15) \]

Hence, electro-magnetic wave function is in 2-dimentional Schwarzschild space-time-

\[ \vec{E} = E_0 \sin \Phi, \vec{B} = B_0 \sin \Phi \]

\[ \Phi = \omega_0 \left[ t - \frac{r}{c} - \frac{2GM}{c^3} \ln \left| \frac{r}{c} - \frac{2GM}{c^2} \right| \right] \]  \hspace{1cm} (16) \]

The electro-magnetic wave equation-Eq(14) is satisfied by the electro-magnetic wave function-Eq(16).
3. CONCLUSION

We find the electro-magnetic wave (CMB) equation and function in Robertson-Walker space-time. We find the electro-magnetic wave equation and function in Schwarzschild space-time.

REFERENCES