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Nonexistence of Plane Symmetric Micro Model in Presence of Cosmological Constant in Barber's Modified Theory of General Relativity

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Abstract: *In Barber's modified theory of general relativity (Gen. Rel. Grav. 14, 117(1982)), the homogeneous and anisotropic plane symmetric space-time in presence of micro matter field with cosmological constant-Λ is considered. It is shown that the cosmological constant generate solutions and models only when it is positive. But the models generated by cosmological constant are not of micro models. Some physical and geometrical features of the models are also studied.*

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1. INTRODUCTION

Einstein's theory of general relativity serves as a basis for constructing mathematical models of the universe though the theory has some controversies and lapses. To overcome these controversies and lapses, various alternative and modified theories of it have been proposed by authors from time to time to unify gravitation and matter fields in various forms. So in an attempt to produce continuous creation theory, Barber (1982) modified the Einstein's theory of general relativity to a variable G-theory, which creates the universe out of self contained gravitational and matter fields in various forms. This theory predicts local effects which are within the observational limits and the scalar field 'ϕ' does not gravitate directly but simply divides the matter tensor acting as a reciprocal to gravitational constant G.

The micro matter fields represented by scalar meson fields represent matter fields with spin less quanta. There are two types of scalar fields' viz. zero rest mass scalar fields and massive scalar fields. The zero rest mass scalar fields describe long range interactions, whereas massive scalar fields describe short range interactions. The study of scalar meson fields in general relativity has a focal point for the researchers due to its physical importance in particle physics. The micro matter field in relativistic mechanics yields some significant results as regards to the singularities. The micro matter field being a field of a single variable 'v' (say) is the special case of general field and the expression given by

$$T_{ij} = v_i v_j - \tfrac{1}{2} g_{ij}\left(v_k v^k - m^2 v^2\right)$$

is the energy-momentum tensor of Yukawa (1935) fields (spin zero meson particle) for the metric of (+2) signature in flat space time .

The Klein-Gorden equation takes the form

$$g^{ij}\, v_{;\, ij} + m^2 v = 0$$

where ‘v’ is the real scalar field and ‘m’ is the rest-mass of scalar meson field. Here (;) semicolon followed by an index denotes covariant differentiation. When m = 0, the scalar field ‘v’ is known as massless scalar field or micro matter field.

The cosmological constant Λ has been introduced in 1917 by Einstein to modify his own equations of general relativity. Now this Λ term remains a focal point of interest in the context of quantum field theories, quantum gravity, super gravity theories, Kaluza-Klein theories and in the inflationary universe scenario. A number of observations suggest that the universe possess a non-zero cosmological constant (Krauss and Turner,(1995)). The cosmological term which is a measure of the energy of empty space, provides a repulsive force opposing the gravitational pull between the galaxies. If it exists, then the energy it represents counts as mass because Einstein has shown that mass and energy are equivalent. Moreover, if the cosmological constant is large value then the energy involved due to the matter in the universe, can sum up to the number that inflation predicts. In inflationary era it is very important to study on the role of cosmological constant. But recent research suggests that the cosmological costant corresponds to a very small value of the order 10^{-58} cm^{-2} (Jhori and Chandra, (1983)).

Some of the authors those have studied second self-creation theory in various angles taking different space times in presence of different gravitating fields are Pimentel (1985), Soleng (1987a,b), Venkateswarlu and Reddy(1990), Shanti and Rao(1991), Carvalho(1996), Shri Ram and Singh(1998),Mohanty et al.(2000,2002,2003,2004-05),Panigrahi and Sahu(2002,2003,2004), Sahu and Panigrahi(2003), Sahu and Mohanty (2006), Sahu and Mahapatra (2009a,b) and recently Sahu et al.(2010,2012). But to our knowledge none of the authors has studied the plane symmetric space-time in the context of second self-creation theory when the gravitational field in presence of cosmological constant is a micro matter field represented by massless scalar field. So in the present paper we have considered this problem to study the role of Λ (cosmological constant) for deriving mesonic solutions .The work presented in this paper is treated as an extension work of Sahu and Panigrahi(2003).

2. Field Equations in Barber’s Modified Theory of General Relativity

We consider the homogeneous and anisotropic plane symmetric metric in the form

$$ds^2 = dt^2 - A^2(dx^2 + dy^2) - B^2 dz^2 \tag{1}$$

where A and B are functions of cosmic time‘t’ .

The field equations in Barber’s modified theory of general relativity for micro matter field represented by massless scalar field with cosmological term Λgij may be written as

$$G_{ij} \equiv R_{ij} - \frac{1}{2} g_{ij} R + \Lambda g_{ij} = -8\pi\varphi^{-1} T_{ij}^{v} \tag{2}$$

and

$$\phi = \frac{8\pi}{3}\lambda T^{\gamma} \tag{3}$$

where $\phi = \phi_{;k}^{;k}$ is invariant D’Alembertian, Tv is the trace of the energy-momentum tensor T_{ij}^{v} ,ϕ is the Barber’s scalar and λ is the coupling constant to be evaluated from the experiment. The measurements of the deflection of light restricts the value of coupling to $|\lambda| \le 10^{-1}$. In the limit $\lambda \to 0$ this theory approaches the Einstein’s general relativity theory in every respect.

The energy momentum tensor T_{ij}^{v} for micro matter field representing massless scalar field is taken as

$$T_{ij}^{V} = v_i v_j - \frac{1}{2} g_{ij} v_k v^k \qquad (4)$$

together with

$$g^{ij} v^{ij} = \sigma \qquad (5)$$

where v is the massless scalar field, σ is the source density of massless scalar field and (;) is the covariant differentiation. Also v and σ are both function of cosmic time .Here the equation (2) connects the distribution of matter and energy with geometry of the space-time by relating energy momentum tensor T_{ij}^{v} to the fundamental metric tensor g_{ij} and its derivatives. It is the business of relativistic mechanics to investigate with the help of this equation, the principle which govern the energy momentum tensor and hence determines the behavior of energy and matter.

By using eqn. (4) the field equations (2) and (3) for the metric (1) are obtained as

$$\frac{A_{44}}{A} + \frac{A_4 B_4}{AB} + \frac{B_{44}}{B} - \Lambda = -4\pi \varphi^{-1} v_4^2, \qquad (6)$$

$$2\frac{A_{44}}{A} + \frac{A_4^2}{A^2} - \Lambda = -4\pi \varphi^{-1} v_4^2, \qquad (7)$$

$$\frac{A_4^2}{A^2} + 2\frac{A_4 B_4}{AB} - \Lambda = +4\pi \Phi^{-1} \vartheta_4^2 \qquad (8)$$

$$\phi_{44} + \phi_4 \left[2\frac{A_4}{A} + \frac{B_4}{B} \right] = -\frac{8\pi}{3} \lambda v_4^2 \qquad (9)$$

where the subscript '4' denotes the ordinary differentiation with respect to time.

Adding two times of eqn. (6) with eqn.(7) and three times of eqn.(8), we obtain

$$(A^2 B)_{44} - 3\Lambda (A^2 B) = 0 \qquad (10)$$

3. Solutions and Models

In order to find exact and explicit solutions of A and B from four field equations (6) to (9), we consider the following three cases :

(i) $\Lambda = 0$ (ii) $\Lambda > 0$ (iii) $\Lambda < 0$

Case i: When $\Lambda = 0$.

In this case the result reduces to that of already studied by Panigerahi and Sahu (2003).

Case ii: When $\Lambda > 0$.

In this case equation (10) after integration yields

$$A^2 B = \alpha_1 e^{\sqrt{3\Lambda} t} + \alpha_2 e^{-\sqrt{3\Lambda} t} \qquad (11)$$

where α_1 and α_2 are constants of integration.

From eqn. (11), we can write the explicit form of A and B as

$$A = \left(\alpha_1 e^{\sqrt{3\Lambda} t} + \alpha_2 e^{-\sqrt{3\Lambda} t} \right)^{n_1}$$

and

$$B = \left(\alpha_1 e^{\sqrt{3\Lambda} t} + \alpha_2 e^{-\sqrt{3\Lambda} t} \right)^{n_2} \qquad (12)$$

where $n_i:, i = 1,2$ are real constants and satisfies the relation

$$2n_1 + n_2 = 1. \qquad (13)$$

Here the over determinacy for determining three unknowns A and B from three field eqns. (6)-(8) can be settled by actual substitution of the values of A and B from eqn.(12) in eqn.(8). Thus we obtain

$$n_1^2 + 2n_1 n_2 = \frac{[4\pi\varphi^{-1}\vartheta_4^2 + \Lambda]}{3\Lambda}.\left[\frac{\alpha_1 e^{\sqrt{3\Lambda}t} - \alpha_2 e^{-\sqrt{3\Lambda}t}}{\alpha_1 e^{\sqrt{3\Lambda}t} + \alpha_2 e^{-\sqrt{3\Lambda}t}}\right]^2 \quad (14)$$

As n_i, i = 1, 2 are real constants, so also $n_1^2 + 2n_1n_2$ is a real constant. But this relation cannot be hold good in eqn. (14) as its L.H.S part is constant but R.H.S part is a function of ‘t’.

To make the relation consistent in eqn. (14), we take the following two sub cases:

Sub case1.

When $\alpha_1 \neq 0$, $\alpha_2 = 0$ and $\frac{4\pi\varphi^{-1}v_4^2 + \Lambda}{\Lambda} = 1$. (15a, b, c)

By use of (15), eqn (14) reduces to $n_1^2 + 2n_1n_2 = \frac{1}{3}$. (16)

Thus from eqn. (15c), we obtain

$$\nu = \alpha_3, \quad (17)$$

where α_3 is the constant of integration.

Solving (13) and (16), we find

$$n_1 = n_2 = \frac{1}{3}. \quad (18)$$

Now use of (18) and (15b), eqn. (12) yields

$$A = B = \left(\alpha_1 e^{\sqrt{3\Lambda}t}\right)^{\frac{1}{3}}. \quad (19)$$

Use of (19) and (17) in eqn. (9) and then integrating, we get

$$\emptyset = -\frac{\propto_4}{\sqrt{3\Lambda}\propto_1} . e^{-\sqrt{3\Lambda}t} + \propto_5 , \quad (20)$$

where $\propto_4 \neq 0$ and $\propto_5$ are the constants of integration.

The energy density associated with ‘v ‘(Anderson,(1967) is given as

$$\rho = \frac{1}{2}v_4^2. \quad (21)$$

The source density σ of the scalar field v given by eqn.(5) is

$$\sigma = -v_{44} + v_4 \frac{(A^2B)}{AB} \quad (22)$$

Using eqn.(17) in (21) and (22) separately, we obtain

$$\rho = 0 \quad (23)$$

and

$$\sigma = 0. \quad (24)$$

Thus the corresponding metric of our solution can be expressed as

$$ds^2 = dt^2 - \left(\alpha_1 e^{\sqrt{3\Lambda}t}\right)^{\frac{2}{3}}.(dx^2 + dy^2 + dz^2) \quad (25)$$

Sub case 2.

when $\alpha_1 = 0$, $\alpha_2 \neq 0$ and $\frac{4\pi\varphi^{-1}v_4^2 + \Lambda}{\Lambda} = 1$. (26a,b,c)

By use of (26), eq^n (14) reduces to

$$n_1^2 + 2n_1n_2 = \frac{1}{3}. \quad (27)$$

From (26a, b, c), (27), (13) and (12) as solved in previous case-1, we obtain

$$\nu = \alpha_3 \;, \quad n_1 = n_2 = \frac{1}{3} \text{ and } \quad A = B = \left(\alpha_2\, e^{-\sqrt{3\Lambda}t}\right)^{\frac{1}{3}}. \quad (28a,b,c)$$

Use of (28 a,c) in eqn. (9) and then integrating , we get

$$\Phi = \frac{\propto_6}{\propto_2 \sqrt{3\Lambda}} \,.\, e^{\sqrt{3\Lambda}t} + \propto_7 \quad (29)$$

where $\propto_6 \neq 0$ and $\propto_7$ are the constants of integration.

As in sub case-1, here also we can find

$\rho = 0$ and $\sigma = 0$.

Thus the geometry of our universe for space time (1) can be written as

$$ds^2 = dt^2 - \left(\alpha_1 e^{\sqrt{3\Lambda t}}\right)^{\frac{2}{3}}.(dx^2 + dy^2 + dz^2) \quad (30)$$

Case iii. When $\Lambda < 0$, say $\Lambda = -\frac{\omega^2}{3}$.

In this case eqn. (10) reduces to

$$\left(A^2B\right)_{44} + \omega^2(A^2B) = 0 \quad (31)$$

Solving (31),we find

$$A^2B = \beta_1 \cos\sqrt{-3\Lambda}t + \beta_2 \sin\sqrt{-3\Lambda}t \quad (32)$$

where β_1 and β_2 are the constants of integration.

From (32), A and B can be expressed in explicit form as

$$A = (\beta_1 \cos\sqrt{-3\Lambda}t + \beta_2 \sin\sqrt{-3\Lambda}t)^{m_1},$$
$$B = (\beta_1 \cos\sqrt{-3\Lambda}t + \beta_2 \sin\sqrt{-3\Lambda}t)^{m_2} \quad (33)$$

where m_1 and m_2 are real constants such that

$$2m_1 + m_2 = 1. \quad (34)$$

Use of (33) in eqn. (8), we get

$$m_1^2 + 2m_1m_2 = \frac{[4\pi\varphi^{-1}v_4^2 + \Lambda]}{-3\Lambda}.\left[\frac{\beta_1 cos\sqrt{-3\Lambda t} + \beta_2 sin\sqrt{-3\Lambda t}}{\beta_2 cos\sqrt{-3\Lambda t} - \beta_1 sin\sqrt{-3\Lambda t}}\right]^2 \quad (35)$$

In eqn. (35), L.H.S is a real constant but R.H.S is a function of time though 1st term of R.H.S can be considered as constant.

Thus the relation in (35) is not possible and hence the solution cannot be determined. So for $\Lambda < 0$, the micro model of the universe does not exist in Barber's second self creation theory.

4. Some Physical and Geometrical Features of the Models

Case i: Here we intended to study the following properties of the model (25).

1. The spatial volume in the model is given by $V = (-g)^{1/2} = (A^2B) = \alpha_1 e^{\sqrt{3\Lambda}t}$.

Now $V \to \alpha_1$ as $t \to 0$ and $V \to \infty$ as $t \to \infty$. Thus the model of the universe starts expanding from a constant volume and becomes infinite large or blows up at infinite future.

2. The scalar expansion θ and the anisotropy $|\sigma|$ are defined by (Raychoudhuri, (1955) as $\theta = u^i_{;i} = \frac{V_4}{V}$,

where V is the volume element and

$$\sigma^2 = \frac{1}{12}\left[\left(\frac{g_{11,4}}{g_{11}} - \frac{g_{22,4}}{g_{22}}\right)^2 + \left(\frac{g_{22,4}}{g_{22}} - \frac{g_{33,4}}{g_{33}}\right)^2 + \left(\frac{g_{33,4}}{g_{33}} - \frac{g_{11,4}}{g_{11}}\right)^2\right].$$

Here the scalar expansion θ and anisotropy $|\sigma|$ in the model are found as

$\theta = \sqrt{3\Lambda} > 0$ and $\sigma^2 = 0$. Thus $\lim_{t\to\infty} \frac{\sigma}{\theta} = 0$

Thus it is evident from the above result that the universe is expanding in nature with constant rate of expansion. Also the universe is non-shearing and isotropic throughout the evolution.

3. The massless scalar field v and Barbers scalar φ are given by (17) and (20) respectively.

   Thus $\phi \to$ a constant as $t \to 0$ and $\phi \to$ a constant as $t \to \infty$ subject to the condition $\alpha_5 \neq 0$ or $\alpha_5 = 0$. So Barber's scalar has no singularity in this model. As the massless scalar field v in this model is found to be constant, so micro model of the universe does not exist.

4. The Kretschmann curvature invariant defined by $L = R_{hijk} R^{hijk}$, where R_{hijk} is the Riemann curvature tensor. Here L is found to be

$$L = \left(\frac{5}{3}\alpha_1^4\Lambda^2\right).\frac{1}{(\alpha_1 e^{\sqrt{3\Lambda}t})^4} > 0.$$

   Now $L \to$ a constant as $t \to 0$ and $L \to 0$ as $t \to \infty$. Since L is constant, the result confirms that the model has no geometrical singularity.

5. As the Hubble's parameter H is constant, so the model is of steady state model.

Case ii: Here we intended to study the following properties of the model (30).

1. The spatial volume in the model is given by $V = \alpha_2 e^{-\sqrt{3\Lambda}t}$.

   Here $V \to \alpha_2$ as $t \to 0$ and $V \to 0$ as $t \to \infty$. Thus the model of the universe starts expanding from a constant volume but volume of the universe gradually decreases as time increases. So at infinite future, volume of the universe becomes zero and the universe may collapse.

2. As in case-1, here also the scalar expansion θ and anisotropy $|\sigma|$ in the model are found as $\theta = -\sqrt{3\Lambda} < 0$ and $\sigma^2 = 0$. Thus $\lim_{t\to\infty} \frac{\sigma}{\theta} = 0$.

   Thus it is evident from the above result that the expansion of the universe is contracting in nature and isotropic throughout the evolution.

3. The massless scalar field v and Barbers scalar φ are given by (17) and (29) respectively.

   As in case-1 here also the micro model of the universe does not exist.

4. The Kretschmann curvature invariant is found to be

$$L = \left(\frac{5}{3}\alpha_2{}^4\Lambda^2 \right).(\alpha_2 e^{\sqrt{3\Lambda}t})^4$$
.

L$\rightarrow$ a constant as t $\rightarrow$ 0 and L $\rightarrow$ 0 as t $\rightarrow \infty$. Since L is constant, the result confirms that the model has no geometrical singularity.

5. As the Hubble's parameter H is constant, the model given by (30) is of steady state model.

5. Conclusion

In this paper we have described the plane symmetric space-time in modified theory of general relativity in presence of massless scalar field with cosmological constant as source matter. It is observed that plane symmetric cosmological models of the universe exist only when the cosmological constant Λ is positive. The model found in subcase-1 is expanding in nature, isotropic throughout the evolution, steady state and free from geometrical singularity. However the model found in sub case-2 is contracting in nature, isotropic throughout the evolution, steady state and free from geometrical singularity. It is interesting that massless scalar field v does not survive in both the models. Hence micro model of the plane symmetric universe does not exist in modified theory of general relativity.

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