# Investigation of Energy Staggering in Superdeformed Bands of Doubly Odd <sup>192,194</sup>Tl Nuclei

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**Abstract:** Ten superdeformed rotational bands (SDRB's) in <sup>192,194</sup>Tl nuclei have been studied with a four inertial parameters formula based on collective rotational model plus perturbed term linearly dependent on spin. The band head spins have been assigned by performing a fit with Harris parameterization to the experimental dynamical moment of inertia  $J^{(2)}$  as a function of rotational frequency h $\omega$  using a simulated search program. Using these assigned spin values, the calculated E2 transition energies agree with the experimental data very will. The kinematic  $J^{(1)}$  and dynamic  $J^{(2)}$  moments of inertia have been calculated and examined. The SD bands 1 and 2 in <sup>192</sup>Tl have been assigned odd and even respectively considering that the two bands are signature partner pair. These two bands have flat  $J^{(2)}$  which are nearly constant with h $\omega$ , they are assigned as double intruder configuration v  $j_{15/2} \otimes \pi i_{13/2}$ . The other SD bands 3 and 4 in <sup>192</sup>Tl and the six bands of <sup>194</sup>Tl all show characteristic rise of  $J^{(2)}$  with increasing h $\omega$  as for all observed SD bands 1, 3, 5 in <sup>194</sup>Tl exhibits  $\Delta I = 2$ staggering with amplitude of the same order of magnitude as that seen in the other SDRB's in the mass region A ~ 190. Also, we investigated the  $\Delta I = 1$  staggering splitting in the signature partner pair in <sup>192</sup>Tl (SD1, SD2) by extracted the differences between the average transitions  $I + 2 \rightarrow I \rightarrow I - 2$  energies in one band and the transition  $I + 1 \rightarrow I - 1$  energies in its signature partner.

**Keywords:** Superdeformed Rotational Bands, Harris Model, Staggering in Superdeformed Rotational Bands, Signature Partner Pair.

## **1. INTRODUCTION**

Since the first observation of discrete superdeformed rotational bands (SDRB) in <sup>152</sup>Dy [1], the super deformation (SD) at high spin remains one of the most challenging topics of nuclear structure [2]. More than 335 yrast and excited SD bands were observed in nuclei not only in the A ~ 150 mass region but also for A ~ 60, 80, 130, 140, 190 [3, 4]. At present, although a general understanding of SD has been achieved, there are still many open problems, such as spin assignment, evolution of dynamical moment of inertia  $J^{(2)}$  with rotational frequency h $\omega$  and the phenomenon of  $\Delta I = 2$  energy staggering. For most SD bands, the spin have not been established, only  $J^{(2)}$  can be extracted from the experimental differences in gamma transition energies  $\Delta E_{\gamma}$ .

Because of the regular behavior of  $E_{\gamma}$ , their spins were predicted theoretically in terms of their observed gamma ray energies by various approaches [5-11]. Moreover, the spins of the five SDRB's <sup>194</sup>Hg (SD1, SD3), <sup>194</sup>Pb (SD1) and <sup>193</sup>Tl (SD1, SD2) were established experimentally [12] which are in agreement with the prediction [8],

For most SD bands of even-even and odd-A nuclei in mass region A ~ 190 the dynamical moment of inertia  $J^{(2)}$  exhibits a smooth increase with increasing rotational frequency  $\hbar\omega$ , this is due to the gradual alignment of quasiparticles occupying high N-intruder orbitals originating from the  $i_{13/2}$  proton and  $j_{15/2}$  neutron subshells in the presence of pair correlations. In doubly odd nuclei a quite good part of  $J^{(2)}$  keep constant or flat. By now, it is commonly accepted that the dependence of  $J^{(2)}$  on  $\hbar\omega$  depend sensitively on the number of occupied high N-intruder orbitals.

In  $\Delta I = 2$  energy staggering, the SD rotational sequences split into two sequences of state in which spins differ by 4ħ from level to level and small energy displacement occurs between the two sets. This  $\Delta I = 2$  staggering was referred to as  $\Delta I = 4$  bifurcation or as C<sub>4</sub> oscillation hereby suggesting the presence of a fourfold rotational symmetry seen in the SD nuclear Hamiltonian [13], in which it is invariant under a rotation of 90° about the rotation axis as opposite to 180° which result in the normal  $\Delta I = 2$  sequence. Also  $\Delta I = 2$  staggering is associated with the alignment of the total nuclear angular momentum along the axis perpendicular to the long deformation axis of the prolate nucleus [14]. Our group tried to understand the  $\Delta I = 2$  staggering phenomenon in framework of phenomenological models [15-20].

There is another kind of staggering often occur in SDRB's of odd-A nuclei, it is the  $\Delta I = 1$  staggering. It was seen that most SDRB's in odd-A nuclei are signature partners and exhibit a  $\Delta I = 1$  signature splitting [21-24]. Recently signature partners have been found in doubly odd SD nuclei.

In the present paper, the  $\Delta I = 2$  and  $\Delta I = 1$  energy staggering in doubly odd <sup>192,194</sup>Tl nuclei are examined in framework of four parameter formula based on collective rotational model with adding a perturbed staggering term. In next section the formalism of the approach will be sketched. In section 3 we define the types of moment of inertia as theoretical tools. In section 4, the Harris method to assign the band head spins of the SDRB's is illustrated. Section 5 is devoted to explore the  $\Delta I = 2$  and  $\Delta I = 1$  energy staggering in SDRB's. In section 6 we present the numerical result and discussion. Finally conclusion is given in section 7.

## 2. PROPOSED MODEL TO PREDICT ENERGY STAGGERING IN SDRB'S

The deviation of excitation energy E(I) from the quadratic term, which represents the rigid rotor behavior, can be attributed to the dependence of the nuclear moment of inertia on the angular momentum. Several factors give the deviation of moment of inertia from the rigid rotor value like the Coriolis force, the centrifugal stretching, antipairing and alignment of angular momentum. However, the presence of higher-order terms is very important. For the SD bands, only gamma ray transition energies  $E_{\gamma}$  are determined. To understand the structure of SDRB's firstly a third order polynomial in angular momentum is considered to parameterize the variation of gamma ray transition energies  $E_{\gamma}$  for quadrupole transition between states of spin I and I - 2:

$$E_{\nu}(I) = AI^{3} + BI^{2} + CI + D \tag{1}$$

Where I is specifies the final state of the transition and A, B, C, D are higher order inertial parameters and  $E_{\gamma}(I)$  is defined as:

$$E_{v}(I) = E(I) - E(I-2)$$
<sup>(2)</sup>

To illustrate the staggering clearly and to measure how much each transition is perturbed, we suggested a perturbation reference linearly dependent on spin I:

$$\delta E_{\gamma}(I) = \begin{cases} \alpha I + \beta for & \frac{4,8,12,\dots}{5,9,13,\dots} \\ \gamma & for & \frac{6,10,14,\dots}{7,11,15,\dots} \end{cases}$$
(3)

### **3. MOMENTS OF INERTIA**

Two types of moments of inertia are usually discussed. The kinematic  $J^{(1)}$  (or first) moment of inertia and the dynamic  $J^{(2)}$  (or second) moment of inertia.  $J^{(1)}$  is defined as the first order derivative of the excitation energy E(I) with respect to spin  $\hat{I}$ :

$$\frac{J^{(1)}}{\hbar^2} = \hat{I} \left[ \frac{dE(I)}{d\hat{I}} \right]^{-1}, \quad \hat{I} = \sqrt{I(I+1)}$$

$$= \frac{\hat{I}}{\hbar \omega}$$
(4)

The dynamic moment of inertia  $J^{(2)}$  is a quantity related to the curvature of the excitation energy as a function of spin, it can be extracted from the difference between two consecutive quadrupole transitions in the band and is given by:

$$\frac{J^{(2)}}{\hbar^{2}} = \left[\frac{d^{2}E(I)}{d\hat{I}^{2}}\right]^{-1} = \left[\frac{d\omega}{d\hat{I}}\right]^{-1} = \left[\frac{d\omega}{d\hat{I}}\right]^{-1} = \frac{1}{\hbar}\frac{d\hat{I}}{d\omega} = \frac{1}{\hbar}\frac{d}{d\omega}\left(\frac{1}{\hbar}\omega J^{(1)}\right) = \frac{1}{\hbar}\left(J^{(1)} + \omega\frac{dJ^{(1)}}{d\omega}\right) \tag{5}$$

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For approximation

$$\frac{J^{(2)}}{\hbar^2} = \frac{d\hat{I}}{d\,\omega} \simeq \frac{2}{\Delta E_{\gamma}/2} = \frac{4}{\Delta E_{\gamma}}$$
(6)

with

$$\Delta E_{\gamma} = E_{\gamma}(I+2) - E_{\gamma}(I) \tag{7}$$

 $J^{(2)}$  does not depend on the knowledge of spin I, but only on the measured gamma ray energies. For the structureless rotation, which is characterized by  $dJ^{(1)} / d\omega = 0$ , we have  $J^{(2)} = J^{(1)} = \text{constant}$ . In particular for a rigid nuclear rotor, we should obtain  $J^{(2)} = J^{(1)} = J_{rigid}$ .

The kinematic moment of inertia  $J^{(1)}$  may be extracted by the experimentally gamma transition energies themselves:

$$J^{(1)} = \frac{2I - 1}{E_{\gamma}(I)}$$
(8)

## 4. SPIN ASSIGNMENT FOR SD BANDS USING HARRIS THREE PARAMETER EXPANSION

Harris [25] has presented expansion for nuclear rotational energy E in terms of even powers of the rotational frequency  $\omega$ , the expansion for E as a function of  $\omega$  (up to  $\omega^6$ ) is:

$$E = \frac{1}{2}c_1\omega^2 + \frac{3}{4}c_2\omega^4 + \frac{5}{6}c_3\omega^6$$
(9)

where  $c_1$ ,  $c_2$  and  $c_3$  are expansion parameters. Using the equations in previous section the corresponding expansion for  $J^{(2)}$  is:

$$J^{(2)} = c_1 + 3 c_2 \omega^2 + 5 c_3 \omega^4 \tag{10}$$

which leads to expression for Î as:

$$\hbar \hat{I} = \int J^{(2)} d\omega$$

$$= c_1 + c_2 \omega^3 + c_3 \omega^5$$
(11)

It is an odd power of  $\omega$ .

#### 5. Analysis of $\Delta = 2$ and $\Delta = 1$ Staggering in SDRB's

In the  $\Delta I = 2$  staggering the SDRB's split into two sequences of state I, I+4, I+8, ... and I+2, I+6, I+10, ... in which spins differ by 4ħ from level to level and small energy displacement occurs between the two sets. The effect of perturbation on  $\Delta E_{\gamma}$  can be determined by comparing the  $\Delta E_{\gamma}$  values with a smooth reference representing the fourth order derivative of gamma ray energies given by [26]:

$$\Delta^{4}E_{\gamma}(I) = \frac{1}{16} \left[ E_{\gamma}(I+4) - 4E_{\gamma}(I+2) + 6E_{\gamma}(I) - 4E_{\gamma}(I-2) + E_{\gamma}(I-4) \right]$$
(12)

$$S^{(4)} = \Delta^{4} E_{\gamma}(I) - (\Delta^{4} E_{\gamma}(I))^{\text{ref}}$$
(13)

This formula contains five consecutive transition energies which are denoted as the five-point formula.

To analysis the behavior of  $\Delta I = 1$  staggering in signature partner pairs, one may calculate the differences between the average transition  $I + 2 \rightarrow I \rightarrow I - 2$  energies in one band and the transition  $I + 1 \rightarrow I - 1$  energies in its signature partner:

$$S^{(2)}(I) = \frac{1}{2} \left\{ \frac{1}{2} [E_{\gamma}(I+2 \to I) + E_{\gamma}(I \to I-2)] - E_{\gamma}(I+1 \to I-1) \right\}$$
  
=  $\frac{1}{4} [E_{\gamma}(I+2) - 2E_{\gamma}(I+1) + E_{\gamma}(I)]$  (14)

#### 6. CALCULATED RESULTS AND DISCUSSION

In our calculation, the spin assignments of our selected SDRB's in <sup>192,194</sup>Tl nuclei are determined from Harris formula equation (11) and have been taken from our previous works [18]. The optimized best model parameters A, B, C, D,  $\alpha$ ,  $\beta$ ,  $\gamma$  have been calculated by using a search program in order to minimize the root mean square (rms) deviation  $\chi$  between the calculated and observed [2] transition energies. We adopted  $\chi$  by the common definition:

$$\chi^2 = \frac{1}{N} \sum_{j} \left( \frac{E_{\gamma}^{\exp(j) - E_{\gamma}^{cal}(j)}}{\Box E_{\gamma}(j)} \right)^2$$

where  $\Delta E_{\gamma}(j)$  is the experimental error and N is the number of the experimental points entering into the fitting procedure. Tables (1, 2) summarize the values of the adopted band head spin I<sub>o</sub>, the lowest transition energies  $E_{\gamma}$  (I<sub>o</sub> + 2  $\rightarrow$  I<sub>o</sub>) and the model parameters in KeV obtained by best fitting procedure.

**Table1.** The calculated optimized best model parameters A, B, C, D,  $\alpha$ ,  $\beta$  and  $\gamma$  (all in KeV) and suggested band head spin  $I_o$  for SDRB's in <sup>192, 194</sup>Tl nuclei which exhibit staggering in energies.

parameters	<sup>192</sup> Tl (SD2)	<sup>194</sup> Tl (SD1)	<sup>194</sup> Tl (SD3)	<sup>194</sup> Tl (SD5)
А	-1.6924 x 10 <sup>-4</sup>	-9.12 x 10 <sup>-4</sup>	-1.5844 x 10 <sup>-3</sup>	-9.6824 x 10 <sup>-4</sup>
В	2.5387 x 10 <sup>-4</sup>	1.368 x 10 <sup>-3</sup>	2.3767 x 10 <sup>-3</sup>	1.4523 x 10 <sup>-3</sup>
С	19.3106	20.0071	20.9904	19.7001
D	-9.6555	-10.0031	-10.4956	-9.8503
α	7.3529 x 10 <sup>-3</sup>	0.01665	0.01704	8.5084 x 10 <sup>-3</sup>
В	-0.0147	-0.0333	-0.03409	-0.01701
γ	0.0220	0.04995	0.05113	0.02552
Io	16	12	10	8
$E_{\gamma} (I_o + 2 \rightarrow I_o)$	337.5	268.0	240.5	187.9

**Table2.** The calculated best model parameters A, B, C and D (in KeV) and suggested band head spin  $I_o$  for SDRB's in <sup>192, 194</sup>Tl nuclei.

parameters	<sup>192</sup> Tl (SD1)	<sup>192</sup> Tl (SD3)	<sup>192</sup> Tl (SD4)
А	-2.7795 x 10 <sup>-5</sup>	-1.0539 x 10 <sup>-3</sup>	-1.0845 x 10 <sup>-3</sup>
В	4.1693 x 10 <sup>-5</sup>	1.5808 x 10 <sup>-3</sup>	1.6268 x 10 <sup>-3</sup>
С	19.4559	20.4048	20.4194
D	-9.7279	-10.2026	-10.2099
Io	13	10	9
$E_{\gamma} (I_o + 2 \rightarrow I_o)$	283.0	233.4	213.4
parameters	<sup>194</sup> Tl (SD2)	<sup>194</sup> Tl (SD4)	<sup>194</sup> Tl (SD6)
А	-9.442 x 10 <sup>-4</sup>	-1.3305 x 10 <sup>-3</sup>	-8.435 x 10 <sup>-4</sup>
В	1.4163 x 10 <sup>-3</sup>	1.9958 x 10 <sup>-3</sup>	1.2652 x 10 <sup>-3</sup>
С	20.0108	20.9224	19.7245
D	-10.0056	-10.4615	-9.8625
Io	9	9	9
$E_{\gamma} (I_o + 2 \rightarrow I_o)$	209.3	220.3	207.0

Using the adopted parameters listed in Tables (1, 2) we calculated the transition energies  $E_{\gamma}$  and the moments of inertia  $J^{(2)}$  and  $J^{(1)}$  the results of  $E_{\gamma}$  are given in Table (3) and compared with experimental data [3], very good agreement has been obtained. Figures (1, 2) illustrate the variation of the calculated kinematic  $J^{(1)}$  (open circles) and dynamic  $J^{(2)}$  (solid curves) moments of inertia and the experimental  $J^{(2)}$  (closed circles with error bars) as a function of rotational frequency  $\hbar\omega$  for the various SDRB's in <sup>192,194</sup>Tl nuclei. It is seen that all bands except bands 1 and 2 in <sup>192</sup>Tl experience a smooth rise in  $J^{(2)}$  with  $\hbar\omega$  which is similar to that observed in most bands of the A ~ 190 region. Bands 1, 2 in <sup>192</sup>Tl have flat  $J^{(2)}$  which are nearly constant with  $\hbar\omega$ . The  $\Delta I = 2$  staggering, i.e the curve found by smoothly interpolating the band energy of the spin sequence  $I = I_0 + 4n$  ( $n = 0, 1 \dots$ ) is somewhat displaced from the corresponding curve of the sequence  $\tilde{I} = I_0 + 4n + 2$ . The magnitude of the displacement in SD bands which experimentally emerges as  $\Delta I = 2$  staggering in the gamma ray transition energies is found to be in the range of some hundred eV to a few KeV. This is illustrated in Table (4) and Figure (3), where the staggering parameter  $S^{(4)}$  has been calculated by using the five point formula and plotted against the spin I. A significant anomalous staggering has been observed in the SDRB's <sup>192</sup>Tl (SD2) and <sup>194</sup>Tl (SD1, SD3, SD5). The two bands 1, 2 in <sup>192</sup>Tl have been interpreted as signature partner pairs. In Table (5) and Figure (4), the  $\Delta I = 1$  energy staggering parameters  $\Delta^2 E_{\gamma}$  in

signature partner pair <sup>192</sup>Tl (SD1, SD2) are given as a function of nuclear spin I, a clear large amplitude staggering is seen at high spins.



**Figure1.** The calculated results of the kinematic  $J^{(1)}$  (open circles) and dynamic  $J^{(2)}$  (solid curves) moments of inertia plotted as a function of rotational frequency h $\omega$  for the SDRB's in odd-odd <sup>192</sup>Tl nuclei and comparison with the experimental data for  $J^{(2)}$  (closed circles with error bars). The calculated results extracted from the proposed four parameters formula with perturbed term and the experimental data are extracted from ref. [3].

	103		102				100		102			
	<sup>192</sup> Tl (	SD1)	<sup>192</sup> Tl (SD2)				<sup>192</sup> Tl (SD3)			<sup>192</sup> Tl (SD4)		
I(ħ)	Ε <sub>γ</sub> (	I) (KeV)	I(ħ)	$\overline{E_{\gamma}(I)(KeV)}$		I(ħ)	Ε <sub>γ</sub> (1	$E_{\gamma}$ (I) (KeV)		$E_{\gamma}$ (I) (KeV)		
1(11)	EXP	CAL	1(11)	EXP	CAL	1(11)	EXP	CAL	1(11)	EXP	CAL	
15	283.0	282.0265	18	337.5	337.0228	12	233.4	233.0615	11	213.4	213.1571	
17	320.8	230.8983	20	374.9	375.4033	14	273.8	272.8826	13	253.7	253.1349	
19	359.0	359.7590	22	413.4	413.4840	16	313.0	312.3622	15	293.3	292.7872	
21	397.8	398.6075	24	451.1	451.7273	18	351.6	351.4497	17	332.2	322.0622	
23	437.1	437.4422	26	489.6	489.5958	20	390.4	390.0945	19	371.0	370.9076	
25	476.1	476.2620	28	527.4	527.6694	22	427.9	428.2462	21	409.3	409.2715	
27	515.2	515.0654	30	565.5	565.2932	24	465.4	465.8540	23	446.4	447.1017	
29	554.4	553.8511	32	603.1	603.1647	26	501.8	502.8674	25	482.6	484.3464	
31	593.0	592.6177	34	640.9	640.5114	28	537.8	539.2358	27	519.9	520.9532	
33	632.0	631.3641	36	677.7	678.1483	30	573.0	574.9086	29	555.3	556.8703	
35	670.4	670.0887	38	715.0	715.1852	32	607.2	609.8353	31	591.2	592.0455	
37	707.9	708.7905				34	642.6	643.9651	33	625.2	626.4269	
						36	676.8	677.2476	35	659.7	659.9622	

**Table3.** The calculated transition energies  $E_{\gamma}(I)$  for our selected SDRB's and comparison with experimental data. The model parameters and the band head spin are listed in Tables 1, 2.

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			38	712.5	709.6323	37	693.8	692.5995
			40	744.7	741.0683	39	727.3	724.2867

 Table3. Continued

	<sup>194</sup> Tl (SD1)	)		<sup>194</sup> Tl (SD2)		<sup>194</sup> Tl (SD3)			
I(ħ)	Ε <sub>γ</sub> (Ι)	(KeV)	I(ħ)	I(	ħ)	E <sub>γ</sub> (I)	I(	(ħ)	
1(11)	EXP	CAL		EXP	CAL	(KeV)	EXP	CAL	
14	268.0	268.0628	11	209.3	209.0281	12	240.5	239.1634	
16	307.0	307.2252	13	248.4	248.3000	14	280.0	279.5391	
18	345.1	345.5168	15	287.5	287.2888	16	318.8	319.7076	
20	384.2	383.8905	17	326.0	325.9489	18	358.1	358.9121	
22	421.0	421.4387	19	364.4	364.2351	20	397.2	397.8938	
24	457.0	458.8484	21	401.7	402.1021	22	425.3	435.6231	
26	494.9	495.4783	23	439.3	439.5045	24	473.0	473.1139	
28	530.9	531.7489	25	475.9	476.3970	26	510.9	509.0641	
30	567.0	567.2854	27	512.0	512.7344	28	546.6	544.7601	
32	601.2	601.2417	29	548.0	548.4713	30	582.2	578.6261	
34	634.9	636.5095	31	583.5	583.5623	32	617.4	612.2223	
36	669.8	669.9767	33	617.5	617.9621	34	652.0	643.7001	
38	703.6	702.8004	35	652.0	651.6255	36	685.5	674.8935	
			37	685.9	684.5071	38	717.5	703.6781	

## Table3. Continued

	<sup>194</sup> Tl (SD4)	)		<sup>194</sup> Tl (SD5)		<sup>194</sup> Tl (SD6)			
I(ħ)	E <sub>γ</sub> (I)	(KeV)	I(ħ)	I(	(ħ)	$E_{\gamma}(I)$	I	(ħ)	
1(11)	EXP	CAL	1(11)	EXP	CAL	(KeV)	EXP	CAL	
11	220.3	218.1555	10	187.9	186.3535	11	207.0	206.1384	
13	259.4	258.9438	12	226.3	225.1720	13	245.4	244.9179	
15	299.7	299.3330	14	264.0	263.6045	15	283.7	283.4443	
17	338.7	339.2591	16	302.0	301.8761	17	321.8	321.6771	
19	378.3	378.6584	18	339.2	339.6015	19	358.2	359.5759	
21	415.5	417.4668	20	376.6	377.1401	21	396.2	397.1003	
23	454.2	455.6206	22	413.7	413.9715	23	432.5	434.2096	
25	491.5	493.0560	24	450.0	450.5911	25	470.1	470.8633	
27	527.8	529.7089	26	486.1	486.3425	27	505.2	507.0212	
29	564.0	565.5156	28	521.8	521.8582	29	543.7	542.6425	
31	599.7	600.4122	30	558.4	556.3435	31	579.1	577.6870	
33	633.7	634.3347	32	593.7	590.5692	33	613.0	612.138	
35	669.2	667.2195	34	627.7	623.6025				
37	703.4	699.0026							

Table4.	<i>The calculated</i> $\Delta I = 2$	$r^{\circ}$ staggering parameter $S^{\circ}$	<sup>4)</sup> (in KeV) obtained	l by the five point f	ormula for SDRB's
$^{192}Tl$ (SL	D2), <sup>194</sup> Tl (SDS1, DS3,	D5) and comparison with	n experiment.		·

<sup>192</sup> Tl (SD2)			<sup>194</sup> Tl (SD1)			<sup>194</sup> Tl (SD3)			<sup>194</sup> Tl (SD5)		
т		<b>S</b> <sup>(4)</sup>	т		S <sup>(4)</sup>	т		S <sup>(4)</sup>	т		S <sup>(4)</sup>
1	EXP	CAL	1	EXP	CAL	1	EXP	CAL	1	EXP	CAL
22	3.5	1.0000	18	-5.2	1.7316	16	-1.9	1.5	14	-2.1	0.6126
24	-3.1	-1.1176	20	4.8	-1.9980	18	-4.2	-1.7727	16	2.1	-0.7487
26	2.5	1.2352	22	1.2	2.2644	20	1.8	2.0454	18	-1.5	0.8848
28	-1,8	-1.3529	24	-6.5	-2.5080	22	-0.6	-2.3181	20	0.0	-1.0210
30	1.5	1.4705	26	5.8	2.7972	24	-2.6	2.5909	22	1.1	1.1571
32	-1.9	-15882	28	-4.0	-3.0636	26	4.4	-2.8636	24	-0.8	-1.2932
34	2.7	1.7058	30	3.4	3.3300	28	-2.4	3.1363	26	1.5	1.4294
			32	0.3	-3.5964	30	0.1	-3.4090	28	-3.5	-1.5655
			34	-4.0	3.8628	32	-0.3	3.6818	30	2.2	1.7016

**Table5.** The calculated  $\Delta I = 1$  staggering parameter  $\Delta^{(4)} E_{\gamma}$  (in KeV) in signature partner pairs <sup>192</sup>Tl (SD1, SD2).

I 18 19 20 21 22 23 24 25 26 27
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Investigation of Energy Staggering in Superdeformed Bands of Doubly Odd <sup>192,194</sup>Tl Nuclei

$\Delta^{(4)} E_{\gamma}$	2.40	-2.80	3.50	-3.65	4.05	-4.85	5.50	-5.75	6.05	-6.70
Ι	28	29	30	31	32	33	34	35	36	37
$\Delta^{(4)}E_{\gamma}$	7.40	-7.95	8.20	-8.70	9.40	-10.00	10.3	-10.40	11.45	-11.55
		130 120 (	<sup>44</sup> TL (SD1) ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓	h collev) 0.3	13 12 12 12 12 12 12 12 12 12 12 12 12 12	0 194TL (SD2) 0 0 0 0 0 0 0 0 0 0 104 0 0 0 0 0 0 0 0 0 0 0 0 0	h co(MeV) 0.3	0.4		
		130 120 PPE 110 100 900- 0.1		Romey) 0.3	(, \August Lange and a set of the			↓ ↓ ↓ ↓ ↓ ↓		
		130,			1:	30				
		ر ( مهم معرف المعرف ا 100 معرف المعرف المع	<sup>94</sup> TL (SD5)		1: (, ^ every, est ) (, ^ ever					

Figure 2. The same as figure (1) but for <sup>194</sup>Tl nuclei



**Figure3.** The calculated  $\Delta I = 2$  Staggering parameter  $S^{(4)}$  (closed circles with solid line) obtained by the five point formula plotted as a function of spin I and comparison with experiment (Open circles with dotted lines) for the SDRB's <sup>192</sup>Tl (SD2), <sup>194</sup>Tl (SD1, SD3, SD5).



**Figure4.** Calculated  $\Delta I = 1$  energy staggering parameter  $\Delta^{(4)} E_{\gamma}$  (signature splitting) a function of nuclear spin I for the signature partner pair <sup>192</sup>Tl (SD1, SD2).

#### 7. CONCLUSION

The SDRB'S in <sup>192,194</sup>Tl nuclei have been studied in framework of four parameters formula with perturbed term for gamma ray transition energy. The band head spins and the model parameters are a adopted by using a simulated fitting search program. The calculated results agree very well with the corresponding experimental ones. It indicates that our approach is efficient for not only the yrast SD bands but also the excited SD bands of odd-odd nuclei. Bands 1, 2 in <sup>192</sup>Tl have flat dynamical moment of inertia  $J^{(2)}$  against rotational frequency h $\omega$  and are considered as signature partner pair, they exhibits  $\Delta I = 1$  energy staggering. The rest of our SDRB's have a smooth increase in  $J^{(2)}$  with h $\omega$  due to the gradual alignment of quasinucleons occupying high N intruder orbitals in the presence of the pair correlations. We found a  $\Delta I = 2$  energy staggering in bands 1, 3, 5 in <sup>194</sup>Tl by performing staggering parameter analysis.

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