# Investigation of Energy Staggering in Superdeformed Bands of Doubly Odd ${ }^{192,194} \mathbf{T l}$ Nuclei 

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#### Abstract

Ten superdeformed rotational bands (SDRB's) in ${ }^{192,194} \mathrm{Tl}$ nuclei have been studied with a four inertial parameters formula based on collective rotational model plus perturbed term linearly dependent on spin. The band head spins have been assigned by performing a fit with Harris parameterization to the experimental dynamical moment of inertia $J^{(2)}$ as a function of rotational frequency $\hbar \omega$ using a simulated search program. Using these assigned spin values, the calculated E2 transition energies agree with the experimental data very will. The kinematic $J^{(1)}$ and dynamic $J^{(2)}$ moments of inertia have been calculated and examined. The $S D$ bands 1 and 2 in ${ }^{192} \mathrm{Tl}$ have been assigned odd and even respectively considering that the two bands are signature partner pair. These two bands have flat $J^{(2)}$ which are nearly constant with $\hbar \omega$, they are assigned as double intruder configuration $v j_{15 / 2} \otimes \pi i_{13 / 2}$. The other $S D$ bands 3 and 4 in ${ }^{192} \mathrm{Tl}$ and the six bands of ${ }^{194} \mathrm{Tl}$ all show characteristic rise of $J^{(2)}$ with increasing $\hbar \omega$ as for all observed $S D$ bands in other nuclei in this region. They are strongly coupled over the entire range. Band 2 in ${ }^{192} \mathrm{Tl}$ and bands $1,3,5$ in ${ }^{194} \mathrm{Tl}$ exhibits $\Delta I=2$ staggering with amplitude of the same order of magnitude as that seen in the other SDRB's in the mass region A ~ 190. Also, we investigated the $\Delta I=1$ staggering splitting in the signature partner pair in ${ }^{192} \mathrm{Tl}$ (SD1, SD2) by extracted the differences between the average transitions $I+2 \rightarrow I \rightarrow I-2$ energies in one band and the transition $\quad I+1 \rightarrow I-1$ energies in its signature partner.


Keywords: Superdeformed Rotational Bands, Harris Model, Staggering in Superdeformed Rotational Bands, Signature Partner Pair.

## 1. Introduction

Since the first observation of discrete superdeformed rotational bands (SDRB) in ${ }^{152}$ Dy [1], the super deformation (SD) at high spin remains one of the most challenging topics of nuclear structure [2]. More than 335 yrast and excited SD bands were observed in nuclei not only in the A ~ 150 mass region but also for A ~ 60, 80, 130, 140, 190 [3, 4]. At present, although a general understanding of SD has been achieved, there are still many open problems, such as spin assignment, evolution of dynamical moment of inertia $J^{(2)}$ with rotational frequency $\hbar \omega$ and the phenomenon of $\Delta I=2$ energy staggering. For most SD bands, the spin have not been established, only $\mathrm{J}^{(2)}$ can be extracted from the experimental differences in gamma transition energies $\Delta \mathrm{E}_{\gamma}$.
Because of the regular behavior of $\mathrm{E}_{\gamma}$, their spins were predicted theoretically in terms of their observed gamma ray energies by various approaches [5-11]. Moreover, the spins of the five SDRB's ${ }^{194} \mathrm{Hg}(\mathrm{SD} 1, \mathrm{SD} 3),{ }^{194} \mathrm{~Pb}$ (SD1) and ${ }^{193} \mathrm{Tl}$ (SD1, SD2) were established experimentally [12] which are in agreement with the prediction [8],
For most SD bands of even-even and odd-A nuclei in mass region A ~ 190 the dynamical moment of inertia $\mathbf{J}^{(2)}$ exhibits a smooth increase with increasing rotational frequency $\hbar \omega$, this is due to the gradual alignment of quasiparticles occupying high N -intruder orbitals originating from the $\mathrm{i}_{13 / 2}$ proton and $\mathrm{j}_{15 / 2}$ neutron subshells in the presence of pair correlations. In doubly odd nuclei a quite good part of $\mathbf{J}^{(2)}$ keep constant or flat. By now, it is commonly accepted that the dependence of $\mathbf{J}^{(2)}$ on $\hbar \omega$ depend sensitively on the number of occupied high N -intruder orbitals.

In $\Delta \mathrm{I}=2$ energy staggering, the SD rotational sequences split into two sequences of state in which spins differ by $4 \hbar$ from level to level and small energy displacement occurs between the two sets. This $\Delta \mathrm{I}=2$ staggering was referred to as $\Delta \mathrm{I}=4$ bifurcation or as $\mathrm{C}_{4}$ oscillation hereby suggesting the

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presence of a fourfold rotational symmetry seen in the SD nuclear Hamiltonian [13], in which it is invariant under a rotation of $90^{\circ}$ about the rotation axis as opposite to $180^{\circ}$ which result in the normal $\Delta \mathrm{I}=2$ sequence. Also $\Delta \mathrm{I}=2$ staggering is associated with the alignment of the total nuclear angular momentum along the axis perpendicular to the long deformation axis of the prolate nucleus [14]. Our group tried to understand the $\Delta \mathrm{I}=2$ staggering phenomenon in framework of phenomenological models [15-20].

There is another kind of staggering often occur in SDRB's of odd-A nuclei, it is the $\Delta \mathrm{I}=1$ staggering. It was seen that most SDRB's in odd-A nuclei are signature partners and exhibit a $\Delta \mathrm{I}=1$ signature splitting [21-24]. Recently signature partners have been found in doubly odd SD nuclei.
In the present paper, the $\Delta \mathrm{I}=2$ and $\Delta \mathrm{I}=1$ energy staggering in doubly odd ${ }^{192,194} \mathrm{Tl}$ nuclei are examined in framework of four parameter formula based on collective rotational model with adding a perturbed staggering term. In next section the formalism of the approach will be sketched. In section 3 we define the types of moment of inertia as theoretical tools. In section 4, the Harris method to assign the band head spins of the SDRB's is illustrated. Section 5 is devoted to explore the $\Delta \mathrm{I}=2$ and $\Delta \mathrm{I}=1$ energy staggering in SDRB's. In section 6 we present the numerical result and discussion. Finally conclusion is given in section 7.

## 2. Proposed Model to Predict Energy Staggering in SDRB's

The deviation of excitation energy $\mathrm{E}(\mathrm{I})$ from the quadratic term, which represents the rigid rotor behavior, can be attributed to the dependence of the nuclear moment of inertia on the angular momentum. Several factors give the deviation of moment of inertia from the rigid rotor value like the Coriolis force, the centrifugal stretching, antipairing and alignment of angular momentum. However, the presence of higher-order terms is very important. For the SD bands, only gamma ray transition energies $\mathrm{E}_{\gamma}$ are determined. To understand the structure of SDRB's firstly a third order polynomial in angular momentum is considered to parameterize the variation of gamma ray transition energies $\mathrm{E}_{\gamma}$ for quadrupole transition between states of spin I and I-2:

$$
\begin{equation*}
E_{\gamma}(I)=A I^{3}+B I^{2}+C I+D \tag{1}
\end{equation*}
$$

Where I is specifies the final state of the transition and $\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}$ are higher order inertial parameters and $\mathrm{E}_{\gamma}(\mathrm{I})$ is defined as:

$$
\begin{equation*}
E_{\gamma}(I)=E(I)-E(I-2) \tag{2}
\end{equation*}
$$

To illustrate the staggering clearly and to measure how much each transition is perturbed, we suggested a perturbation reference linearly dependent on spin I:
$\delta \mathrm{E}_{\gamma}(\mathrm{I})=\left\{\begin{array}{cc}\alpha I+\beta \text { for } & \begin{array}{c}4,8,12, \ldots \\ 5,9, \ldots \\ 6, \ldots \ldots \\ 6,10,14, \ldots \\ 7,11,5, \ldots\end{array}\end{array}\right.$

## 3. Moments of Inertia

Two types of moments of inertia are usually discussed. The kinematic $J^{(1)}$ (or first) moment of inertia and the dynamic $J^{(2)}$ (or second) moment of inertia. $\mathrm{J}^{(1)}$ is defined as the first order derivative of the excitation energy $\mathrm{E}(\mathrm{I})$ with respect to spin $\hat{I}$ :

$$
\begin{align*}
\frac{J^{(1)}}{\hbar^{2}} & =\hat{I}\left[\frac{d E(I)}{d \hat{I}}\right]^{-1}, \quad \hat{I}=\sqrt{I(I+1)}  \tag{4}\\
& =\frac{\hat{I}}{\hbar \omega}
\end{align*}
$$

The dynamic moment of inertia $\mathrm{J}^{(2)}$ is a quantity related to the curvature of the excitation energy as a function of spin, it can be extracted from the difference between two consecutive quadrupole transitions in the band and is given by:

$$
\begin{align*}
\frac{J^{(2)}}{\hbar^{2}} & =\left[\frac{d^{2} E(I)}{d \hat{I}^{2}}\right]^{-1} \\
& =\left[\frac{d}{d \hat{I}} \frac{d E(I)}{d \hat{I}}\right]^{-1}=\left[\frac{d \omega}{d \hat{I}}\right]^{-1}=\frac{1}{\hbar} \frac{d \hat{I}}{d \omega}  \tag{5}\\
& =\frac{1}{\hbar} \frac{d}{d \omega}\left(\frac{1}{\hbar} \omega J^{(1)}\right)=\frac{1}{\hbar}\left(J^{(1)}+\omega \frac{d J^{(1)}}{d \omega}\right)
\end{align*}
$$

For approximation

$$
\begin{equation*}
\frac{J^{(2)}}{\hbar^{2}}=\frac{d \hat{I}}{d \omega} \simeq \frac{2}{\Delta E_{\gamma} / 2}=\frac{4}{\Delta E_{\gamma}} \tag{6}
\end{equation*}
$$

with

$$
\begin{equation*}
\Delta E_{\gamma}=E_{\gamma}(I+2)-E_{\gamma}(I) \tag{7}
\end{equation*}
$$

$\mathbf{J}^{(2)}$ does not depend on the knowledge of spin I, but only on the measured gamma ray energies. For the structureless rotation, which is characterized by $\mathrm{dJ}^{(1)} / \mathrm{d} \omega=0$, we have $\mathbf{J}^{(2)}=\mathbf{J}^{(1)}=$ constant. In particular for a rigid nuclear rotor, we should obtain $J^{(2)}=J^{(1)}=J_{\text {rigid }}$.
The kinematic moment of inertia $\mathbf{J}^{(1)}$ may be extracted by the experimentally gamma transition energies themselves:
$J^{(1)}=\frac{2 I-1}{E_{\gamma}(I)}$

## 4. Spin Assignment for SD Bands using Harris Three Parameter Expansion

Harris [25] has presented expansion for nuclear rotational energy E in terms of even powers of the rotational frequency $\omega$, the expansion for E as a function of $\omega$ (up to $\omega^{6}$ ) is:

$$
\begin{equation*}
E=\frac{1}{2} c_{1} \omega^{2}+\frac{3}{4} c_{2} \omega^{4}+\frac{5}{6} c_{3} \omega^{6} \tag{9}
\end{equation*}
$$

where $c_{1}, c_{2}$ and $c_{3}$ are expansion parameters. Using the equations in previous section the corresponding expansion for $\mathrm{J}^{(2)}$ is:

$$
\begin{equation*}
J^{(2)}=c_{1}+3 c_{2} \omega^{2}+5 c_{3} \omega^{4} \tag{10}
\end{equation*}
$$

which leads to expression for $\hat{I}$ as:

$$
\begin{align*}
\hbar \hat{I} & =\int J^{(2)} d \omega  \tag{11}\\
& =c_{1}+c_{2} \omega^{3}+c_{3} \omega^{5}
\end{align*}
$$

It is an odd power of $\omega$.

## 5. Analysis of $\Delta=2$ And $\Delta=1$ Staggering in SDRB's

In the $\Delta \mathrm{I}=2$ staggering the SDRB's split into two sequences of state $\mathrm{I}, \mathrm{I}+4, \mathrm{I}+8, \ldots$ and $\mathrm{I}+2, \mathrm{I}+6$, $\mathrm{I}+10, \ldots$ in which spins differ by $4 \hbar$ from level to level and small energy displacement occurs between the two sets. The effect of perturbation on $\Delta \mathrm{E}_{\gamma}$ can be determined by comparing the $\Delta \mathrm{E}_{\gamma}$ values with a smooth reference representing the fourth order derivative of gamma ray energies given by [26]:

$$
\begin{align*}
& \Delta^{4} E_{\gamma}(I)=\frac{1}{16}\left[E_{\gamma}(I+4)-4 E_{\gamma}(I+2)+6 E_{\gamma}(I)-4 E_{\gamma}(I-2)+E_{\gamma}(I-4)\right]  \tag{12}\\
& S^{(4)}=\Delta^{4} E_{\gamma}(I)-\left(\Delta^{4} E_{\gamma}(I)\right)^{\text {lef }} \tag{13}
\end{align*}
$$

This formula contains five consecutive transition energies which are denoted as the five-point formula.
To analysis the behavior of $\Delta \mathrm{I}=1$ staggering in signature partner pairs, one may calculate the differences between the average transition $\mathrm{I}+2 \rightarrow \mathrm{I} \rightarrow \mathrm{I}-2$ energies in one band and the transition I $+1 \rightarrow \mathrm{I}-1$ energies in its signature partner:

$$
\begin{align*}
S^{(2)}(I) & =\frac{1}{2}\left\{\frac{1}{2}\left[E_{\gamma}(I+2 \rightarrow I)+E_{\gamma}(I \rightarrow I-2)\right]-E_{\gamma}(I+1 \rightarrow I-1)\right\}  \tag{14}\\
& =\frac{1}{4}\left[E_{\gamma}(I+2)-2 E_{\gamma}(I+1)+E_{\gamma}(I)\right]
\end{align*}
$$

## 6. Calculated Results and Discussion

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In our calculation, the spin assignments of our selected SDRB's in ${ }^{192,194} \mathrm{Tl}$ nuclei are determined from Harris formula equation (11) and have been taken from our previous works [18]. The optimized best model parameters $\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}, \alpha, \beta, \gamma$ have been calculated by using a search program in order to minimize the root mean square (rms) deviation $\chi$ between the calculated and observed [2] transition energies. We adopted $\chi$ by the common definition:

$$
\chi^{2}=\frac{1}{N} \sum_{j}\left(\frac{E_{\gamma}^{\exp }(j)-E_{\gamma}^{c a l}(j)}{\square E_{\gamma}(j)}\right)^{2}
$$

where $\Delta \mathrm{E}_{\gamma}(\mathrm{j})$ is the experimental error and N is the number of the experimental points entering into the fitting procedure. Tables $(1,2)$ summarize the values of the adopted band head spin $I_{0}$, the lowest transition energies $\mathrm{E}_{\gamma}\left(\mathrm{I}_{0}+2 \rightarrow \mathrm{I}_{\mathrm{o}}\right)$ and the model parameters in KeV obtained by best fitting procedure.

Table1. The calculated optimized best model parameters $A, B, C, D, \alpha, \beta$ and $\gamma$ (all in KeV) and suggested band head spin $I_{o}$ for SDRB's in ${ }^{192,194}$ Tl nuclei which exhibit staggering in energies.

| parameters | ${ }^{192} \mathbf{T l}$ (SD2) | ${ }^{194} \mathbf{T l}$ (SD1) | ${ }^{194} \mathbf{T l}$ (SD3) | ${ }^{194} \mathbf{T l}$ (SD5) |
| :---: | :---: | :---: | :---: | :---: |
| A | $-1.6924 \times 10^{-4}$ | $-9.12 \times 10^{-4}$ | $-1.5844 \times 10^{-3}$ | $-9.6824 \times 10^{-4}$ |
| B | $2.5387 \times 10^{-4}$ | $1.368 \times 10^{-3}$ | $2.3767 \times 10^{-3}$ | $1.4523 \times 10^{-3}$ |
| C | 19.3106 | 20.0071 | 20.9904 | 19.7001 |
| D | -9.6555 | -10.0031 | -10.4956 | -9.8503 |
| $\alpha$ | $7.3529 \times 10^{-3}$ | 0.01665 | 0.01704 | $8.5084 \times 10^{-3}$ |
| B | -0.0147 | -0.0333 | -0.03409 | -0.01701 |
| $\gamma$ | 0.0220 | 0.04995 | 0.05113 | 0.02552 |
| $\mathrm{I}_{\mathrm{o}}$ | 16 | 12 | 10 | 8 |
| $\mathrm{E}_{\gamma}\left(\mathrm{I}_{0}+2 \rightarrow \mathrm{I}_{\mathrm{o}}\right)$ | 337.5 | 268.0 | 240.5 | 187.9 |

Table2. The calculated best model parameters $A, B, C$ and $D$ (in $K e V$ ) and suggested band head spin $I_{o}$ for SDRB's in ${ }^{192,194} \mathrm{Tl}$ nuclei.

| parameters | ${ }^{192} \mathbf{T l}$ (SD1) | ${ }^{192} \mathbf{T l}$ (SD3) | ${ }^{192} \mathbf{T l}$ (SD4) |
| :---: | :---: | :---: | :---: |
| A | $-2.7795 \times 10^{-5}$ | $-1.0539 \times 10^{-3}$ | $-1.0845 \times 10^{-3}$ |
| B | $4.1693 \times 10^{-5}$ | $1.5808 \times 10^{-3}$ | $1.6268 \times 10^{-3}$ |
| C | 19.4559 | 20.4048 | 20.4194 |
| D | -9.7279 | -10.2026 | -10.2099 |
| $\mathrm{I}_{\mathrm{o}}$ | 13 | 10 | 9 |
| $\mathrm{E}_{\gamma}\left(\mathrm{I}_{\mathrm{o}}+2 \rightarrow \mathrm{I}_{\mathrm{o}}\right)$ | 283.0 | 233.4 | 213.4 |
| parameters | ${ }^{194} \mathrm{Tl}(\mathrm{SD} 2)$ | ${ }^{194} \mathrm{Tl}(\mathrm{SD} 4)$ | ${ }^{194} \mathrm{Tl}$ (SD6) |
| A | $-9.442 \times 10^{-4}$ | $-1.3305 \times 10^{-3}$ | $-8.435 \times 10^{-4}$ |
| B | $1.4163 \times 10^{-3}$ | $1.9958 \times 10^{-3}$ | $1.2652 \times 10^{-3}$ |
| C | 20.0108 | 20.9224 | 19.7245 |
| D | -10.0056 | -10.4615 | -9.8625 |
| $\mathrm{I}_{\mathrm{o}}$ | 9 | 9 | 9 |
| $\mathrm{E}_{\gamma}\left(\mathrm{I}_{\mathrm{o}}+2 \rightarrow \mathrm{I}_{\mathrm{o}}\right)$ | 209.3 | 220.3 | 207.0 |

Using the adopted parameters listed in Tables $(1,2)$ we calculated the transition energies $\mathrm{E}_{\gamma}$ and the moments of inertia $\mathbf{J}^{(2)}$ and $\mathbf{J}^{(1)}$.the results of $\mathrm{E}_{\gamma}$ are given in Table (3) and compared with experimental data [3], very good agreement has been obtained. Figures (1, 2) illustrate the variation of the calculated kinematic $\mathrm{J}^{(1)}$ (open circles) and dynamic $\mathrm{J}^{(2)}$ (solid curves) moments of inertia and the experimental $J^{(2)}$ (closed circles with error bars) as a function of rotational frequency $\hbar \omega$ for the various SDRB's in ${ }^{192,194} \mathrm{Tl}$ nuclei. It is seen that all bands except bands 1 and 2 in ${ }^{192} \mathrm{Tl}$ experience a smooth rise in $\mathrm{J}^{(2)}$ with $\hbar \omega$ which is similar to that observed in most bands of the $\mathrm{A} \sim 190$ region. Bands 1,2 in ${ }^{192} \mathrm{Tl}$ have flat $\mathrm{J}^{(2)}$ which are nearly constant with $\hbar \omega$. The $\Delta \mathrm{I}=2$ staggering, i.e the curve found by smoothly interpolating the band energy of the spin sequence $I=I_{0}+4 n(n=0,1 \ldots)$ is somewhat displaced from the corresponding curve of the sequence $\grave{I}=I_{o}+4 n+2$. The magnitude of the displacement in SD bands which experimentally emerges as $\Delta \mathrm{I}=2$ staggering in the gamma ray transition energies is found to be in the range of some hundred eV to a few KeV . This is illustrated in Table (4) and Figure (3), where the staggering parameter $S^{(4)}$ has been calculated by using the five point formula and plotted against the spin I. A significant anomalous staggering has been observed in the SDRB's ${ }^{192} \mathrm{Tl}$ (SD2) and ${ }^{194} \mathrm{Tl}$ (SD1, SD3, SD5). The two bands 1,2 in ${ }^{192} \mathrm{Tl}$ have been interpreted as signature partner pairs. In Table (5) and Figure (4), the $\Delta I=1$ energy staggering parameters $\Delta^{2} E_{\gamma}$ in

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signature partner pair ${ }^{192} \mathrm{Tl}$ (SD1, SD2) are given as a function of nuclear spin I, a clear large amplitude staggering is seen at high spins.


Figure1. The calculated results of the kinematic $J^{(1)}$ (open circles) and dynamic $J^{(2)}$ (solid curves) moments of inertia plotted as a function of rotational frequency $\hbar \omega$ for the SDRB's in odd-odd ${ }^{192} \mathrm{Tl}$ nuclei and comparison with the experimental data for $J^{(2)}$ (closed circles with error bars). The calculated results extracted from the proposed four parameters formula with perturbed term and the experimental data are extracted from ref. [3].

Table3. The calculated transition energies $E_{\gamma}(I)$ for our selected SDRB's and comparison with experimental data. The model parameters and the band head spin are listed in Tables 1, 2.

| ${ }^{192} \mathrm{Tl}$ (SD1) |  |  | ${ }^{192} \mathrm{Tl}$ (SD2) |  |  | ${ }^{192} \mathrm{Tl}$ (SD3) |  |  | ${ }^{192} \mathrm{Tl}$ (SD4) |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| I(ћ) | $\mathrm{E}_{\gamma}$ (I) (KeV) |  | I(ћ) | $\mathrm{E}_{\gamma}$ (I) (KeV) |  | I(ћ) | $\mathrm{E}_{\gamma}(\mathrm{I})(\mathrm{KeV})$ |  | I( $\dagger$ ) | $\mathrm{E}_{\gamma}$ (I) (KeV) |  |
|  | EXP | CAL |  | EXP | CAL |  | EXP | CAL |  | EXP | CAL |
| 15 | 283.0 | 282.0265 | 18 | 337.5 | 337.0228 | 12 | 233.4 | 233.0615 | 11 | 213.4 | 213.1571 |
| 17 | 320.8 | 230.8983 | 20 | 374.9 | 375.4033 | 14 | 273.8 | 272.8826 | 13 | 253.7 | 253.1349 |
| 19 | 359.0 | 359.7590 | 22 | 413.4 | 413.4840 | 16 | 313.0 | 312.3622 | 15 | 293.3 | 292.7872 |
| 21 | 397.8 | 398.6075 | 24 | 451.1 | 451.7273 | 18 | 351.6 | 351.4497 | 17 | 332.2 | 322.0622 |
| 23 | 437.1 | 437.4422 | 26 | 489.6 | 489.5958 | 20 | 390.4 | 390.0945 | 19 | 371.0 | 370.9076 |
| 25 | 476.1 | 476.2620 | 28 | 527.4 | 527.6694 | 22 | 427.9 | 428.2462 | 21 | 409.3 | 409.2715 |
| 27 | 515.2 | 515.0654 | 30 | 565.5 | 565.2932 | 24 | 465.4 | 465.8540 | 23 | 446.4 | 447.1017 |
| 29 | 554.4 | 553.8511 | 32 | 603.1 | 603.1647 | 26 | 501.8 | 502.8674 | 25 | 482.6 | 484.3464 |
| 31 | 593.0 | 592.6177 | 34 | 640.9 | 640.5114 | 28 | 537.8 | 539.2358 | 27 | 519.9 | 520.9532 |
| 33 | 632.0 | 631.3641 | 36 | 677.7 | 678.1483 | 30 | 573.0 | 574.9086 | 29 | 555.3 | 556.8703 |
| 35 | 670.4 | 670.0887 | 38 | 715.0 | 715.1852 | 32 | 607.2 | 609.8353 | 31 | 591.2 | 592.0455 |
| 37 | 707.9 | 708.7905 |  |  |  | 34 | 642.6 | 643.9651 | 33 | 625.2 | 626.4269 |
|  |  |  |  |  |  | 36 | 676.8 | 677.2476 | 35 | 659.7 | 659.9622 |

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|  |  |  |  |  |  | 38 | 712.5 | 709.6323 | 37 | 693.8 | 692.5995 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |  | 40 | 744.7 | 741.0683 | 39 | 727.3 | 724.2867 |

Table3. Continued

| ${ }^{194} \mathrm{Tl}$ (SD1) |  |  | ${ }^{194} \mathrm{Tl}$ (SD2) |  |  | ${ }^{194} \mathrm{Tl}$ (SD3) |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| I(ћ) | $\mathrm{E}_{\gamma}$ (I) (KeV) |  | I(ћ) | I(ћ) |  | $\begin{aligned} & \hline \mathrm{E}_{\gamma}(\mathrm{I}) \\ & (\mathrm{KeV}) \end{aligned}$ | I(ち) |  |
|  | EXP | CAL |  | EXP | CAL |  | EXP | CAL |
| 14 | 268.0 | 268.0628 | 11 | 209.3 | 209.0281 | 12 | 240.5 | 239.1634 |
| 16 | 307.0 | 307.2252 | 13 | 248.4 | 248.3000 | 14 | 280.0 | 279.5391 |
| 18 | 345.1 | 345.5168 | 15 | 287.5 | 287.2888 | 16 | 318.8 | 319.7076 |
| 20 | 384.2 | 383.8905 | 17 | 326.0 | 325.9489 | 18 | 358.1 | 358.9121 |
| 22 | 421.0 | 421.4387 | 19 | 364.4 | 364.2351 | 20 | 397.2 | 397.8938 |
| 24 | 457.0 | 458.8484 | 21 | 401.7 | 402.1021 | 22 | 425.3 | 435.6231 |
| 26 | 494.9 | 495.4783 | 23 | 439.3 | 439.5045 | 24 | 473.0 | 473.1139 |
| 28 | 530.9 | 531.7489 | 25 | 475.9 | 476.3970 | 26 | 510.9 | 509.0641 |
| 30 | 567.0 | 567.2854 | 27 | 512.0 | 512.7344 | 28 | 546.6 | 544.7601 |
| 32 | 601.2 | 601.2417 | 29 | 548.0 | 548.4713 | 30 | 582.2 | 578.6261 |
| 34 | 634.9 | 636.5095 | 31 | 583.5 | 583.5623 | 32 | 617.4 | 612.2223 |
| 36 | 669.8 | 669.9767 | 33 | 617.5 | 617.9621 | 34 | 652.0 | 643.7001 |
| 38 | 703.6 | 702.8004 | 35 | 652.0 | 651.6255 | 36 | 685.5 | 674.8935 |
|  |  |  | 37 | 685.9 | 684.5071 | 38 | 717.5 | 703.6781 |

Table3. Continued

| ${ }^{194} \mathrm{Tl}$ (SD4) |  |  | ${ }^{194} \mathrm{Tl}$ (SD5) |  |  | ${ }^{194} \mathrm{Tl}$ (SD6) |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| I(ћ) | $\mathrm{E}_{\gamma}$ (I) (KeV) |  | I(ћ) | I(ћ) |  | $\begin{gathered} \mathrm{E}_{\gamma}(\mathrm{I}) \\ (\mathrm{KeV}) \\ \hline \end{gathered}$ | I(ћ) |  |
|  | EXP | CAL |  | EXP | CAL |  | EXP | CAL |
| 11 | 220.3 | 218.1555 | 10 | 187.9 | 186.3535 | 11 | 207.0 | 206.1384 |
| 13 | 259.4 | 258.9438 | 12 | 226.3 | 225.1720 | 13 | 245.4 | 244.9179 |
| 15 | 299.7 | 299.3330 | 14 | 264.0 | 263.6045 | 15 | 283.7 | 283.4443 |
| 17 | 338.7 | 339.2591 | 16 | 302.0 | 301.8761 | 17 | 321.8 | 321.6771 |
| 19 | 378.3 | 378.6584 | 18 | 339.2 | 339.6015 | 19 | 358.2 | 359.5759 |
| 21 | 415.5 | 417.4668 | 20 | 376.6 | 377.1401 | 21 | 396.2 | 397.1003 |
| 23 | 454.2 | 455.6206 | 22 | 413.7 | 413.9715 | 23 | 432.5 | 434.2096 |
| 25 | 491.5 | 493.0560 | 24 | 450.0 | 450.5911 | 25 | 470.1 | 470.8633 |
| 27 | 527.8 | 529.7089 | 26 | 486.1 | 486.3425 | 27 | 505.2 | 507.0212 |
| 29 | 564.0 | 565.5156 | 28 | 521.8 | 521.8582 | 29 | 543.7 | 542.6425 |
| 31 | 599.7 | 600.4122 | 30 | 558.4 | 556.3435 | 31 | 579.1 | 577.6870 |
| 33 | 633.7 | 634.3347 | 32 | 593.7 | 590.5692 | 33 | 613.0 | 612.138 |
| 35 | 669.2 | 667.2195 | 34 | 627.7 | 623.6025 |  |  |  |
| 37 | 703.4 | 699.0026 |  |  |  |  |  |  |

Table4. The calculated $\Delta I=2$ staggering parameter $S^{(4)}$ (in KeV) obtained by the five point formula for SDRB's ${ }^{192}$ Tl (SD2), ${ }^{194}$ Tl (SDS1, DS3, D5) and comparison with experiment.

| ${ }^{192} \mathbf{T l}$ (SD2) |  |  | ${ }^{194} \mathbf{T l}$ (SD1) |  |  | ${ }^{194}$ Tl (SD3) |  |  | ${ }^{194} \mathbf{T l}$ (SD5) |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| I |  | (4) | I | $S^{(4)}$ |  | I | $\mathrm{S}^{(4)}$ |  | I | $S^{(4)}$ |  |
| I | EXP | CAL |  | EXP | CAL |  | EXP | CAL |  | EXP | CAL |
| 22 | 3.5 | 1.0000 | 18 | -5.2 | 1.7316 | 16 | -1.9 | 1.5 | 14 | -2.1 | 0.6126 |
| 24 | -3.1 | -1.1176 | 20 | 4.8 | -1.9980 | 18 | -4.2 | -1.7727 | 16 | 2.1 | -0.7487 |
| 26 | 2.5 | 1.2352 | 22 | 1.2 | 2.2644 | 20 | 1.8 | 2.0454 | 18 | -1.5 | 0.8848 |
| 28 | -1,8 | -1.3529 | 24 | -6.5 | -2.5080 | 22 | -0.6 | -2.3181 | 20 | 0.0 | -1.0210 |
| 30 | 1.5 | 1.4705 | 26 | 5.8 | 2.7972 | 24 | -2.6 | 2.5909 | 22 | 1.1 | 1.1571 |
| 32 | -1.9 | -15882 | 28 | -4.0 | -3.0636 | 26 | 4.4 | -2.8636 | 24 | -0.8 | -1.2932 |
| 34 | 2.7 | 1.7058 | 30 | 3.4 | 3.3300 | 28 | -2.4 | 3.1363 | 26 | 1.5 | 1.4294 |
|  |  |  | 32 | 0.3 | -3.5964 | 30 | 0.1 | -3.4090 | 28 | -3.5 | -1.5655 |
|  |  |  | 34 | -4.0 | 3.8628 | 32 | -0.3 | 3.6818 | 30 | 2.2 | 1.7016 |

Table5. The calculated $\Delta I=1$ staggering parameter $\Delta^{(4)} E_{\gamma}$ (in KeV ) in signature partner pairs ${ }^{192} \mathrm{Tl}$ (SD1, SD2).

| I | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

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| $\Delta^{(4)} \mathrm{E}_{\gamma}$ | 2.40 | -2.80 | 3.50 | -3.65 | 4.05 | -4.85 | 5.50 | -5.75 | 6.05 | -6.70 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| I | 28 | 29 | 30 | 31 | 32 | 33 | 34 | 35 | 36 | 37 |
| $\Delta^{(4)} \mathrm{E}_{\gamma}$ | 7.40 | -7.95 | 8.20 | -8.70 | 9.40 | -10.00 | 10.3 | -10.40 | 11.45 | -11.55 |



Figure2. The same as figure (1) but for ${ }^{194} \mathrm{Tl}$ nuclei


Figure3. The calculated $\Delta I=2$ Staggering parameter $S^{(4)}$ (closed circles with solid line) obtained by the five point formula plotted as a function of spin I and comparison with experiment (Open circles with dotted lines) for the SDRB's ${ }^{192} \mathrm{Tl}$ (SD2), ${ }^{194} \mathrm{Tl}$ (SD1, SD3, SD5).


Figure4. Calculated $\Delta I=1$ energy staggering parameter $\Delta^{(4)} E_{\gamma}$ (signature splitting) a function of nuclear spin I for the signature partner pair ${ }^{192}$ Tl (SD1, SD2).

## 7. Conclusion

The SDRB'S in ${ }^{192,194} \mathrm{Tl}$ nuclei have been studied in framework of four parameters formula with perturbed term for gamma ray transition energy. The band head spins and the model parameters are a adopted by using a simulated fitting search program. The calculated results agree very well with the corresponding experimental ones. It indicates that our approach is efficient for not only the yrast SD bands but also the excited SD bands of odd-odd nuclei. Bands 1,2 in ${ }^{192} \mathrm{Tl}$ have flat dynamical moment of inertia $J^{(2)}$ against rotational frequency $\hbar \omega$ and are considered as signature partner pair, they exhibits $\Delta I=1$ energy staggering. The rest of our SDRB's have a smooth increase in $J^{(2)}$ with $\hbar \omega$ due to the gradual alignment of quasinucleons occupying high N intruder orbitals in the presence of the pair correlations. We found a $\Delta \mathrm{I}=2$ energy staggering in bands $1,3,5$ in ${ }^{194} \mathrm{Tl}$ by performing staggering parameter analysis.

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