Golden Ratio on Discrete Surface

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Abstract: The golden ratio on the discrete surface has been discussed.

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1. INTRODUCTION

In the previous paper [1] the similarity between golden ratio \( \phi = \frac{1+\sqrt{5}}{2} \approx 1.618 \ldots \) and the average hyperbolic – elliptic unit being expressed on the continuous surface \( s_{\text{continuous}}(1) = 2 - \frac{1}{\sqrt{1+\pi^2}} \approx 1.696 \ldots \) was recognized.

The subject of interest of this paper is to compare the golden ratio and the average hyperbolic – elliptic unit being expressed on the discrete surface, too. Here the path is not concluded on the circumference of a circle [2] but on the average perimeter of the most favourable polygons.

2. THE PSUEDO PI (\( \pi^* \))

The perimeter of n-sided regular polygon \( 2\pi^* R \) is shorter than the circumference of a circle \( 2\pi R \) since pseudo pi, denoted \( \pi^* \), of any n-sided polygon is smaller than \( \pi \) of a circle [3]:

\[ \pi^* = \frac{n}{\sin \frac{n}{2}} \pi < \pi \; \text{for} \; n < \infty. \]

\( \pi^* \) of the first three regular polygons are collected in Table 1.

<table>
<thead>
<tr>
<th>Name</th>
<th>Number of sides n</th>
<th>( \pi^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>digon</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>triangle</td>
<td>3</td>
<td>( 3 \sin \frac{\pi}{3} = 3 \frac{\sqrt{3}}{2} = 2.598 )</td>
</tr>
<tr>
<td>square</td>
<td>4</td>
<td>( 4 \sin \frac{\pi}{4} = 2 \sqrt{2} = 2.828 \ldots )</td>
</tr>
<tr>
<td>circle</td>
<td>( \infty )</td>
<td>( \pi )</td>
</tr>
</tbody>
</table>

We can see that \( \pi^* \) rises with number of polygon sides \( n \) becoming equal to \( \pi \) when a polygon at \( n = \infty \) converts to a circle. The increase of \( \pi^* \) is gradually smaller. Consequently the average \( \pi^* \) of two neighbour even-sided polygons is smaller than \( \pi^* \) of the odd-sided polygon in the middle between them. The smallest is the average pi of digon (\( n=2 \)) and square (\( n=4 \)) since it is smaller than \( \pi^* \) of triangle (\( n=3 \)):

\[ \pi^*_{\text{minimal}} = \frac{\pi^*_{\text{digon}} + \pi^*_{\text{square}}}{2} = 2.414 \ldots < \pi^*_{\text{triangle}} = 2.598 \ldots \]

It enables the smallest and thus most favourable path concluded on the average perimeter \( 2\pi^*_{\text{minimal}} R \) of the corresponding polygons on the discrete surface:

\[ \pi^*_{\text{favourable}} = \pi^*_{\text{minimal}} = \frac{\pi^*_{\text{digon}} + \pi^*_{\text{square}}}{2} = \frac{2 + 2\sqrt{2}}{2} = 1 + \sqrt{2}. \]

3. \( \pi^* \) AND THE AVERAGE HYPERBOLIC-ELLPTIC UNIT

The most favourable \( \pi^* \) gives the next ratio of the average hyperbolic – elliptic unit \( s(1) \) to elliptic unit 1 being expressed on the most favourable discrete surface:

\[ \frac{\pi^*}{s(1)} \]
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\[
s_{\text{discrete}}(1) = 2 - \frac{1}{\sqrt{1 + \pi^2_{\text{favourable}}}} = 2 - \frac{1}{\sqrt{1 + (1 + \sqrt{2})^2}} = 2 - \frac{1}{\sqrt{4 + 2\sqrt{2}}} = 1.6173... \tag{4}
\]

4. THE AVERAGE HYPERBOLIC-ELLIPITIC UNIT BEING EXPRESSED ON THE DISCRETE SURFACE COMPARED TO THE GOLDEN RATIO

The average hyperbolic-elliptic unit being expressed on the most favourable average discrete surface \(s_{\text{discrete}}(1) = 2 - \frac{1}{\sqrt{1 + \sqrt{2}}} = 1.6173... \) only on the fourth decimal differs from the golden ratio \(\phi = \frac{1 + \sqrt{5}}{2} = 1.6180...\)

Since:

\[
\phi - s_{\text{discrete}}(1) = 1.6180 - 1.6173 = 0.0007. \tag{5}
\]

5. THE AVERAGE HYPERBOLIC-ELLIPITIC UNIT BEING EXPRESSED ON THE DISCRETE AS WELL AS CONTINUOUS SURFACE COMPARED TO THE GOLDEN RATIO

\[
s_{\text{discrete}}(1) = 1.6173 < \phi = 1.6180 < s_{\text{continuous}}(1) = 1.6967. \tag{6}
\]

The golden ratio lies within the interval defined by the discrete and the continuous unit.

6. CONCLUSION

The golden ratio almost equals the average hyperbolic – elliptic unit if the latter is expressed on the most favourable average discrete surface which is characterized by the value of pseudo pi yielding \(\pi^*_{\text{favourable}} = 1 + \sqrt{2}\).

DEDICATION

To Mahatma Gandhi and his quote: “True beauty after all consists in purity of heart.”

Figure 1. About true beauty [4]

REFERENCES


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