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Convection in Viscoelastic Fluid Coupled with Cross –Diffusions

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Abstract: *The present investigation mathematically establishes that the viscoelastic double diffusive convection coupled with cross diffusions of the Veronis' type cannot manifest as oscillatory motion of growing amplitude in an initially bottom heavy configuration if the thermal Rayleigh number* R_S *, the Lewis number* τ *, the Prandtl number* σ *and the viscoelastic parameter F satisfy the inequality* $R'_S \le \frac{27\pi^4 k_1}{4}\left\{1+\frac{\tau k_2}{\sigma}(1-F)\right\},\ 0<F<1$. *A similar mathematical theorem is also proved for Stern's type configuration. Further, the results derived herein are uniformly valid for the quite general nature of bounding surfaces.*

Keywords: *Double-diffusive convection; Rayleigh numbers; Prandtl number; Lewis number; Soret and Dufour effects; Rivlin –Ericksen viscoelastic fluid.*

1. INTRODUCTION

Double diffusive convection, with its archetypal case of heat and salt, generally referred to as thermohaline convection, has been intensively studied in recent past on account of its interesting complexities as well as its direct relevance in many problems of practical interest in the fields of Limnology, Oceanography, Geophysics, Astrophysics and Chemical Engineering etc. Two fundamental configurations have been studied in this context, the first one by Stern [1] wherein the temperature gradient was stabilizing while the concentration gradient was destabilizing and the second one by Veronis' [2] wherein the temperature gradient was destabilizing while the concentration gradient was stabilizing. The main results derived by Stern and Veronis' for their respective problems are that instability might occur in the configurations through a stationary pattern of motions or oscillatory motions provided the destabilizing concentration gradient or temperature gradient is sufficiently large even when the total density field is gravitationally stable. Thus, oscillatory motions of growing amplitude can occur in a thermohaline configuration

of the Veronis' type wherein the total density field is either gravitationally stable or unstable as indicated by the analysis of Veronis' notwithstanding the respective character of his work with respect to the nature of the bounding surfaces.

The stability properties of binary fluids are quite different from pure fluids because of Soret and Dufour effects [3], [4]. An externally imposed temperature gradient produces a chemical potential gradient and the phenomenon known as the Soret effect, arises when the mass flux contains a term that depends upon the temperature gradient. The analogous effect that arises from a concentration gradient dependent term in the heat flux is called the Dufour effect. Although it is clear that the thermosolutal and Soret-Dufour problems are quite closely related, their relationship has never been carefully elucidated. They are in fact, formally identical and identification is done by means of a linear transformation that takes the equations and boundary conditions for the latter problem into those for the former. The analysis of double diffusive convection becomes complicated in case when the diffusivity of one property is much greater than the other. Further, when two transport processes take place simultaneously, they interfere with each other and produce cross diffusion effect. The Soret and Dufour coefficients describe the flux of mass caused by temperature gradient and the flux of heat caused by concentration gradient respectively. The coupling of the fluxes of the stratifying agents is a prevalent feature in multicomponent fluid systems. In general, the stability of such systems are also affected by the cross-diffusion terms. Generally, it is assumed that the effect of cross diffusions on the stability criteria is negligible. However, there are liquid mixtures for which cross diffusions are of the same order of magnitude as the diffusivities. There are only few studies available on the effect of cross diffusion on double diffusion convection largely because of the complexity in determining these coefficients. Hurle and Jakeman [5] have studied the effect of Soret coefficient on the double–diffusive convection. They have reported that the magnitude and sign of the Soret coefficient were changed by varying the composition of the mixture. McDougall [6] has made an in depth study of double diffusive convection where in both Soret and Dufour effects are important.

In all the above studies, the fluid has been considered to be Newtonian. However, with the growing importance of non-Newtonian fluids in modern technology and industries, the investigations on such fluids are desirable. The Rivlin-Ericksen [7] fluid is such fluid. Many research workers have paid their attention towards the study of Rivlin-Ericksen fluid. Johri [8] has discussed the viscoelastic Rivlin-Ericksen incompressible fluid under time dependent pressure gradient. Sisodia and Gupta [9] and Srivastava and Singh [10] have studied the unsteady flow of a dusty elastico-viscous Rivlin-Ericksen fluid through channel of different cross-sections in the presence of the time dependent pressure gradient. Sharma and Kumar [11] have studied the thermal instability of a layer of Rivlin-Ericksen elastico-viscous fluid acted on by a uniform rotation and found that rotation has a stabilizing effect and introduces oscillatory modes in the system. Sharma and Kumar [12] have studied the thermal instability in Rivlin-Ericksen elastico-viscous fluid in hydromagnetics.

In the present paper, therefore, an attempt is made to establish a mathematical theorem disproving the existence of neutral or unstable oscillatory motions in an initially bottom heavy/top heavy thermosolutal convection configuration of the Veronis'/ Stern type in a layer of Rivlin-Ericksen viscoelastic fluid in the presence of Soret and Dufour effects..

2. Mathematical Formulation of the Problem

The relevant governing equations and boundary conditions of thermosolutal convection of a Rivlin-Ericksen viscoelastic fluid coupled with cross-diffusions are given by

$$\left(D^2-a^2\right)\left((D^2-a^2)-\frac{p}{\sigma}(1-F)\right)w = R\ \ a^2\theta - R_s a^2\phi \ \ , \tag{1}$$

$$\left(D^2-a^2-p\right)\theta + D_T(D^2-a^2)\,\phi = -w \ \ , \tag{2}$$

$$\left(D^2-a^2-\frac{p}{\tau}\right)\phi + S_T(D^2-a^2)\,\theta = -\frac{w}{\tau} \ \ , \tag{3}$$

With $w = 0 = \theta = \phi = Dw$ at z=0 and z=1 (on rigid boundaries) (4)

$w = 0 = D^2 w = \theta = \phi$ at z=0 and z=1 (on a dynamical free boundaries), (5)

In (1)–(5), z is real independent variable such that $0 \le z \le 1$, $D = \frac{d}{dz}$ is differentiation w.r.t z , a^2 >0 is a constant, $\sigma > 0$ is a constant, $\tau > 0$ is a constant, $0 < F < 1$ is a constant, R_T and R_S are positive constants for the Veronis' configuration and negative constant for Stern's configuration, $p = p_r + ip_i$ is complex constant in general such that p_r and p_i are real constants and as a consequence the dependent variables $w(z) = w_r(z) + iw_i(z)$, $\theta(z) = \theta_r(z) + i\theta_i(z)$ and $\phi(z) = \phi_r(z) + i\phi_i(z)$ are complex valued functions(and their real and imaginary parts are real valued). The meanings of symbols from the physical point of view are as follows; z is the vertical coordinate, d/dz is differentiation along the vertical direction, a^2 is square of horizontal wave number, $\sigma = \frac{\upsilon}{\kappa}$ is the thermal Prandtl number, $\tau = \dfrac{\eta_1}{\kappa}$ is the Lewis number, $F = \dfrac{\upsilon_0}{d^2}$ is the viscoelastic parameter, $R = \dfrac{g\alpha\beta_1 d^4}{\kappa\upsilon}$ is the thermal Rayleigh number, $R_S = \dfrac{g\alpha\beta_2 d^4}{\kappa\upsilon}$ is the concentration Rayleigh number, , $D_T = \dfrac{\beta_2 D_f}{\beta_1\kappa}$ is the Dufour number, $S_T = \dfrac{\beta_1 S_f}{\beta_2\eta_1}$ is the Soret number, ϕ is the concentration, θ is the temperature, p is the complex growth rate and w is the vertical velocity.

3. The Linear Transformation and Mathematical Analysis

The nature of the system (1)-(3) is clearly qualitatively different from those of double-diffusive convection problems ($D_T = 0 = S_T$) as now we have coupling between all the three eigen-functions $w, \theta, and\ \phi$ in all the three equations. Consequently, they behave nastily and obstruct any attempt for the elegant extension of the earlier results for the double-diffusive convection problems to the present generalized set up. The nasty behaviour of these equations is mollified by the linear transformations given by:

$$\tilde{w} = (S_T + B)\, w$$
$$\tilde{\theta} = E\theta + F\phi$$
$$\tilde{\phi} = S_T\theta + B\phi \tag{6}$$

where

$$\mathrm{B} = -\frac{1}{\tau}A, \quad \mathrm{E} = \frac{S_T + B}{D_T + A}A, \quad \mathrm{F} = \frac{S_T + B}{D_T + A}D_T$$

And A is a positive root of the equation

$$A^2 + (\tau - 1)A - \tau\, S_T D_T = 0\,.$$

The system (1)-(3) together with boundary conditions (4)-(5), upon using the transformations as defined above takes the following form:

$$\left(D^2 - a^2\right)\left((D^2 - a^2) - \frac{p}{\sigma}(1-F)\right)w = R^{/}\, a^2\theta - R_S^{\ /}\ a^2\phi\ , \tag{7}$$

$$\left(k_1\left\langle D^2 - a^2\ \right\rangle - p\right)\theta = -w\,, \tag{8}$$

$$\left(k_2(D^2 - a^2) - \frac{p}{\tau}\right)\phi = -\frac{w}{\tau}\ , \tag{9}$$

With

$$w = 0 = Dw = \theta = \phi \qquad \text{at z =0 and z = 1} \tag{10}$$

or $w = 0 = Dw = \theta = \phi$ at z =0 and z = 1 (11)

Or

$$\left.\begin{aligned} w = 0 = Dw = \theta = \phi \quad & at\ z = 0 \\ w = 0 = D^2 w = \theta = \phi \quad & at\ z = 1 \end{aligned}\right\} \tag{12}$$

Or

$$\left.\begin{aligned} w = 0 = D^2 w = \theta = \phi \quad & at\ z = 0 \\ w = 0 = Dw = \theta = \phi \quad & at\ z = 1 \end{aligned}\right\}, \tag{13}$$

Where

$$k_1 = 1 + \frac{\tau\, D_T S_T}{A},\; k_2 = 1 - \frac{S_T D_T}{A}\; are\;\; positive\, cons\tan ts$$

$$and\;\; R^{/} = \frac{(D_T + A)(R\;\; B + R_S S_T)}{BA - S_T D_T},\; R_S^{/} = \frac{(S_T + B)(R_S A + R\;\; D_{T)}}{BA - S_T D_T}$$

$$are\;\;\; respectively\; the\; \mathrm{mod}ified\; thermal\; Rayleigh\, number\; and\; the\; \mathrm{mod}ified$$
$$concentration\; Rayleigh\, number.$$

The sign tilde has been omitted for simplicity.

It may further be noted that equations (7)-(13) describe an eigenvalue problem for p and govern double-diffusive instability of Rivlin-Ericksen viscoelastic fluid coupled with cross-diffusions for any combination of dynamically free and rigid boundaries.

We now prove the following theorem

Theorem 1: If $R' > 0, R_S' > 0$, $p_r \ \geq\ 0$, $p_i\ \neq\ 0$, $F < 1$ and $R_S' \leq \dfrac{27\pi^4 k_1}{4}\left[1 + \dfrac{\tau k_2}{\sigma}(1 - F)\right]$,

then a necessary condition for the existence of a non-trivial solution (w, θ, ϕ, p) of equations (7)-(9) together with boundary conditions (10)-(13) is that

$$R_S' < R' \tag{14}$$

Proof: Multiplying equation (7) by w* (the complex conjugate of w) throughout and integrating the resulting equation over the vertical range of z, we get

$$\int_0^1 w^*\left(D^2 - a^2\right)\left[D^2 - a^2 - \frac{p}{\sigma}(1 - F)\right]w\, dz = R' a^2 \int_0^1 w^* \theta\, dz - R_S' a^2 \int_0^1 w^* \phi\, dz \tag{15}$$

Taking the complex conjugate of (8) and (9) and using the resulting equations in (15), we get

$$\int_0^1 w^*(D^2 - a^2)\,(D^2 - a^2 - \frac{p}{\sigma}\langle 1 - F\ \rangle)\, w\, dz = -\, R_T' a^2 \int_0^1 \theta\left[k_1(D^2 - a^2) - p^*\right]\theta^* dz$$

$$+ R_S' a^2 \tau \int_0^1 \phi\left[k_2(D^2 - a^2) - \frac{p^*}{\tau}\right]\phi^* dz \tag{16}$$

Integrating the various terms of equation (16) by parts for an appropriate number of times and making use of either of the boundary conditions (10) – (13), and one of the following inequalities

$$\int_0^1 \psi^* D^{2n}\psi\, dz = (-1)^n 1 \int_0 \left|D^{2n}\psi\right|^2 dz\,, \tag{17}$$

Where,

$\psi = \theta = \phi,$ for n = 0, 1 and $\psi = w,$ for n = 0, 1, 2,

We have

$$\int_0^1\left(\left|D^2 w\right|^2 + 2a^2\left|Dw\right|^2 + a^4\left|w\right|^2\right)dz + \frac{p(1 - F)}{\sigma}\int_0^1\left(\left|Dw\right|^2 + a^2\left|w\right|^2\right)dz$$

$$= R' a^2 \int_0^1 [k_1\left(\left|D\theta\right|^2 + a^2\left|\theta\right|^2\right) + p^*\left|\theta\right|^2\ \ ]dz - R_S' a^2\, \tau \int_0^1 [k_2(\ \left|D\phi\right|^2 + a^2\left|\phi\right|^2) + \frac{p^*}{\tau}\left|\phi\right|^2\ \ ]dz \tag{18}$$

Equating the real and imaginary parts of both sides of equation (18) and canceling $p_i \neq 0$ throughout from the imaginary part, we get

$$\int_0^1\left(\left|D^2w\right|^2+2a^2\left|Dw\right|^2+a^4\left|w\right|^2\right)dz+\frac{p_r(1-F)}{\sigma}\int_0^1\left(\left|Dw\right|^2+a^2\left|w\right|^2\right)dz$$

$$=R'a^2\int_0^1\left[k_1\left(\left|D\theta\right|^2+a^2\left|\theta\right|^2\right)+p_r\left|\theta\right|^2\right]dz-R'_Sa^2\,\tau\int_0^1\left[k_2\left(\left|D\phi\right|^2+a^2\left|\phi\right|^2\right)+\frac{p_r}{\tau}\left|\phi\right|^2\right]dz \quad (19)$$

And

$$\frac{(1-F)}{\sigma}\int_0^1\left(\left|Dw\right|^2+a^2\left|w\right|^2\right)dz+R'a^2\int_0^1\left|\theta\right|^2dz-R'_Sa^2\int_0^1\left|\phi\right|^2dz=0 \quad (20)$$

Equation (19) can be written in the alternative form as

$$\int_0^1\left(\left|D^2w\right|^2+2a^2\left|Dw\right|^2+a^4\left|w\right|^2\right)dz+\frac{p_r(1-F)}{\sigma}\int_0^1\left(\left|Dw\right|^2+a^2\left|w\right|^2\right)dz$$

$$=R'a^2\int_0^1k_1\left(\left|D\theta\right|^2+a^2\left|\theta\right|^2\right)dz-R'_Sa^2\tau\int_0^1k_2\left(\left|D\phi\right|^2+a^2\left|\phi\right|^2\right)dz+p_ra^2\left[R'\int_0^1\left|\theta\right|^2dz-R'_S\int_0^1\left|\phi\right|^2dz\right], \quad (21)$$

And we derive the validity of the theorem from the resulting inequality obtained by replacing each one of the terms of this equation by its appropriate estimate.

Since $w,\theta\ and\phi$ vanish at z = 0 and z = 1, therefore Rayliegh-Ritz inequality [13] yields

$$\int_0^1\left|Dw\right|^2dz\geq\pi^2\int_0^1\left|w\right|^2dz \quad (22)$$

$$\int_0^1\left|D\theta\right|^2dz\geq\pi^2\int_0^1\left|\theta\right|^2dz \quad (23)$$

$$\int_0^1\left|D\phi\right|^2dz\geq\pi^2\int_0^1\left|\phi\right|^2dz \quad (24)$$

And

$$\int_0^1\left|D^2w\right|^2dz\geq\pi^4\int_0^1\left|w\right|^2dz \quad (25)$$

Utilizing inequalities (22) and (25), we get

$$\int_0^1\left(\left|D^2w\right|^2+2a^2\left|Dw\right|^2+a^4\left|w\right|^2\right)dz\geq\left(\pi^2+a^2\right)^2\int_0^1\left|w\right|^2dz\ . \quad (26)$$

Further, since $p_r \geq 0$, therefore, we have

$$\frac{p_r(1-F)}{\sigma}\int_0^1\left(\left|Dw\right|^2+a^2\left|w\right|^2\right)dz\geq 0\ . \quad (27)$$

Multiplying (8) by its complex conjugate and integrating the resulting equation over the vertical range of z, we get

$$\int_0^1 \left[\left(k_1\left(D^2-a^2\right)-p\right)\theta\left(k_1\left(D^2-a^2\right)-p^*\right)\theta^*\right]dz=\int_0^1 ww^*\,dz.$$

Integrating the above equation by parts an appropriate number of times and using either of the boundary conditions, we get

$$\int_0^1 \left|k^2{}_1\left(D^2-a^2\right)\theta\right|^2 + 2p_r k\int_0^1\left(\left|(D\theta)\right|^2+a^2|\theta|^2\right)dz+|p|^2\int_0^1|\theta|^2\,dz=\int_0^1|w|^2\,dz. \tag{28}$$

Since $p_r \geq 0$, therefore from (28), we have

$$k^2{}_1\int_0^1\left|\left(D^2-a^2\right)\theta\right|^2 dz \leq \int_0^1|w|^2\,dz \tag{29}$$

Also emulating the derivation of inequalities (26) and (27) we derive the following inequality

$$\int_0^1\left|\left(D^2-a^2\right)\theta\right|^2 dz=\int_0^1\left|D^2\theta\right|^2+2a^2|D\theta|^2+a^4|\theta|^2\,dz \geq \left(\pi^2+a^2\right)^2\int_0^1|\theta|^2\,dz \tag{30}$$

Combining inequalities (29) and (30), we get

$$\int_0^1|w|^2\,dz \geq \left(\pi^2+a^2\right)^2 k_1{}^2\int_0^1|\theta|^2\,dz \tag{31}$$

Also, we know

$$\int_0^1|w|^2=\int_0^1\left(\left|w^2\right|\right)^{\frac{1}{2}}\int_0^1\left(|w|^2\right)^{\frac{1}{2}}$$

Which upon using inequalities (29) and (30) yields

$$\int_0^1|w|^2\,dz \geq k_1{}^2\left(\pi^2+a^2\right)\left\{\int_0^1\left|\left(D^2-a^2\right)\theta\right|^2 dz\right\}^{\frac{1}{2}}\left\{\int_0^1|\theta|^2\right\}^{\frac{1}{2}}dz \tag{32}$$

$$\geq k_1{}^2\left(\pi^2+a^2\right)\left|-\int_0^1\theta^*\left(D^2-a^2\right)\theta\,dz\right| \qquad \text{(Using Schwartz inequality)}$$

$$\geq\left(\pi^2+a^2\right)k_1{}^2\int_0^1\left\{|D\theta|^2+a^2|\theta|^2\right\}dz \tag{33}$$

Further, using inequality (24), we have

$$\int_0^1\left(|D\phi|^2+a^2|\phi|^2\right)dz \geq \left(\pi^2+a^2\right)\int_0^1|\phi|^2\,dz\quad. \tag{34}$$

Also it follows from equation (20) that

$$R'_S a^2\int_0^1|\phi|^2\,dz \geq \frac{(1-F)}{\sigma}\int_0^1\left(|Dw|^2+a^2|w|^2\right)dz\ . \tag{35}$$

Combining inequalities (24) and (25) and using inequality (17), we get

$$\int_0^1 \left(|D\phi|^2 + a^2|\phi|^2\right)dz \geq \frac{(\pi^2+a^2)(1-F)}{\sigma R'_S a^2}\int_0^1 \left(|Dw|^2 + a^2|w|^2\right)dz \tag{36}$$

$$\geq \frac{(\pi^2+a^2)^2(1-F)}{\sigma R'_S a^2}\int_0^1 |w|^2 dz$$

Also, from equation (21) and the fact that $p_r \geq 0$, we obtain,

$$p_r a^2\left\{R'\int_0^1 |\theta|^2 dz - R'_S\int_0^1 |\phi|^2 dz\right\} \leq 0 \tag{37}$$

Now if permissible, let $R'_S \geq R'$. Then, in that case, we derive from equation (21) and inequalities (26), (27), (33), (36) and (37) that

$$\left\{(\pi^2+a^2)^2 + \frac{(1-F)\pi k_2(\pi^2+a^2)^2}{\sigma} - \frac{R'_S a^2}{(\pi^2+a^2)k_1}\right\}\int_0^1 |w|^2 dz < 0 \text{ or}$$

$$\left\{(\pi^2+a^2)^2\left(1+\frac{(1-F)\pi k_2}{\sigma}\right) - \frac{R'_S a^2}{(\pi^2+a^2)k_1}\right\}\int_0^1 |w|^2 dz < 0 \tag{38}$$

Or $\left\{\frac{(\pi^2+a^2)^3}{a^2}\left(1+\frac{(1-F)\pi k_2}{\sigma}\right) - \frac{R'_S}{k_1}\right\}\int_0^1 |w|^2 dz < 0$,

Which implies that?

$$R'_S > \frac{(\pi^2+a^2)^3 k_1}{a^2}\left(1+\frac{(1-F)\pi k_2}{\sigma}\right),$$

And thus we necessarily have

$$R'_S > \frac{27\pi^4 k_1}{4}\left(1+\frac{(1-F)\pi k_2}{\sigma}\right),$$

Since the minimum value of $\frac{(\pi^2+a^2)^3}{a^2}$ for $a^2 > 0$ is $\frac{27\pi^4}{4}$.

Hence, if $R'_S \leq \frac{27\pi^4 k_1}{4}\left(1+\frac{(1-F)\pi k_2}{\sigma}\right)$, then we must have

$$R'_S < R'$$

And this completes the proof of the theorem.

Theorem 1 implies from the physical point of view that the thermosolutal convection of the Veronis' type in the Rivlin-Ericksen viscoelastic fluid coupled with cross –diffusions cannot manifest as an oscillatory motions of growing amplitude in an initially bottom heavy configuration if

$$R'_S \leq \frac{27\pi^4 k_1}{4}\left(1+\frac{(1-F)\pi k_2}{\sigma}\right).$$

Further this result is uniformly valid for the quite general nature of the bounding surfaces.

Special Case1. For the case when F = 0 (Newtonian Fluid) Theorem 1 can be restated as:

If $R' > 0, R'_S > 0$, $p_r \geq 0$, $p_i \neq 0$, and $R'_S \leq \frac{27\pi^4 k_1}{4}\left[1+\frac{\tau k_2}{\sigma}\right]$, then a necessary condition for the existence of a non-trivial solution (w, θ, ϕ, p) of equations (7) – (9) together with boundary conditions (10) – (13) is that:

$R'_S < R'$.

Theorem2. If $R' < 0, R'_S < 0$, $p_r \geq 0$, pi $\neq 0$, $F < 1$ and $|R'| \leq \frac{27\pi^4 k_1}{4}\left[1+\frac{1}{\sigma}(1-F)k_2\right]$, then a necessary condition for the existence of a non-trivial solution (w, θ, ϕ, p) of equations (7)-(9) together with boundary conditions (10)-(13) is that

$$|R'| < |R'_S| \quad . \tag{39}$$

Proof: Putting $R' = -|R'| and\ R'_S = -|R'_S|$ in equation (7) and following the same process as is done in theorem1, inequality (38) in the present case assumes the form

$$\left[\left(\pi^2+a^2\right)^2\left\{1+\frac{(1-F)k_2}{\sigma}\right\}-\frac{|R'|a^2}{\left(\pi^2+a^2\right)k_1}\right]\int_0^1 |w|^2 dz < 0, \tag{40}$$

Or $\left[\frac{\left(\pi^2+a^2\right)^3}{a^2}\left\{1+\frac{(1-F)k_2}{\sigma}\right\}-|R'|\frac{1}{k_1}\right]\int_0^1 |w|^2 dz < 0,$

Which implies that?

$$|R'| > \frac{\left(\pi^2+a^2\right)^3 k_1}{a^2}\left(1+\frac{(1-F)k_2}{\sigma}\right),$$

And thus we necessarily have

$$|R'| > \frac{27\pi^4 k_1}{4}\left(1+\frac{(1-F)k_2}{\sigma}\right),$$

Since the minimum value of $\frac{\left(\pi^2+a^2\right)^3}{a^2}$ for $a^2 > 0$ *is* $\frac{27\pi^4}{4}$

Hence, if $|R| \leq \frac{27\pi^4 k_1}{4}\left(1+\frac{(1-F)k_2}{\sigma}\right)$, then we must have

$|R'| < |R'_S|$,

And this completes the proof of the theorem.

Theorem 2 implies from the physical point of view that the thermosolutal convection of the Stern's type in the Rivlin-Ericksen viscoelastic fluid coupled with cross-diffusions cannot manifest as an oscillatory motions of growing amplitude in an initially top heavy configuration if

$$|R'| \leq \frac{27\pi^4 k_1}{4}\left(1+\frac{(1-F)k_2}{\sigma}\right).$$

Further this result is uniformly valid for the quite general nature of the bounding surfaces.

Special Case2. For the case when F = 0 (Newtonian Fluid) Theorem 2 can be restated as:

If $R' < 0, R'_S < 0$, $p_r \geq 0$, pi $\neq$ 0, and $|R'| \leq \frac{27\pi^4 k_1}{4}\left[1+\frac{1}{\sigma}k_2\right]$, then a necessary condition for the existence of a non-trivial solution (w,θ,ϕ,p) of equations (7) – (9) together with boundary conditions (10) – (13) is that

$|R'| < |R'_S|$.

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